Neap:

HSC Trial Examination 2017

Mathematics Extension 1

General Instructions

Reading time – 5 minutes

Working time – 2 hours

Write using black pen

Board-approved calculators may be used

A reference sheet is provided at the back of this paper.

In Questions 11–14, show relevant mathematical reasoning and/or calculations

Total marks – 70

Section I Pages 2–4
10 marks

Attempt Questions 1–10

Allow about 15 minutes for this section

Section II Pages 5–8
60 marks

Attempt Questions 11–14

Allow about 1 hour and 45 minutes for this section

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2017 HSC Mathematics Extension 1 Examination.

Neap Trial Exams are licensed to be photocopied or placed on the school intranet and used only within the confines of the school purchasing them, for the purpose of examining that school's students only. They may not be otherwise reproduced or distributed. The copyright of Neap Trial Exams remains with Neap. No Neap Trial Exam or any part thereof is to be issued or passed on by any person to any party inclusive of other schools, non-practising teachers, coaching colleges, tutors, parents, students, publishing agencies or websites without the express written consent of Neap.

Copyright © 2017 Neap ABN 49 910 906 643 PO Box 214 St Leonards NSW 1590 Tel: (02) 9438 1386 Fax: (02) 9438 1385 TENME1_QA_17.FM

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

---

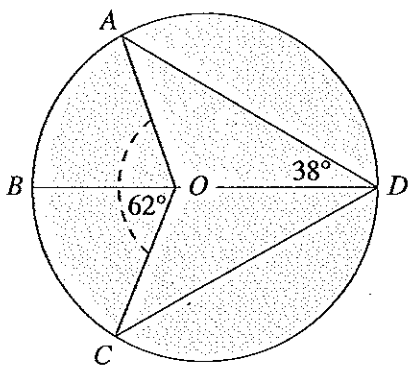
1. Solving $\dfrac{4-x}{x+3} > 0$ gives

(A) $x < -3$ or $x > 4$
(B) $x < -4$ or $x > 3$
(C) $-4 < x < 3$
(D) $-3 < x < 4$

2. The sum, $\displaystyle\sum_{k=1}^{10} 2^{r-1}$ equals

(A) $0 + 1 + 2 + 4 + \ldots + 512$
(B) $0 + 1 + 2 + 4 + \ldots + 1024$
(C) $1 + 2 + 4 + \ldots + 512$
(D) $1 + 2 + 4 + \ldots + 1024$

3. Consider the diagram.



$\angle AOC$ (indicated by the dashed line; O is the centre and BD is the diameter) is equal to

(A) 124°
(B) 138°
(C) 152°
(D) 222°

 TENVE1_QA_17.FM Copyright © 2017 Neap

4. The remainder for $\dfrac{2x^3 + x^2 - x + 10}{x + 2}$ is

(A) -12
(B) 0
(C) 10
(D) 28

5. The curve $y = \dfrac{2x}{\sqrt{x-1}}$ has the features

(A) vertical asymptote: $y = 0$ and horizontal asymptote: $x = 2$.
(B) vertical asymptote: $x = -1$ and minimum value: 4.
(C) vertical asymptote: $x = 1$ and horizontal asymptote: none.
(D) vertical asymptote: $x = 1$ and horizontal asymptote: $y = 4$.

6. The general solution to $\sin\theta = \dfrac{1}{2}$ for all values of n is

(A) $\theta = n\pi + (-1)^n \dfrac{\pi}{6}$

(B) $\theta = n\pi \pm \dfrac{\pi}{6}$

(C) $\theta = 2\pi n \pm \dfrac{\pi}{3}$

(D) $\theta = \dfrac{n\pi}{6} \pm 2\pi n$

7. The $\lim\limits_{x \to 0} \dfrac{\sin^2 4x}{4x^2}$ equals

(A) $\dfrac{1}{16}$

(B) $\dfrac{1}{4}$

(C) 1

(D) 4

Copyright © 2017 Neap

8. The expansion of $\cos(2x + 60°)$ equals

(A) $\dfrac{\sqrt{3}\cos 2x}{2} + \dfrac{\sin 2x}{2}$

(B) $\dfrac{\sqrt{3}\cos 2x}{2} - \dfrac{\sin 2x}{2}$

(C) $\dfrac{\cos 2x}{2} - \dfrac{\sqrt{3}\sin 2x}{2}$

(D) $\dfrac{\cos 2x}{2} + \dfrac{\sqrt{3}\sin 2x}{2}$

9. For $y = \cos^{-1}x$, $-1 \le x \le 1$, the exact value of $\cos^{-1}\left(-\frac{1}{2}\right) - \cos^{-1}\left(\frac{1}{2}\right)$ (in terms of π) is

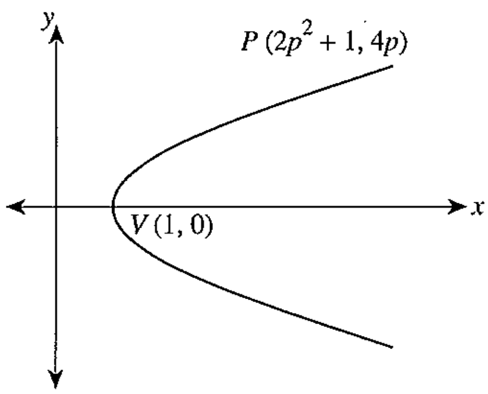
(A) $\dfrac{-4\pi}{3}$

(B) $\dfrac{\pi}{3}$

(C) $\dfrac{2\pi}{3}$

(D) $\dfrac{4\pi}{3}$

10. The diagram shows a parabola.



The Cartesian equation of the parabola is

(A) $y^2 = 8x - 8$

(B) $y^2 = 8x - 1$

(C) $x^2 = 8y - 1$

(D) $x^2 = 8y + 8$

Copyright © 2017 Neap

Section II

60 marks
Attempt Questions 11–14
Allow about 1 hour and 45 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11–14, your responses should include relevant mathematical reasoning and/or calculations.

