PEM

2017

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time - 3 hours
- Write using black pen
- Board–approved calculators may be used
- A reference sheet is provided at the back of this paper
- In Questions 11 – 16, show relevant mathematical reasoning and/or calculations

Total Marks – 100

Section I Questions 1 – 10   **10 marks**
Allow about 15 minutes for this section

Section II Questions 11 – 16   **90 marks**
Allow about 2 hours and 45 minutes for this section

10 marks
Attempt Questions 1-10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10.

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1   What is the basic numeral for 4.5378×10^{-4}, correct to 4 significant figures?

    (A)   0.00045378

    (B)   0.0004538

    (C)   0.4538

    (D)   45380

2   What is the solution on a number line for $-5 < 2x - 3 \le 9$?

    (A)

    (B)

    (C)

    (D)
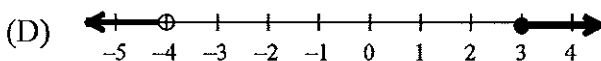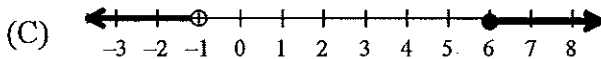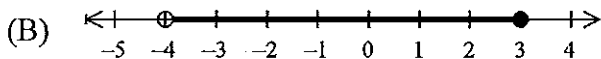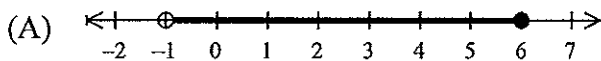
3   What is the solution of $|x-3| = 2x + 1$?

    (A) $x = -4$

    (B) $x = \dfrac{2}{3}$

    (C) $x = \dfrac{4}{3}$

    (D) $x = \dfrac{3}{2}$

4   Which inequalities define the shaded region shown in the diagram?
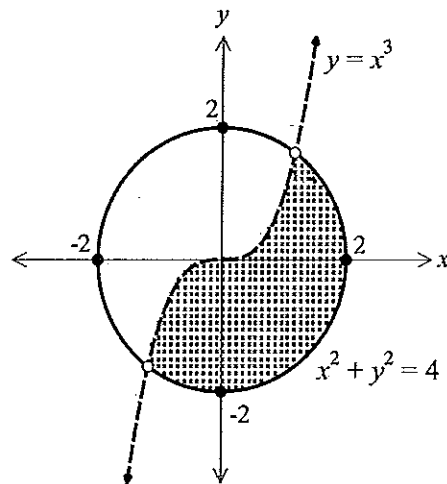
(A) $x^2 + y^2 \geq 4$ and $y < x^3$

(B) $x^2 + y^2 \geq 4$ and $y \geq x^3$

(C) $x^2 + y^2 \leq 4$ and $y > x^3$

(D) $x^2 + y^2 \leq 4$ and $y < x^3$

5   Jane studies 5 different subjects English, Mathematics, Drama, Economics and Biology. Her weekly timetable consists of 5 lessons per subject. Two lessons are cancelled during the week for athletics training.

What is the probability that the 2 lessons cancelled for Jane are both Mathematics lessons?

(A) $\dfrac{5}{25} \times \dfrac{5}{25}$

(B) $\dfrac{2}{5} \times \dfrac{2}{5}$

(C) $\dfrac{5}{25} \times \dfrac{4}{24}$

(D) $\dfrac{2}{5} \times \dfrac{1}{4}$

6   What is the gradient of the normal to the curve $y = \cos 2x$ at the point where $x = \dfrac{\pi}{8}$?

(A) $-\sqrt{2}$

(B) $-\dfrac{\sqrt{2}}{2}$

(C) $\dfrac{\sqrt{2}}{2}$

(D) $\sqrt{2}$

7   A particle that moves in a straight line with a displacement of x metres at time t seconds. When is the particle speeding up?

