

The trigonometric functions

TOPIC TEST

Time allowed: 1 hour

Total marks = 100

SECTION I Multiple-choice questions

10 marks

- Instructions**
- This section consists of 10 multiple-choice questions
 - Each question is worth 1 mark
 - Fill in only ONE CIRCLE
 - Calculators may be used

1 $\frac{3\pi}{5}$ radians = ?

- (A) 72° (B) 108° (C) 216° (D) 300°

2 In radians, in terms of π , $450^\circ = ?$

- (A) $\frac{9\pi}{4}$ (B) 3π (C) $\frac{7\pi}{5}$ (D) $\frac{5\pi}{2}$

3 Which is NOT a solution of the equation $2 \sin x = -1$?

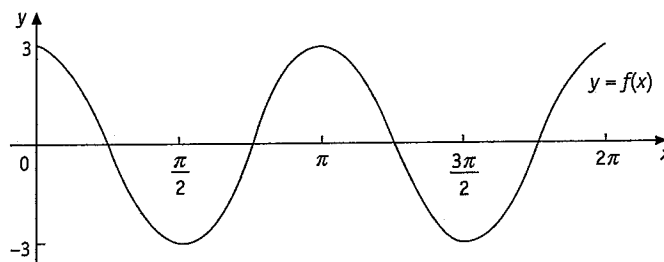
- (A) $x = -\frac{\pi}{6}$ (B) $x = \frac{5\pi}{6}$ (C) $x = \frac{7\pi}{6}$ (D) $x = \frac{11\pi}{6}$

4 The length of an arc of a circle of radius 8 cm subtended by an angle of $\frac{3\pi}{4}$ is:

- (A) $\frac{3\pi}{2}$ cm (B) 3π cm (C) 6π cm (D) 24π cm

5 The diagram shows a sketch of the graph of $y = f(x)$, for $0 \leq x \leq 2\pi$. $f(x) = ?$

- (A) $y = 3 \cos 2x$
(B) $y = 2 \cos 3x$
(C) $y = 3 \cos \frac{x}{2}$
(D) $y = 2 \cos \frac{x}{3}$



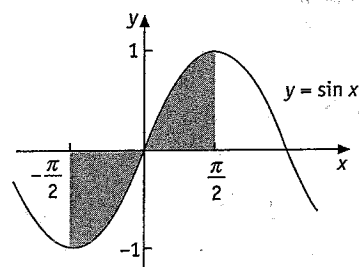
6 $\int 2 \sec^2 \frac{x}{3} dx = ?$

- (A) $\frac{2}{3} \tan \frac{x}{3} + C$ (B) $2 \tan \frac{x}{3} + C$ (C) $6 \tan \frac{x}{3} + C$ (D) $\frac{2}{3} \tan x + C$

7 Which is correct? The shaded area in the diagram is equal to:

I $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin x \, dx$

II $2 \int_0^{\frac{\pi}{2}} \sin x \, dx$



- (A) I only (B) II only
 (C) both I and II (D) neither I nor II

8 The exact value of $\cos \frac{5\pi}{6}$ is?

- (A) $-\frac{\sqrt{3}}{2}$ (B) $-\frac{1}{\sqrt{2}}$ (C) $-\frac{1}{\sqrt{3}}$ (D) $-\sqrt{3}$

9 The period of the graph of $y = 2 \sin \frac{x}{2}$ is?

- (A) π (B) 2π (C) $\frac{\pi}{2}$ (D) 4π

10 $y = 4 \cos \frac{x}{3}$; $\frac{dy}{dx} = ?$

- (A) $-12 \sin \frac{x}{3}$ (B) $-\frac{4}{3} \sin \frac{x}{3}$ (C) $\frac{4}{3} \sin \frac{x}{3}$ (D) $12 \sin \frac{x}{3}$