Marks

Question 11 (15 marks) Use a SEPARATE writing booklet.

(a) On the same number plane, graph $y \geq \frac{1}{4}x^2 - 2$ and $x^2 + (y+1)^2 \leq 4$. Shade the intersecting region. **3**

(b) A fishing boat passes Gabo Island Lighthouse travelling due south. When the boat is due east of the lighthouse, the angle of elevation from the boat to the top of the lighthouse is 30°. After travelling a further 700 metres due south, the angle of elevation to the top of the lighthouse is 5°.

(i) Draw a diagram and display all the relevant information. **1**

(ii) Calculate the height of the lighthouse above the deck of the boat. Give your answer correct to the nearest metre. **3**

(c) Find $\int \frac{x}{\sqrt{1+x}}\,dx,\ x = u^2 - 1$. **3**

(d) The acceleration of a particle moving in a straight line is given by $\ddot{x} = -5x$ m s^{-2}, where x is the displacement from the origin. **3**

Find the velocity at the origin if the velocity is $\sqrt{5}$ m s^{-1} when the displacement is $x = 2$ metres.

(e) A fireworks technician is testing fireworks for an upcoming display. He has 20 fireworks, of which 13 are red and 7 are yellow. He launches 8 of them in a random order. **2**

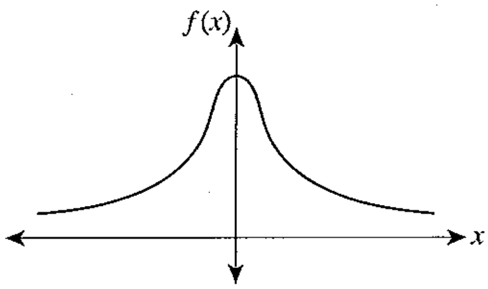
What is the probability that 5 of the launched fireworks are red?

Copyright © 2017 Neap

Marks

Question 12 (15 marks) Use a SEPARATE writing booklet.

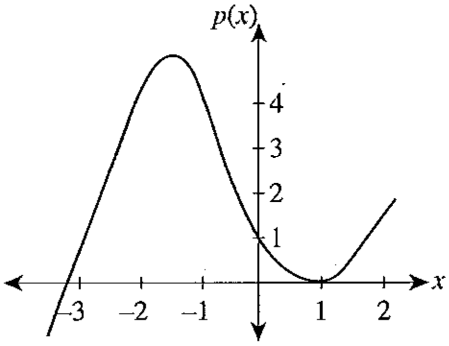
(a) Consider the graph of $f(x)$. **2**



Find the approximate graph of $f'(x)$.

(b) Use two iterations of Newton's method to find a better approximation to the root of $y = 2x - 2\cos 2x$ in the neighbourhood of $x = 0.425$. Give you answer correct to three decimal places. **3**

(c) Consider the graph. **2**



Find the equation of the polynomial in the form $p(x) = ax^3 + bx^2 + cx + d$.

(d) (i) Show that the domain of the curve $f(x) = \sqrt{\frac{x}{2} + 3}$ is $x \geq -6$. **1**

(ii) Show that the inverse function of $f(x) = \sqrt{\frac{x}{2} + 3}$ is $f^{-1}(x) = 2x^2 - 6$. **1**

(iii) On the same number plane, graph $f(x) = +\sqrt{\frac{x}{2} + 3}, -6 \leq x \leq 4$ and $f^{-1}(x) = 2x^2 - 6, 0 \leq x \leq 2.5$. Show that the point of intersection of the two curves is (2, 2). **3**

(iv) Find the acute angle between the tangents to the curves at (2, 2) to the nearest degree. **3**

Copyright © 2017 Neap

Marks

Question 13 (15 marks) Use a SEPARATE writing booklet.

(a) (i) Copy and complete the table for $y = 2\sin\left(\frac{\pi x}{2} - \frac{\pi}{6}\right)$, $0 \le x \le 5$. **2**

x	0	1	2	3	4	5
y						

(ii) Find the maximum and minimum turning points for $y = 2\sin\left(\frac{\pi x}{2} - \frac{\pi}{6}\right)$, $0 \le x \le 5$. **3**

(iii) Solve the equation $2\sin\left(\frac{\pi x}{2} - \frac{\pi}{6}\right) = 0$, $0 \le x \le 5$. **2**

(iv) Graph the equation $y = 2\sin\left(\frac{\pi x}{2} - \frac{\pi}{6}\right)$, $0 \le x \le 5$. Label the vertical intercept, the maximum and minimum turning points, and the horizontal intercepts. **3**

(v) If the area between the curve $y = 2\sin\left(\frac{\pi x}{2} - \frac{\pi}{6}\right)$, $1 \le x \le 2$ and the x-axis is rotated around the x-axis, find the volume of revolution. **3**

(b) $P(x) = x^3 + 2x$ where $x \ge 1$. **2**

If $P(k) = k^3 + 2k = 3N$, where $x = k$ and N is an integer, show by mathematical induction that $P(k+1)$ is divisible by 3.