(A) $\dfrac{dx}{dt} < 0$ and $\dfrac{d^2x}{dt^2} > 0$

(B) $\dfrac{dx}{dt} > 0$ and $\dfrac{d^2x}{dt^2} < 0$

(C) $\dfrac{dx}{dt} > 0$ and $\dfrac{d^2x}{dt^2} = 0$

(D) $\dfrac{dx}{dt} < 0$ and $\dfrac{d^2x}{dt^2} < 0$

8   What is the value of $\displaystyle\sum_{k=5}^{\infty} 5\left(\dfrac{2}{5}\right)^{k-1}$?

(A) $\dfrac{16}{75}$

(B) $\dfrac{8}{25}$

(C) $\dfrac{1031}{125}$

(D) $\dfrac{25}{3}$

9 The diagram shows the graphs of $f(x) = \cos x$ and $g(x)$.

What is the equation of $g(x)$?
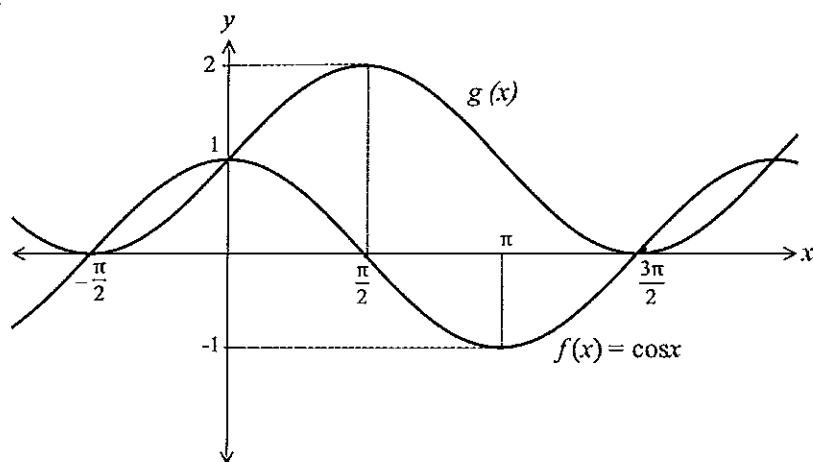
(A) $\quad g(x) = f\left(x - \dfrac{\pi}{2}\right) + 2$

(B) $\quad g(x) = f\left(x + \dfrac{\pi}{2}\right) + 1$

(C) $\quad g(x) = f\left(x - \dfrac{\pi}{2}\right) + 1$

(D) $\quad g(x) = f\left(x + \dfrac{\pi}{2}\right) + 2$

10 Which of the following statements is true if $(2, -5)$ is a minimum turning point of $f(x)$ and $f(x) = -f(-x)$?

(A) $\quad (-2, -5)$ is a maximum turning point of $f(x)$

(B) $\quad (-2, 5)$ is a minimum turning point of $f(x)$

(C) $\quad (-2, -5)$ is a minimum turning point of $f(x)$

(D) $\quad (-2, 5)$ is a maximum turning point of $f(x)$

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90 marks
Attempt Questions 11-16
Allow about 2 hours and 45 minutes for this section

Answer each question in a separate writing booklet.

In Questions 11-16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a Writing Booklet.

(a) Find the primitive function of $(2x-1)^3$.   **1**

(b) Find $\lim\limits_{x \to -1} \dfrac{2x^2+3x+1}{x+1}$.   **2**

(c) Find the value of m given $\sqrt{2}+1+\dfrac{1}{\sqrt{2}+1}=\sqrt{m}$,   **2**

    where m is a positive integer.

(d) Factorise completely the expression $m^3-9m^2-4m+36$.   **3**

(e) Find the values of b and c in the quadratic equation $x^2+bx+c=0$,   **2**

    where the roots are $\sqrt{3}+1$ and $\sqrt{3}-1$.

Question 11 (continued)

(f)   For a certain curve passing through $(1,3)$, $\dfrac{dy}{dx} = \dfrac{1}{2x-1}$.   2

       Find the equation of the curve.

(g)   Solve $8^{x+1} = 2\left(4^{x-1}\right)$.   3

End of Question 11

Please turn over

Question 12 (15 marks) Use a New Writing Booklet.