SECTION II

90 marks

Show all necessary working

11 Express in radians in terms of π :

a 80°

b 36°

2 marks each

12 Express in degrees:

a $\frac{\pi}{12}$

b $\frac{11\pi}{6}$

2 marks each

13 Find the value of $\sin \frac{3\pi}{5}$ correct to four decimal places.

2 marks

14 Write down the exact value of:

a $\sin \frac{2\pi}{3}$

b $\tan \frac{11\pi}{6}$

3 marks each

15 Find all values of θ , $0 \leq \theta \leq 2\pi$, for which:

a $\cos \theta = \frac{\sqrt{3}}{2}$

b $\tan \theta = -1$

4 marks each

c $4 \sin^2 \theta = 3$

4 marks

16 Find the arc length of a sector of radius 9 cm with angle at the centre of 20° .

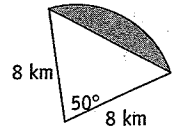
4 marks

17 Find the area of a sector with radius 12 cm and angle at the centre of 240° .

4 marks

18 Find the shaded area correct to one decimal place.

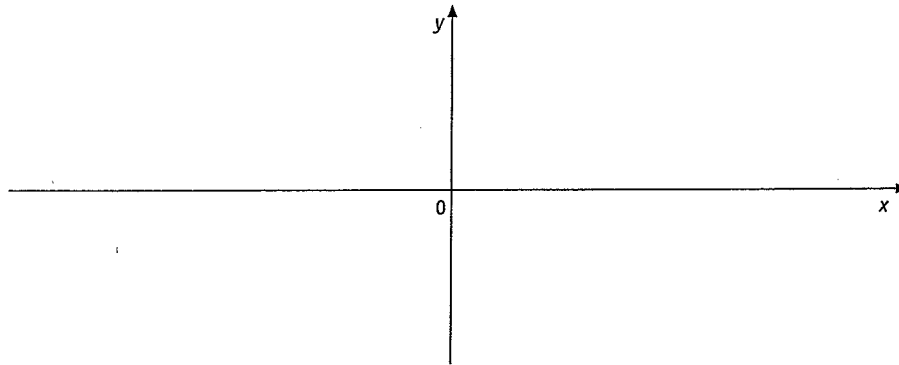
4 marks



19 Sketch the graph of:

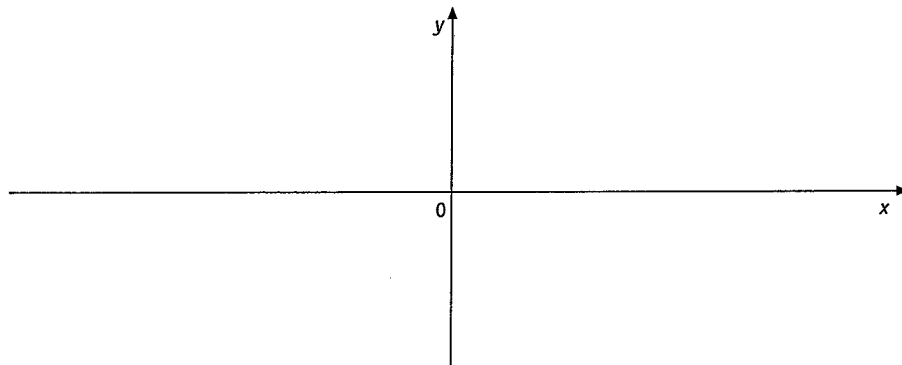
4 marks

a $y = 2 \sin x$



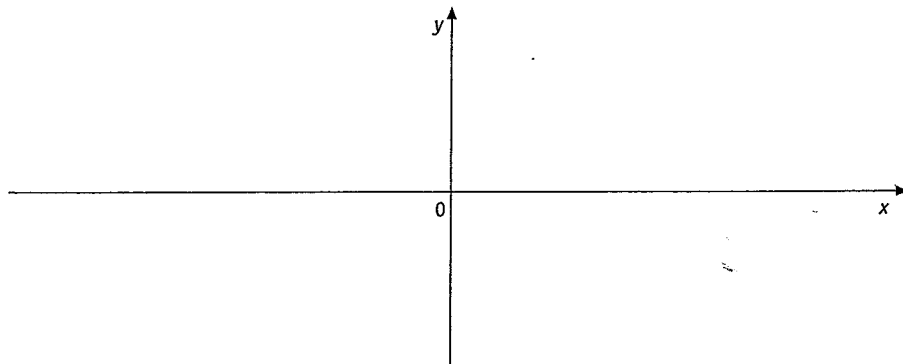
b $y = \cos 2x$

4 marks



c $y = 4 \tan x$

4 marks



20 Differentiate

a $2 \sin 4x$

b $-3 \cos \frac{x}{2}$

3 marks each

c $\tan\left(2x - \frac{\pi}{3}\right)$

d $x \sin x$

3 marks each

21 Find the equation of the tangent to the curve $y = \cos \pi x$ at the point when $x = \frac{1}{2}$

4 marks

22 Find:

a $\int \sin 2x \, dx$