Copyright © 2017 Neap

Marks

Question 14 (15 marks) Use a SEPARATE writing booklet.

(a) A mountain climber is camped at 2000 metres above sea level, where he is boiling water to make tea. At this height water boils at 93.4°C. Assuming that the rate at which the water cools is proportional to the difference between its boiled temperature, T, and the air temperature, A, then $\frac{dT}{dt} = k(T - A)$, where t is the time in minutes and k is a constant.

(i) If the air temperature is 8°C, show that $T = 8 + 85.4e^{kt}$. **3**

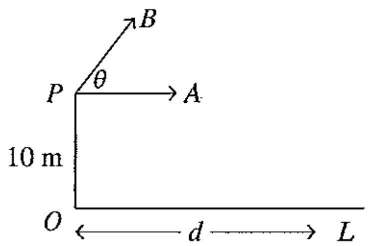
(ii) The tea cools to 75°C five minutes after the water boiled. **2**

Find the value of k correct to three decimal places.

(iii) The mountain climber wishes to drink his tea whilst the temperature is between 75°C and 63°C. **2**

How long does he have to the nearest minute?

(b) At a shooting range the clay targets are shot from a platform 10 metres above ground level.



Target A is shot horizontally. Target B is shot at an angle of θ degrees to the horizontal. Both targets are shot at a velocity of 40 m s^{-1} in the same direction.

The equations of motion are $\ddot{x} = 0$, $\ddot{y} = -g$ and $g = 9.8$ m s^{-2}.

(i) Show that the equations of motion of the targets are respectively: **4**

target A $\quad y = 10 - \frac{g}{3200}x^2$

target B $\quad y = -\frac{g}{3200\cos^2\theta}x^2 + (\tan\theta)x + 10$

(ii) Both targets are missed by the shooter and land at the same position L. **4**

Find the horizontal distance, d, to the landing point, L, and the angle of projection, θ, for target B to the nearest degree.

End of paper

Copyright © 2017 Neap

Mathematics

Factorisation

$a^2 - b^2 = (a+b)(a-b)$

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

Angle sum of a polygon

$S = (n-2) \times 180°$

Equation of a circle

$(x-h)^2 + (y-k)^2 = r^2$

Trigonometric ratios and identities

$\sin\theta = \dfrac{\text{opposite side}}{\text{hypotenuse}}$

$\cos\theta = \dfrac{\text{adjacent side}}{\text{hypotenuse}}$

$\tan\theta = \dfrac{\text{opposite side}}{\text{adjacent side}}$

$\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$

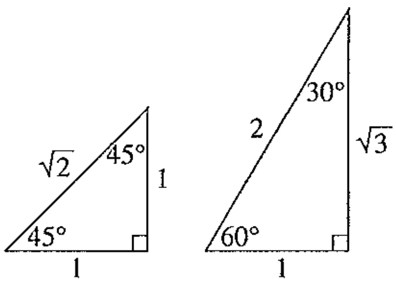
$\sec\theta = \dfrac{1}{\cos\theta}$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

$\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

$\sin^2\theta + \cos^2\theta = 1$

Exact ratios



Sine rule

$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine rule

$c^2 = a^2 + b^2 - 2ab\cos C$

Area of a triangle

$\text{Area} = \dfrac{1}{2}ab\sin C$

Distance between two points

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Perpendicular distance of a point from a line

$d = \dfrac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$

Slope (gradient) of a line

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$

Point–gradient form of the equation of a line

$y - y_1 = m(x - x_1)$

***n*th term of an arithmetic series**

$T_n = a + (n-1)d$

Sum to *n* terms of an arithmetic series

$S_n = \dfrac{n}{2}[2a + (n-1)d]$ or $S_n = \dfrac{n}{2}(a + l)$

***n*th term of a geometric series**

$T_n = ar^{n-1}$

Sum to *n* terms of a geometric series

$S_n = \dfrac{a(r^n - 1)}{r - 1}$ or $S_n = \dfrac{a(1 - r^n)}{1 - r}$

Limiting sum of a geometric series

$S = \dfrac{a}{1 - r}$

Compound interest

$A_n = P\left(1 + \dfrac{r}{100}\right)^n$

Copyright © 2017 Neap

Mathematics (continued)

Differentiation from first principles

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Derivatives

If $y = x^n$, then $\frac{dy}{dx} = nx^{n-1}$

If $y = uv$, then $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

If $y = \frac{u}{v}$, then $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

If $y = F(u)$, then $\frac{dy}{dx} = F'(u)\frac{du}{dx}$

If $y = e^{f(x)}$, then $\frac{dy}{dx} = f'(x)e^{f(x)}$

If $y = \log_e f(x) = \ln f(x)$, then $\frac{dy}{dx} = \frac{f'(x)}{f(x)}$

If $y = \sin f(x)$, then $\frac{dy}{dx} = f'(x)\cos f(x)$

If $y = \cos f(x)$, then $\frac{dy}{dx} = -f'(x)\sin f(x)$

If $y = \tan f(x)$, then $\frac{dy}{dx} = f'(x)\sec^2 f(x)$

Integrals

$$\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)} + C$$

$$\int e^{ax+b} dx = \frac{1}{a}e^{ax+b} + C$$

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + C$$

$$\int \sin(ax+b)dx = -\frac{1}{a}\cos(ax+b) + C$$

$$\int \cos(ax+b)dx = \frac{1}{a}\sin(ax+b) + C$$

$$\int \sec^2(ax+b)dx = \frac{1}{a}\tan(ax+b) + C$$

Trapezoidal rule (one application)

$$\int_a^b f(x)dx \approx \frac{b-a}{2}[f(a) + f(b)]$$

Simpson's rule (one application)

$$\int_a^b f(x)dx \approx \frac{b-a}{6}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right]$$

