# Topic 8. Exercises on Volumes by Slicing
## Level 1

1. By taking slices perpendicular to the axis of rotation, use the method of slicing to find the volume of the solid obtained by rotating the region $\{(x, y) : x \geq 0,\ y \geq 0,\ y + x \leq 1\}$ about the $x$-axis.

$\boxed{\frac{\pi}{3}}$

2. By taking slices perpendicular to the axis of rotation, use the method of slicing to find the volume of the solid obtained by rotating the region $\{(x, y) : 0 \le x \le 1,\ 0 \le y \le x^2\}$ about the $x$-axis.

$\boxed{\frac{\pi}{5}}$

3. By taking slices perpendicular to the axis of rotation, use the method of slicing to find the volume of the solid obtained by rotating the region

$\{(x, y) : 0 \le x \le \frac{\pi}{2}, 0 \le y \le \sin x\}$ about the $x$-axis.

$\boxed{\dfrac{\pi^2}{4}}$

4. By taking slices perpendicular to the axis of rotation, use the method of slicing to find the volume of the solid obtained by rotating the region $\{(x, y) : 0 \leq x \leq 1, 0 \leq y \leq x^2\}$ about the line $x = 1$.

$\boxed{\frac{\pi}{6}}$

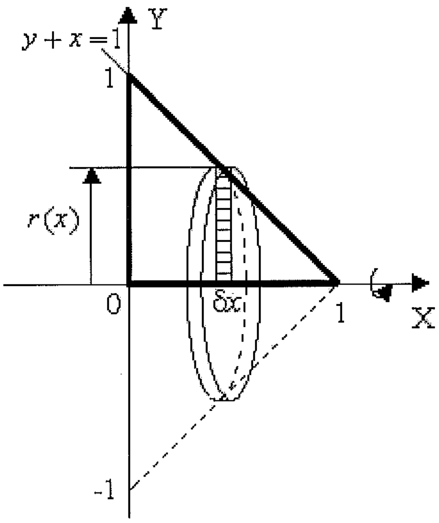
5. By taking slices perpendicular to the axis of rotation, use the method of slicing to find the volume of the solid obtained by rotating the region $\{(x, y) : 0 \leq x \leq 2,\ 4 - x^2 \leq y \leq 4\}$ about the line $x = 2$.

$\boxed{\dfrac{8\pi}{3}}$

6. By taking slices perpendicular to the axis of rotation, use the method of slicing to find the volume of the solid obtained by rotating the region $\{(x, y) : 0 \leq x \leq 2, 4 - x^2 \leq y \leq 4\}$ about the line $y = 4$.

$\boxed{\dfrac{32\pi}{5}}$

Q1.
*Solution*:

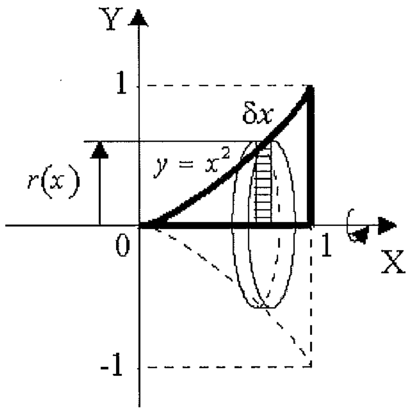



A slice taken perpendicular to the axis of rotation is a disc of thickness $\delta x$ and radius $r = y$. But $y + x = 1$, hence $r(x) = 1 - x$. The slice has volume $\delta V = \pi r^2 \delta x \Rightarrow \delta V = \pi(1-x)^2 \delta x$.

$$\therefore \quad V = \lim_{\delta x \to 0} \sum_{x=0}^{1} \pi(1-x)^2 \delta x = \int_0^1 \pi(1-x)^2 \, dx = \left. \frac{\pi(1-x)^3}{3} \right|_1^0 = \frac{\pi}{3}.$$

$\therefore$ the volume of the solid is $\frac{\pi}{3}$ cubic units.

Q2.
*Solution*:

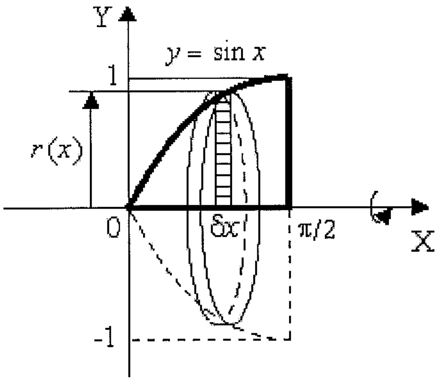



A slice taken perpendicular to the axis of rotation is a disk of thickness $\delta x$ and radius $r(x) = x^2$. The slice has volume $\delta V = \pi r^2 \delta x = \pi x^4 \delta x$.

$$\therefore \quad V = \lim_{\delta x \to 0} \sum_{x=0}^{1} \pi x^4 \delta x = \int_0^1 \pi x^4 \, dx = \left. \frac{\pi x^5}{5} \right|_0^1 = \frac{\pi}{5}.$$

$\therefore$ the volume of the solid is $\frac{\pi}{5}$ cubic units.