(a) The diagram shows a square tile measuring 10 cm by 10 cm. The tile design is of a sector with radius, r cm, and subtending an angle of 50° from B. The sector is white in colour and the surrounding areas are shaded as shown in the diagram. $BX = BY = r$ cm and $AX = CY$.
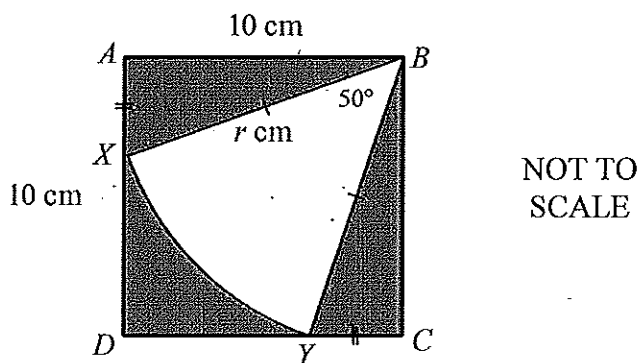
(i)   Show that $\angle ABX = \angle CBY$, giving reasons. **3**

(ii) Find the radius, r cm, of the sector. **1**
      Give the answer correct to 2 decimal places.

(iii) Find the total shaded area of the tile correct to the nearest square centimetre. **2**

(b) School bus A is 150 km due west of school bus B. At the same time, school bus A drives due east at 50 km/h while school bus B drives due south at 40 km/h.

(i)   After x hours of driving, show that **2**

$$d = \sqrt{4100x^2 - 15000x + 22500} \, ,$$

where d is the distance between the buses and $0 < x < 3$.

(ii) Find x correct to 2 decimal places and show that it would give the minimum distance between the buses. **3**

Question 12 continues on page 8

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Question 12 continued

(c) The diagram shows the parabolas $y = (x-a)^2$ and $y = a - x^2$, where $a > 0$. The parabolas intersect **once only** at point A.
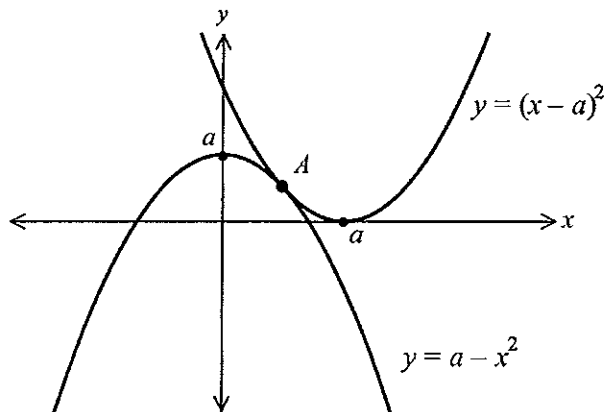
(i) Show that $a = 2$. **2**

(ii) The parabolas $y = (x-a)^2$ and $y = a - x^2$ have a common tangent at point A. Find the coordinates of point A and hence find equation of the tangent. **2**

End of Question 12

- 8 -

Question 13 (15 marks) Use a New Writing Booklet.

(a)   The diagram shows a six pointed star which is drawn using two triangles,
      $\triangle ABC$ and $\triangle FDE$. The intersection of the two triangles is a regular hexagon.

(i)   Show that $\triangle AVU$ is an equilateral triangle.   **2**

(ii)   Similarly $\triangle VEW$, $\triangle WCX$, $\triangle XFY$, $\triangle YBZ$ and $\triangle ZDU$ are all   **2**
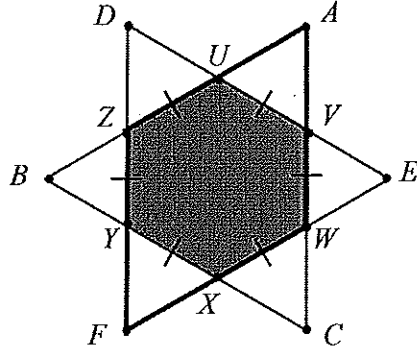   equilateral triangles.
       Prove that $ZAWF$ is a rhombus.

(b)   The diagram shows four points on a Cartesian number plane,

      $A\left(-\dfrac{2}{3},0\right)$, $B(0,2)$, $C(1,5)$ and $D(x,0)$ where $x>0$.
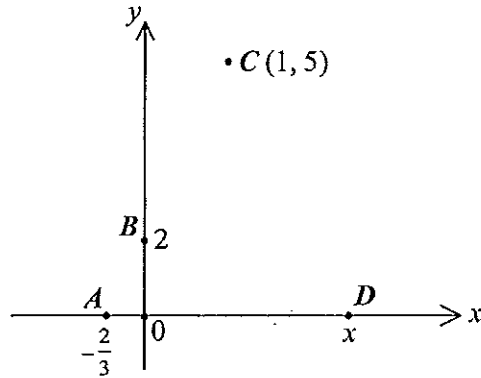
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(i)   Show that the points A, B and C are collinear.   **2**

(ii)   Find the x-coordinate of point D, given that points A, C and D **2**
       form an isosceles triangle, where $AC = CD$.