Solution of a quadratic equation

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Sum and product of roots of a quadratic equation

$\alpha + \beta = -\frac{b}{a}$ $\qquad$ $\alpha\beta = \frac{c}{a}$

Equation of a parabola

$(x-h)^2 = \pm 4a(y-k)$

Logarithms – change of base

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Angle measure

$180° = \pi$ radians

Length of an arc

$l = r\theta$

Area of a sector

$\text{Area} = \frac{1}{2}r^2\theta$

Copyright © 2017 Neap

Mathematics Extension 1

Angle sum identities

$\sin(\theta + \phi) = \sin\theta\cos\phi + \cos\theta\sin\phi$

$\cos(\theta + \phi) = \cos\theta\cos\phi + \sin\theta\sin\phi$

$\tan(\theta + \phi) = \dfrac{\tan\theta + \tan\phi}{1 - \tan\theta\tan\phi}$

***t* formulae**

If $t = \tan\dfrac{\theta}{2}$, then

$$\sin\theta = \frac{2t}{1+t^2}$$

$$\cos\theta = \frac{1-t^2}{1+t^2}$$

$$\tan\theta = \frac{2t}{1-t^2}$$

General solution of trigonometric equations

$\sin\theta = a, \quad \theta = n\pi + (-1)^n \sin^{-1} a$

$\cos\theta = a, \quad \theta = 2n\pi \pm \cos^{-1} a$

$\tan\theta = a, \quad \theta = n\pi + \tan^{-1} a$

Division of an interval in a given ratio

$\left(\dfrac{mx_2 + nx_1}{m+n}, \dfrac{my_2 + ny_1}{m+n}\right)$

Parametric representation of a parabola

For $x^2 = 4ay$,

$$x = 2at, y = at^2$$

At $(2at, at^2)$

tangent: $y = tx - at^2$

normal: $x + ty = at^3 + 2at$

At (x_1, y_1),

tangent: $xx_1 = 2a(y + y_1)$

normal: $y - y_1 = -\dfrac{2a}{x_1}(x - x_1)$

Chord of contact from (x_0, y_0): $xx_0 = 2a(y + y_0)$

Acceleration

$\dfrac{d^2x}{dt^2} = \dfrac{dv}{dt} = v\dfrac{dv}{dx} = \dfrac{d}{dx}\left(\dfrac{1}{2}v^2\right)$

Simple harmonic motion

$x = b + a\cos(nt + \alpha)$

$\ddot{x} = -n^2(x - b)$

Further integrals

$\displaystyle\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \sin^{-1}\frac{x}{a} + C$

$\displaystyle\int \frac{1}{a^2 + x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$

Sum and product of roots of a cubic equation

$\alpha + \beta + \gamma = -\dfrac{b}{a}$

$\alpha\beta + \alpha\gamma + \beta\gamma = \dfrac{c}{a}$

$\alpha\beta\gamma = -\dfrac{d}{a}$

Estimation of roots of a polynomial equation

Newton's method

$x_2 = x_1 - \dfrac{f(x_1)}{f'(x_1)}$

Binomial theorem

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k} = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$$

Copyright © 2017 Neap

Neap:

HSC Trial Examination 2017

Mathematics Extension 1

Solutions and marking guidelines

Neap Trial Exams are licensed to be photocopied or placed on the school intranet and used only within the confines of the school purchasing them, for the purpose of examining that school's students only. They may not be otherwise reproduced or distributed. The copyright of Neap Trial Exams remains with Neap. No Neap Trial Exam or any part thereof is to be issued or passed on by any person to any party inclusive of other schools, non-practising teachers, coaching colleges, tutors, parents, students, publishing agencies or websites without the express written consent of Neap.

Copyright © 2017 Neap ABN 43 910 906 643 PO Box 214 St Leonards NSW 1590 Tel: (02) 9438 1386 Fax: (02) 9438 1385 TENME1_SS_17.FM

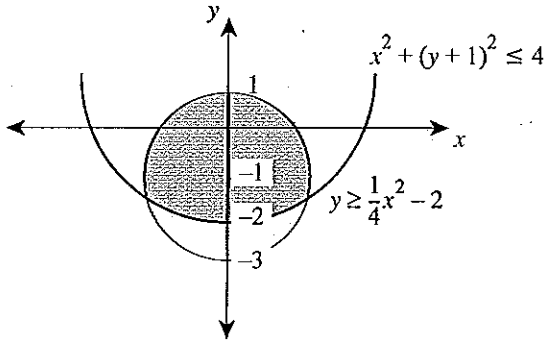
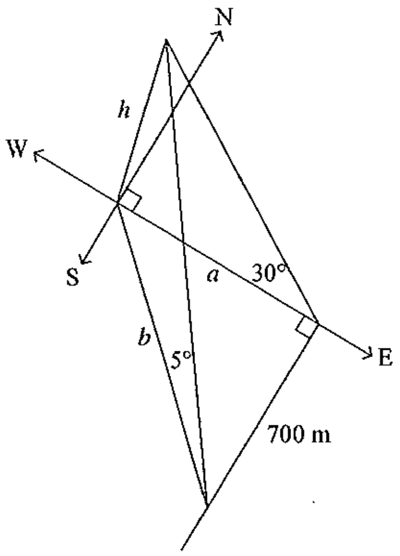
Section I