Q3.
*Solution*:

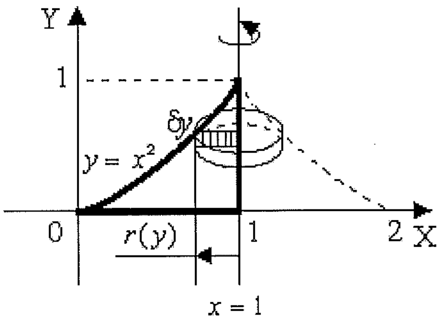



A slice taken perpendicular to the axis of rotation is a disk of thickness $\delta x$ and radius $r(x) = \sin x$. The slice has volume $\delta V = \pi r^2(x)\delta x = \pi \sin^2 x\, \delta x$.

$$\therefore \quad V = \lim_{\delta x \to 0} \sum_{x=0}^{\pi/2} \pi \sin^2 x\, \delta x = \int_0^{\pi/2} \pi \sin^2 x\, dx = \pi \int_0^{\pi/2} \frac{1-\cos 2x}{2} dx = \frac{\pi}{2}\left(x - \frac{\sin 2x}{2}\right)\Bigg|_0^{\pi/2} = \frac{\pi^2}{4}.$$

$\therefore$ the volume of the solid is $\frac{\pi^2}{4}$ cubic units.

Q4.
*Solution*:



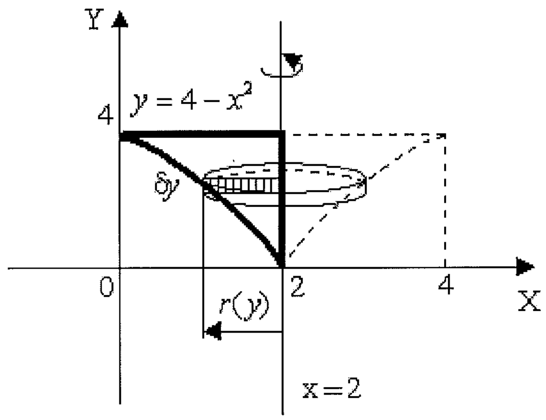

A slice taken perpendicular to the axis of rotation is a disc of thickness $\delta y$ and radius $1 - x$. But $y = x^2$, hence $r(y) = 1 - \sqrt{y}$. The slice has volume $\delta V = \pi r^2 \delta y \Rightarrow \delta V = \pi\left(1 - \sqrt{y}\right)^2 \delta y$.

$$\therefore \quad V = \lim_{\delta y \to 0} \sum_{y=0}^{1} \pi\left(1-\sqrt{y}\right)^2 \delta y = \int_0^1 \pi\left(1-\sqrt{y}\right)^2 dy = \int_0^1 \pi\left(1 - 2\sqrt{y} + y\right) dy$$

$$= \pi\left(y - \frac{2y^{3/2}}{3/2} + \frac{y^2}{2}\right)\Bigg|_0^1 = \frac{\pi}{6}.$$

$\therefore$ the volume of the solid is $\frac{\pi}{6}$ cubic units.

Q5.




A slice taken perpendicular to the axis of rotation is a disk of thickness $\delta y$ and radius $r(y) = 2 - \sqrt{4-y}$.

The slice has volume $\delta V = \pi r^2 \delta y = \pi\left(2 - \sqrt{4-y}\right)^2 \delta y = \pi\left(8 - y - 4\sqrt{4-y}\right)\delta y$.

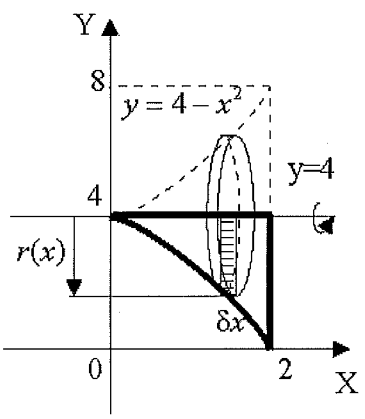
$\therefore \quad V = \lim_{\delta y \to 0} \sum_{y=0}^{4} \pi\left(8 - y - 4\sqrt{4-y}\right)\delta y = \int_0^4 \pi\left(8 - y - 4\sqrt{4-y}\right)dy$.

Substitution $\; y = 4 - y', \quad dy = -dy'\;$ gives

$$V = -\int_4^0 \pi\left(4 + y' - 4\sqrt{y'}\right)dy' = -\pi\left(4y' + \frac{y'^2}{2} - \frac{4y'^{3/2}}{3/2}\right)\Bigg|_4^0 = \frac{8\pi}{3}.$$

$\therefore \quad$ the volume of the solid is $\frac{8\pi}{3}$ cubic units.

Q6.




A slice taken perpendicular to the axis of rotation is a disk of thickness $\delta x$ and radius $r(x) = x^2$.

The slice has volume $\delta V = \pi r^2 \delta x = \pi x^4 \delta x$.

$\therefore \quad V = \lim_{\delta x \to 0} \sum_{x=0}^{2} \pi x^4 \delta x = \int_0^2 \pi x^4 dx = \frac{\pi x^5}{5}\Bigg|_0^2 = \frac{32\pi}{5}$.

$\therefore \quad$ the volume of the solid is $\frac{32\pi}{5}$ cubic units.