(iii) Find $\angle CAD, \angle CDA$ and $\angle ACD$ of the isosceles triangle ACD.   **2**
       Give the answers correct to the nearest minute.

Question 13 (continued)

(c)   A photocopier was purchased for $3600. It has an additional ongoing
maintenance fee of $200 for the first year which then increases by $50
per year thereafter. The maintenance fees form an arithmetic sequence
given by,

$$\$200, \$250, \$300, \ldots\ldots\ldots .$$

(i)   During which year will the maintenance fee exceed $980?   2

(ii) Show that the total cost (C) of the photocopier is given by,   2

$$C = 25n^2 + 175n + 3600,$$

where n is the number of years.

(iii) Find the number of years it will take for the total cost, C, of the   1
photocopier to accumulate to $8000.
Give the answer correct to 2 decimal places.

End of Question 13

**Question 14** (15 marks) Use a New Writing Booklet.

(a)   A school has two printers. On any one day, printer A has an 18% chance
of malfunctioning and printer B has a 15% chance of malfunctioning.

    (i)   Find the probability that on any one day exactly one printer will   **2**
be working effectively.

    (ii)   After five years the printers have deteriorated further such that, the   **1**
probability that at least one printer is working effectively is 83%.
Find the probability that on any one day both printers will malfunction.

(b)   (i)   The curve $y = \ln\left(x^2 + 2\right)$ has one stationary point.   **2**
Find its coordinates and determine its nature.

    (ii)   For the curve $y = \ln\left(x^2 + 2\right)$, find the points of inflexion.   **3**

    (iii)   Sketch the graph of $y = \ln\left(x^2 + 2\right)$, clearly showing the stationary   **2**
point and the points of inflexion.

(c)   A particle moves in a straight line. Its displacement x in metres is given by

$$x = 70e^{-\frac{t}{10}} - 20t,$$

where t is the time in seconds.

    (i)   Find the initial displacement of the particle.   **1**

    (ii)   Will the particle ever come to rest?   **2**
Justify your answer using appropriate calculations.

    (iii)   Find the distance travelled by the particle in the first 3 seconds.   **1**
Give the answer correct to 2 decimal places.

    (iv)   Find the speed that the particle is approaching as $t \to \infty$.   **1**

End of Question 14

Question 15 (15 marks) Use a New Writing Booklet.

(a) A bath tub is in the shape of half a cylinder with a radius of 50 cm and a length of 1.8 m. The tub is filled with water up to 10 cm from the top of the tub.

    (i) A cross section of the bath tub is shown in the diagram below, where O is the centre and AB is the water level in the tub.
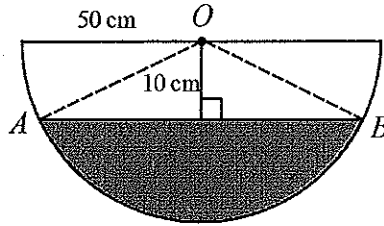
         Show that $\angle AOB = 2.74$ radians, correct to 2 decimal places.   **1**

    (ii) Find the volume of water in the tub, correct to the nearest litre,   **2**
given $1\,\text{cm}^3 = 1\,\text{mL}$.

(b) The amount of caffeine, $C(t)$, in milligrams in your system after drinking a medium cappuccino is given by,

$$C(t) = 105e^{-kt},$$

where k is a constant and t is the time in hours that have passed since drinking the cappuccino.

    (i) After one hour the caffeine in your system has decreased by 40%.   **2**
Find the exact value of k.

    (ii) When will there be 10 milligrams of caffeine remaining in your system?   **1**
Give the answer correct to 2 significant figures.