Sample answer	Syllabus outcomes and marking guide
Question 1 **D** $(x+3)^2\left(\frac{4-x}{x+3}\right) > 0 \times (x+3)^2$ $(x+3)(4-x) > 0$ $(x+3)(x-4) < 0$ $-3 < x < 4$	PE3, Band E3
Question 2 **C** $\sum_{k=1}^{10} 2^{r-1} = 2^0 + 2^1 + 2^2 + \ldots + 2^9$ $= 1 + 2 + 4 + \ldots + 512$	PE6, Band E2
Question 3 **B** $\angle AOB = 2 \times \angle ADB$ $= 2 \times 38$ $= 76$ The angle at the centre of a circle is twice the angle at the circumference standing on the same arc. $\therefore \angle AOC = 76 + 62$ $= 138°$	PE3, Band E2
Question 4 **B** Substitute $x = -2$. $2 \times (-2)^3 + (-2)^2 - (-2) + 10 = 0$	PE3, Band E2
Question 5 **C** The curve exists for $x > 1$. There is a vertical asymptote at $x = 1$ only.	PE1, Band E3
Question 6 **A** $\sin\theta = \frac{1}{2}$ $\theta = \frac{\pi}{6}, \frac{5\pi}{6}$ $= n\pi + (-1)^n\frac{\pi}{6}$ Answer A generates all correct solutions in the first and second quadrants as required.	HE7, Band E2
Question 7 **D** $\lim_{x\to 0} \frac{\sin^2 4x}{4x^2} = 4 \lim_{x \to 0}\left(\frac{\sin 4x}{4x}\right)^2$ $= 4$	H5, Band E2

 TEXME1_SS_17.FM Copyright © 2017 Neap

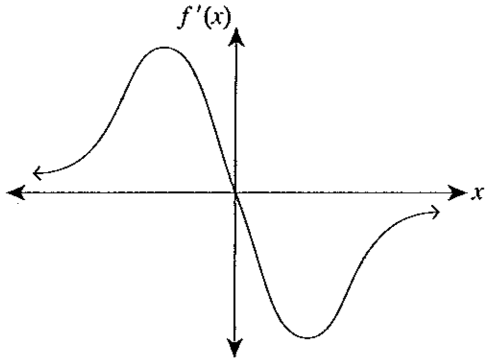
Sample answer	Syllabus outcomes and marking guide
Question 8 **C** $\cos(2x + 60°) = \cos 2x \cos 60° - \sin 2x \sin 60°$ $= \frac{\cos 2x}{2} - \frac{\sqrt{3}\sin 2x}{2}$	PE1, Band E2
Question 9 **B** $\cos^{-1}\left(-\frac{1}{2}\right) - \cos^{-1}\left(\frac{1}{2}\right) = \frac{2\pi}{3} - \frac{\pi}{3}$ $= \frac{\pi}{3}$	HE4, Band E3
Question 10 **A** $(y-k)^2 = 4a(x-h) \qquad k = 0, h = 1$ $(4p-0)^2 = 4a(2p^2 + 1 - 1)$ $16p^2 = 8ap^2 \qquad \therefore a = 2$ $y^2 = 8x - 8$	PE3, Band E3

Section II

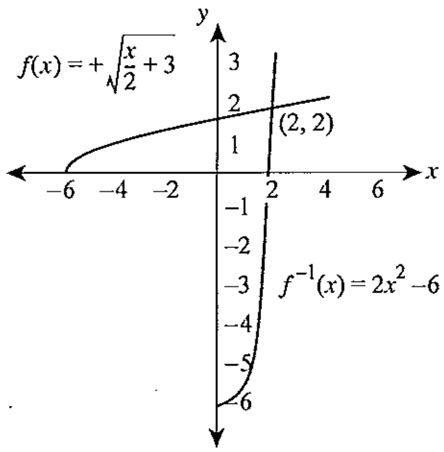
Sample answer	Syllabus outcomes and marking guide
Question 11	
(a) 	PE3, Band E3 • Gives the correct graphs. AND • Shades intersecting region .... 3 • Correctly graphs both equations........ 2 • Draws ONE correct graph 1
(b) (i) 	H5, Band E3 • Draws correct diagram 1
(ii) $b^2 - a^2 = 700^2, \frac{h}{a} = \tan 30°, \frac{h}{b} = \tan 5°$ $a^2 = \left(\frac{h}{\tan 30°}\right)^2, b^2 = \left(\frac{h}{\tan 5°}\right)^2$ $\left(\frac{h}{\tan 5°}\right)^2 - \left(\frac{h}{\tan 30°}\right)^2 = 700^2$ $h^2 = \frac{700(\tan^2 30° \times \tan^2 5°)}{\tan^2 30° - \tan^2 5°}$ $\therefore h = 62$ m	H5, Band E3 • Gives the correct solution............. 3 • Makes a substantial attempt at solution .. 2 • Identifies Pythagoras relationship. OR • Identifies tan relationships 1