b $\int 3 \cos \frac{x}{2} \, dx$

3 marks each

23 Find the exact value of each integral, expressing the answer as a single fraction:

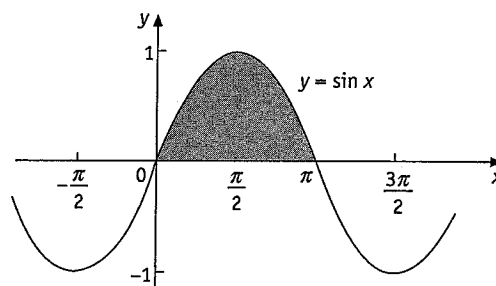
a $\int_0^{\frac{\pi}{4}} (1 - \cos x) dx$

b $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sec^2 x dx$

4 marks each

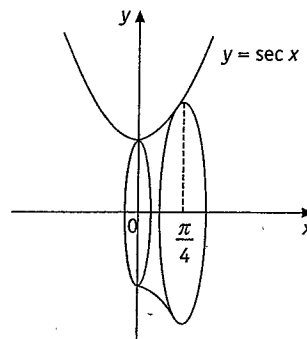
24 Find the area shaded in the diagram.

4 marks



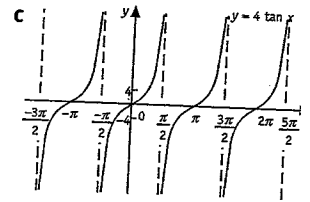
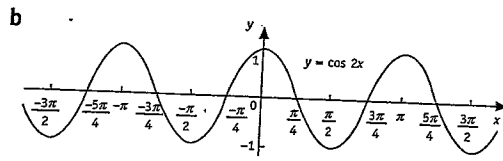
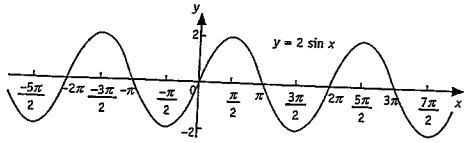
25 Find the volume of the solid formed when the section of the curve $y = \sec x$ between $x = 0$ and $x = \frac{\pi}{4}$ is rotated about the x-axis.

4 marks



- ANSWERS -

Pages 89-94 1 B 2 D 3 B 4 C 5 A 6 C 7 B 8 A 9 D 10 B 11 a $\frac{4\pi}{9}$ b $\frac{\pi}{5}$ 12 a 15° b 330° 13 0.9511 14 a $\frac{\sqrt{2}}{2}$
 b $-\frac{1}{\sqrt{3}}$ 15 a $\frac{\pi}{6}$ or $\frac{11\pi}{6}$ b $\frac{3\pi}{4}$ or $\frac{7\pi}{4}$ c $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}$ or $\frac{5\pi}{3}$ 16 π cm 17 96π cm² 18 3.4 km²



20 a $8 \cos 4x$ b $\frac{3}{2} \sin \frac{x}{2}$ c $2 \sec^2 \left(2x - \frac{\pi}{3} \right)$ d $x \cos x + \sin x$ 21 $y = -\pi x + \frac{\pi}{2}$ 22 a $-\frac{1}{2} \cos 2x + C$ b $6 \sin \frac{x}{2} + C$
 23 a $\frac{\pi - 2\sqrt{2}}{4}$ b $\frac{2\sqrt{3}}{3}$ 24 2 units² 25 π units³