Question 15 (continued)

(c) The diagram shows the graph of $y = \ln(x+2)$. The shaded region is bounded by $y = \ln(x+2)$, $x = 3$ and the coordinate axes.
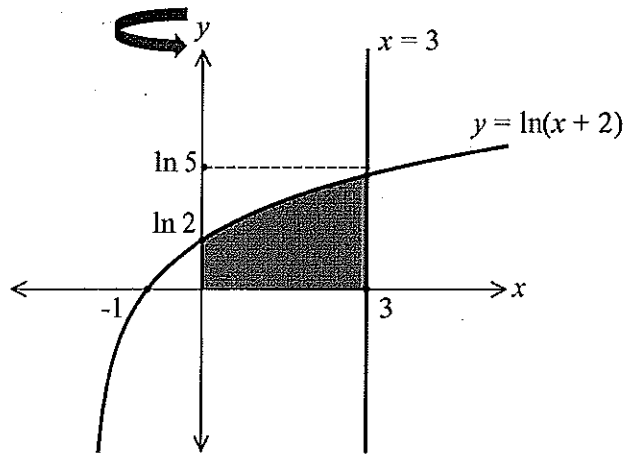
(i) Rewrite $y = \ln(x+2)$ in the form $x = e^y + b$, where b is a constant. **1**

(ii) Find the volume of the solid formed when the shaded area is rotated about the y - axis. Give the answer correct to 1 decimal place. **3**

(d) (i) A new sports car was purchased for $\$P$ and every year that passes it depreciates by 2.5%. The value of the car each year since its purchase forms a geometric sequence.

(α) Show that the nth term of the sequence is given by **1**

$$T_n = (0.975)^n P,$$

where n is the number of years since the car was purchased.

(β) Find the purchase price of the car if its value after 18 years is $\$158\,497.75$. Give the answer to the nearest dollar. **1**

(ii) A classic car was purchased for $\$125\,000$ and every year that passes **3** it appreciates by 3.1%. The value of the car each year since its purchase forms a geometric sequence. The nth term is given by

$$T_n = 128875(1.031)^{n-1}.$$

When will the value of the new sports car and the classic car be the same? Give the answer correct to 2 decimal places.

End of Question 15

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Question 16 (15 marks) Use a New Writing Booklet.

(a) The rate at which a Jacaranda tree grows is given by

$$\frac{dh}{dt} = \frac{110}{(t+4)^2} \text{ metres per year},$$

where h is the height of the tree in metres and t is the number of years that have passed since the tree was planted from an established seedling, 0.5 m in height.

(i) Find the rate at which the tree is growing when $t = 5$.   1

(ii) Find the height of the tree when $t = 5$, correct to 1 decimal place.   3

(b) The diagram shows the graph of $y = 8\sin\frac{\pi}{12}(x-3)$. The shaded area is bounded by $y = 8\sin\frac{\pi}{12}(x-3)$, $y = 8$ and the y-axis.
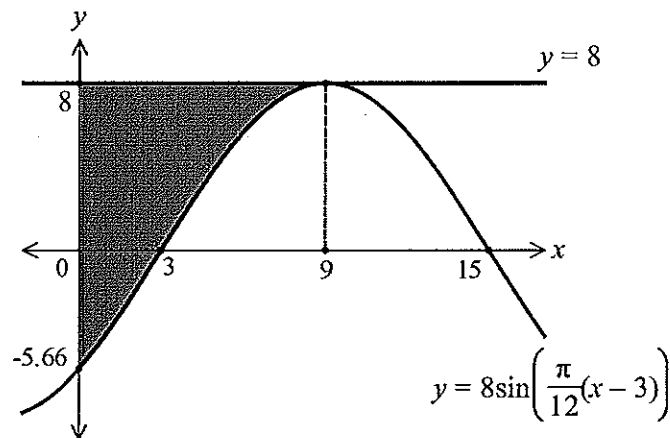
Find the shaded area, giving the answer in simplified exact form.   3

Question 16 (continued)