 Copyright © 2017 Neap

Sample answer	Syllabus outcomes and marking guide
(c) $I=\int\frac{x}{\sqrt{1+x^2}}dx$ Let $x=u^2-1,\ dx=2udu$. $u^2=x+1$, $u=\pm\sqrt{x+1}$ $=\pm(x+1)^{\frac{1}{2}}$ $=\int\frac{u^2-1}{\sqrt{u^2}}2udu$ $=\int 2u^2-2du$ $=\frac{2u^3}{3}-2u+C$ $=\pm\left(\frac{2(x+1)^{\frac{3}{2}}}{3}-2(x+1)^{\frac{1}{2}}\right)+C$ $=\pm 2\left(\frac{\sqrt{(x+1)}^3}{3}-\sqrt{(x+1)}\right)+C$	HE6, Band E2 • Gives the correct solution . . . . .3 • Gives ONE correct integral . . . . .2 • Gives the correct integrand 1
(d) $\ddot{x}=-5x$ $\frac{d\left(\frac{1}{2}v^2\right)}{dx}=-5x$ $\frac{1}{2}v^2=-\frac{5x^2}{2}+\frac{C}{2}$ $v^2=-5x^2+C$ When $v=\sqrt{5},\ x=2$. $(\sqrt{5})^2=-5\times 2^2+C$ $\therefore C=25$ $v^2=-5x^2+25$ For the velocity at the origin, $x=0$. $v^2=-5\times 0^2+25$ $\therefore v=5\text{ m s}^{-1}$	HE3, Band E2 • Gives the correct solution . . . . .3 • Correctly finds $C=25$ . . . . .2 • Correct initial integration. 1
(e) If 5 are red then 3 are yellow. $\text{P(5 red, 3 yellow)}=\frac{{}^{13}C_5\times{}^{7}C_3}{{}^{20}C_8}$ $=\frac{1287\times 35}{125\ 970}$ $=\frac{231}{646}$	PE3, Band E3 • Gives the correct solution . . . . .2 • Makes a substantial attempt at solution. . .1

Sample answer	Syllabus outcomes and marking guide
Question 12	
(a) 	PE6, Band E3 • Gives the correct solution .... 2 • Displays the correct asymptotes. OR • Displays the correct turning points. OR • Displays the graph passing through origin 1
(b) $P(x) = y$ $= 2x - 2\cos 2x$ $P'(x) = y$ $= 2 + 4\sin 2x$ $x_1 = x_0 - \frac{P(x_0)}{P'(x_0)}$ $= 0.425 - \frac{2\times 0.425 - 2\cos(2\times 0.425)}{2 + 4\sin(2\times 0.425)}$ $= 0.5189$ $x_2 = 0.5189 - \frac{2\times 0.5189 - 2\cos(2\times 0.5189)}{2 + 4\sin(2\times 0.5189)}$ $= 0.5149$ $= 0.515$	HE4, Band E3 • Gives the correct solution .... 3 • Gives the correct first iteration .... 2 • Gives the correct differentiation 1
(c) $p(x) = a(x-1)^2(x+3)$ $= a(x^2 - 2x + 1)(x+3)$ $= a(x^3 + x^2 - 5x + 3)$ $p(0) = 1$ $1 = a(0^3 + 0^2 - 0 + 3)$ $1 = 3a$ $\therefore a = \frac{1}{3}$ $p(x) = \frac{1}{3}(x^3 + x^2 - 5x + 3)$ $= \frac{1}{3}x^3 + \frac{1}{3}x^2 - \frac{5}{3}x + 1$	PE3, Band E2 • Gives the correct solution .... 2 • Gives the correct calculation of a 1
(d) (i) $\sqrt{\frac{x}{2} + 3}$ exists for $\frac{x}{2} + 3 \geq 0$. $\frac{x}{2} \geq -3$ $\therefore x \geq -6$	HE4, Band E2 • Gives the correct proof 1

Copyright © 2017 Neap

Sample answer	Syllabus outcomes and marking guide
(ii) $f(x) = y$ $= \sqrt{\frac{x}{2} + 3}$ Interchange x and y. $x = \sqrt{\frac{y}{2} + 3}$ $x^2 = \frac{y}{2} + 3$ $2x^2 = y + 6$ $\therefore y = f^{-1}(x)$ $= 2x^2 - 6$	HE4, Band E2 • Gives the correct proof 1
(iii)  $f(2) = \sqrt{\frac{2}{2} + 3}$ $= 2$ and $f^{-1}(2) = 2(2)^2 - 6$ $= 2$ Therefore the intersection is at (2, 2). *Note: Teacher to exercise judgement.*	H9, Band E2 • Gives the correct solution . . . . .3 • Gives the correct graphs . . . . .2 • Correctly shows point of intersection 1

Sample answer	Syllabus outcomes and marking guide
(iv) $y=\left(\frac{x}{2}+3\right)^{\frac{1}{2}}$ $\frac{dy}{dx}=\frac{1}{2}\left(\frac{x}{2}+3\right)^{-\frac{1}{2}}\times\frac{1}{2}$ $=\frac{1}{4\sqrt{\frac{x}{2}+3}}$ $x=2$ $\frac{dy}{dx}=\frac{1}{4\sqrt{4}}$ $=\frac{1}{8}$ $=m_1$ $y=2x^2-6$ $\frac{dy}{dx}=4x$ $x=2$ $\frac{dy}{dx}=4\times 2$ $=8$ $=m_2$ $\tan\theta=\frac{m_1-m_2}{1+m_1m_2}$ $=\frac{8-\frac{1}{8}}{1+8\times\frac{1}{8}}$ $=\frac{7\frac{7}{8}}{2}$ $=\frac{63}{16}$ $\therefore\theta=\tan^{-1}\frac{63}{16}$ $=75°45'$ $=76°$	HE5, Band E3 • Gives the correct solution. . . . . 3 • Makes a substantial attempt at solution . . 2 • Gives the correct differentiation. 1

Copyright © 2017 Neap

Sample answer	Syllabus outcomes and marking guide

Question 13

(a) (i) $y = 2\sin\left(\frac{\pi x}{2} - \frac{\pi}{6}\right)$

x	0	1	2	3	4	5
y	-1	$\sqrt{3}$	1	$-\sqrt{3}$	-1	$-\sqrt{3}$

HE7, Band E2
- Gives the correct solution 2
- Gives at least THREE correct values. 1

(ii) $$y = 2\sin\left(\pi \frac{x}{2} - \frac{\pi}{6}\right), 0 \le x \le 5$$

$$\frac{dy}{dx} = \frac{2\pi}{2} \times \cos\left(\frac{\pi x}{2} - \frac{\pi}{6}\right) = 0$$

$$\cos\left(\frac{\pi x}{2} - \frac{\pi}{6}\right) = 0$$

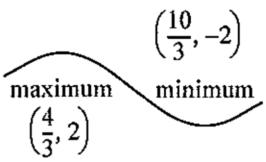
$$\left(\frac{\pi x}{2} - \frac{\pi}{6}\right) = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$$

$$\frac{\pi x}{2} = \frac{2\pi}{3}, \frac{5\pi}{3}, \frac{8\pi}{3}$$

$$x = \frac{4}{3}, \frac{10}{3}, \frac{16}{3}$$

For $0 \le x \le 5$:

x	1	$\frac{4}{3}$	3	$\frac{10}{3}$	4
$\frac{dy}{dx}$	positive	0	negative	0	positive



HE5, Band E3
- Gives the correct solution 3
- Correctly finds the x-coordinates of the stationary points 2
- Gives the correct differentiation 1

(iii) $y = 2\sin\left(\frac{\pi x}{2} - \frac{\pi}{6}\right), \quad 0 \le x \le 5$

When $y = 0$, $\sin\left(\frac{\pi x}{2} - \frac{\pi}{6}\right) = 0, \ 0 \le x \le 5$

$$\left(\frac{\pi x}{2} - \frac{\pi}{6}\right) = k\pi, \ k = 0, 1, 2$$

$$\frac{x}{2} = k + \frac{1}{6}, \ k = 0, 1, 2$$

$$x = 2k + \frac{1}{3}, \ k = 0, 1, 2$$

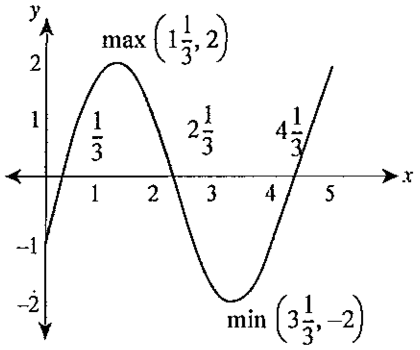
$$\therefore x = \frac{1}{3}, 2\frac{1}{3}, 4\frac{1}{3}$$

HE7, Band E3
- Gives the correct solutions 2
- Gives ONE correct solution. 1

Copyright © 2017 Neap

Sample answer	Syllabus outcomes and marking guide

(iv)



PE6, Band E2
- Gives the correct graph (all information labelled) 3
- Displays the correct roots.

OR
- Displays the correct turning points...... 2
- Gives the correct y-intercept AND one other coordinate 1

(v)

$$y = 2\sin\left(\frac{\pi x}{2} - \frac{\pi}{6}\right)$$

$$y^2 = 4\left(\sin\left(\frac{\pi x}{2} - \frac{\pi}{6}\right)\right)^2,\ 4\sin\theta^2 = (2 - 2\cos(2\theta))$$

$$= 2 - 2\cos\left(\pi x - \frac{\pi}{3}\right)$$

$$V = \pi\int_1^2 y^2\,dx$$

$$= \pi\int_1^2 2 - 2\cos\left(\pi x - \frac{\pi}{3}\right)dx$$

$$= \pi\left[2x - \frac{2}{\pi}\sin\left(\pi x - \frac{\pi}{3}\right)\right]_1^2$$

$$= \pi\left\{\left[4 - \frac{2}{\pi}\sin\left(2\pi - \frac{\pi}{3}\right)\right] - \left[2 - \frac{2}{\pi}\sin\left(\pi - \frac{\pi}{3}\right)\right]\right\}$$

$$= \pi\left\{\left[4 - \frac{2}{\pi} \times -\frac{\sqrt{3}}{2}\right] - \left[2 - \frac{2}{\pi} \times \frac{\sqrt{3}}{2}\right]\right\}$$

$$= \pi\left\{2 + \frac{\sqrt{3}}{\pi} + \frac{\sqrt{3}}{\pi}\right\}$$

$$= 2\pi + 2\sqrt{3}\ \text{units}^3$$

HE7, Band E4
- Gives the correct solution............. 3
- Gives the correct integral 2
- Gives the correct integrand............ 1

(b) $P(k) = k^3 + 2k = 3N$ where N is an integer.

Show $P(k + 1)$ is divisible by 3.

$$P(k+1) = (k+1)^3 + 2(k+1)$$

$$= (k+1)(k^2 + 2k + 1) + 2k + 2$$

$$= k^3 + 2k^2 + k + k^2 + 2k + 1 + 2k + 2$$

$$= k^3 + 3k^2 + 5k + 3$$

$$= k^3 + 2k + 3k^2 + 3k + 3$$

$$= 3N + 3(k^2 + k + 1)$$

$$= 3(N + k^2 + k + 1)$$

$\therefore$ As $N + k^2 + k + 1$ is an integer, $P(k + 1)$ is divisible by 3.

HE2, Band E3
- Gives the correct proof............... 2
- Makes a substantial attempt at solution .. 1

 TENVE1_SS_17.FM Copyright © 2017 Neap

Sample answer	Syllabus outcomes and marking guide
Question 14	
(a) (i) $\frac{dT}{dt}=k(T-A)$ $\int\frac{dT}{T-A}=\int k\,dt$ $\ln(T-A)=kt+c$ $T-A=e^{kt+c}$ $T=A+e^{c}e^{kt}$ $=A+Ce^{kt}$ where $C=e^{c}$ When $t=0$, $A=8$ and $T=93.4$. $93.4=8+Ce^{k\times 0}$ $\therefore C=85.4$ Hence, $T=8+85.4e^{kt}$.	HE3, Band E3 • Gives the correct solution .... 3 • Makes a substantial attempt at solution. . 2 • Correctly separates variables.......... 1
(ii) $T=8+85.4e^{kt}$ When $t=5$, $T=75$. $75=8+85.4e^{k\times 5}$ $e^{5k}=\frac{67}{85.4}$ $\therefore k=\frac{1}{5}\ln\frac{67}{85.4}$ $=-0.0485$ $=-0.049$	HE3, Band E2 • Gives the correct solution .... 2 • Correctly substitutes 1
(iii) $T=8+85.4e^{-0.049t}$ $63=8+85.4e^{-0.049t}$ $e^{-0.049t}=\frac{55}{85.4}$ $\therefore t=-\frac{1}{0.049}\ln\left(\frac{55}{85.4}\right)$ $=8.97\approx 9$ minutes $\therefore$ He has $9-5=4$ minutes to drink his tea.	HE3, Band E2 • Gives the correct solution .... 2 • Correctly substitutes 1

Copyright © 2017 Neap

		Sample answer	Syllabus outcomes and marking guide
(b)	(i)	Target A: $\dot{x} = V\cos\theta$ $x = Vt\cos(\theta) + C$ $t = 0, x = 0, C = 0$ $x = Vt\cos(\theta)$ $V = 40, \theta = 0$ $x = 40t\cos(0)$ $= 40t$ $\dot{y} = -gt + V\sin\theta$ $y = -\frac{1}{2}gt^2 + Vt\sin\theta + C$ $t = 0, y = 10, C = 10$ $y = -\frac{1}{2}gt^2 + Vt\sin\theta + 10$ $V = 40, \theta = 0$ $y = -\frac{1}{2}gt^2 + 40t\sin 0 + 10$ $= -\frac{1}{2}gt^2 + 10$ For $t = \frac{x}{40}$: $y = -\frac{1}{2}g\left(\frac{x}{40}\right)^2 + 10$ $= 10 - \frac{gx^2}{3200}$ Target B: $x = 40t\cos\theta$ $y = -\frac{1}{2}gt^2 + 40t\sin\theta + 10$ For $t = \frac{x}{40\cos\theta}$: $y = -\frac{1}{2}g\left(\frac{x}{40\cos\theta}\right)^2 + 40 \times \frac{x}{40\cos\theta} \times \sin\theta + 10$ $= \frac{-gx^2}{3200\cos^2\theta} + x\tan\theta + 10$	HE3, Band E4 • Gives the correct solutions for target A AND • Gives the correct solutions for target B . . 4 • Gives the correct solution for target A AND makes a substantial attempt at solution for target B. OR • Gives the correct solution for target B AND makes a substantial attempt at solution for target A. . . . . 3 • Makes a substantial attempt at solution for target A. AND • Makes a substantial attempt at solution for target B. . . . . 2 • Makes a substantial attempt at solution for target A. OR • Makes a substantial attempt at solution for target B. 1

Copyright © 2017 Neap

Sample answer	Syllabus outcomes and marking guide
(ii) Target A: $y = 10 - \frac{gx^2}{3200}$ For $y = 0, x = d$. $0 = 10 - \frac{gd^2}{3200}$ $d^2 = \frac{32\,000}{g}$ $\therefore d = 80\sqrt{\frac{5}{g}}$ $= 57.14$ m Target B: $y = \frac{-gx^2}{3200\cos^2\theta} + x\tan\theta + 10$ For $y = 0, x = d$. $0 = \frac{-gd^2}{3200\cos^2\theta} + d\tan\theta + 10 \quad ...(1)$ Substitute $d = 80\sqrt{\frac{5}{g}}$ and $\frac{1}{\cos^2\theta} = \tan^2\theta + 1$ into (1). $0 = -\frac{g}{3200} \times \frac{32\,000}{g}(\tan^2\theta + 1) + 80\sqrt{\frac{5}{g}}\tan\theta + 10$ $= -10\tan^2\theta - 10 + 80\sqrt{\frac{5}{g}}\tan\theta + 10$ $= \tan^2\theta - 8\sqrt{\frac{5}{g}}\tan\theta$ $= \tan\theta \times \left(\tan\theta - 8\sqrt{\frac{5}{g}}\right)$ Target A: $\theta = 0$, Target B: $\theta = \tan^{-1} 8\sqrt{\frac{5}{g}} \approx 80°$	HE3, Band E4 • Gives the correct angle .... 4 • Gives the correct quadratic equation ..... 3 • Substitutes correctly .... 2 • Gives the correct distance 1