(c)   The diagram shows a sun shade that is to be installed in the playground
        of a pre-school. AB is 30 m long and is divided into 6 equal intervals.
        AB divides the sun shade into the north facing and south facing sections.
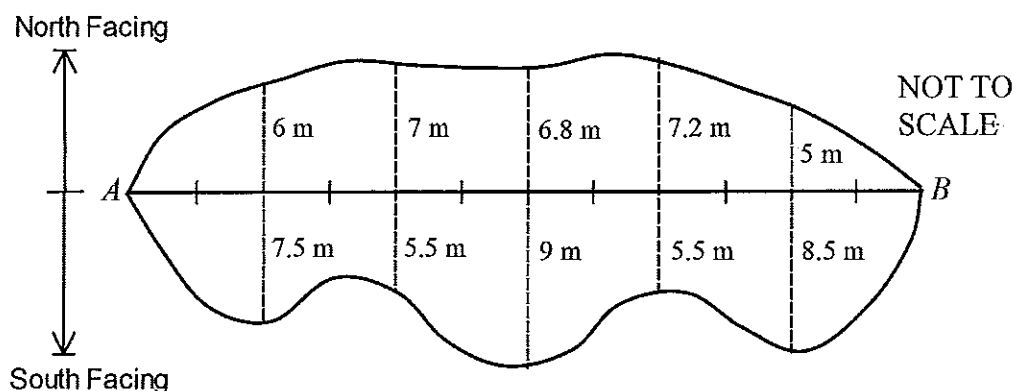
        The installers of the sun shade have measured the vertical lengths from AB
        to the edges of the north facing and south facing sections.

        In order to calculate an approximation for the area of the sun shade the
        installers decided to use the Trapezoidal rule for the north facing section
        and the Simpson's rule for the south facing section.

        (i)   Give a reason as to why the Simpson's rule is a better option than the   1
                Trapezoidal rule for calculating the approximate area of the south
                facing section of the sun shade.

        (ii)   Find the approximate area of the sun shade calculated by the installers   2
                giving the answer to the nearest square metre.

Question 16 (continued)

(d)   Peter and Gina bought a penthouse apartment for $2 500 000. They had
        a deposit of $1 000 000 and borrowed the remainder from a lender who
        offered them an interest rate of 5.2% p.a. compounded fortnightly for 30 years.

        (i)   Show that their fortnightly repayment is $3799.70.   3

        (ii)   After 15 years, the market value of Peter and Gina's apartment is
                 $6 500 000. They decide to borrow against the equity in their
                 apartment from their lender who will advance them 85% of their equity
                 for their next purchase.

                 (α) Find the loan outstanding after 15 years.   1

                 (β)   Find the amount of money they can obtain for their next purchase.   1

END OF PAPER

2017 TRIAL HIGHER CERTIFICATE EXAMINATION

MATHEMATICS – SUGGESTED SOLUTIONS

Section I - 10 MARKS

Questions 1 – 10 (1 mark each)

Question	Answer	Outcomes Assessed	Targeted Performance Bands
1	B	P3,	2
2	A	P3, P4	2
3	B	P4	2-3
4	D	P4, P5	2-3
5	C	H5	3
6	C	P7, H6	3-4
7	D	H4, H9	4
8	A	H5	4
9	C	P5, H5	4-5
10	D	P5, H5, H6	5-6

Section II – 90 Marks

Question 11 (15marks)

(a) 1 mark

Outcomes Assessed: H5

Targeted Performance Bands: 3

Criteria	Marks
• Gives correct expression	1

Sample Answer

$$\int (2x-1)^3 \, dx = \frac{(2x-1)^4}{4 \times 2} + C$$
$$= \frac{(2x-1)^4}{8} + C$$

11(b) 2 marks

Outcomes Assessed: P4, H5

Targeted Performance Bands: 2-3

Criteria	Marks
• Gives the correct solution	2
• Factorises numerator correctly	1

Sample Answer

$$\lim_{x \to -1} \left[\frac{(2x+1)(x+1)}{x+1} \right] = \lim_{x \to -1} (2x+1)$$
$$= -2 + 1$$
$$= -1$$

11(c) 3 marks

Outcomes Assessed: P4

Targeted Performance Bands: 2

Criteria	Marks
• Gives correct solution	2
• Rationalises the denominator correctly	1

Sample Answer

$$\sqrt{2}+1+\left[\frac{1}{\sqrt{2}+1}\times\frac{\sqrt{2}-1}{\sqrt{2}-1}\right]$$

$$=\sqrt{2}+1+\sqrt{2}-1$$

$$=2\sqrt{2}$$

$$=\sqrt{8}$$

$$\therefore m=8$$

11(d) 3 marks

Outcomes Assessed: P3, P4

Targeted Performance Bands: 3

Criteria	Marks
• Gives the correct factorised expression	3
• Gives the factorised expression $(m-9)(m^2-4)$	2
• Gives the factorised expression $m^2(m-9)-4(m-9)$	1

Sample Answer

$$m^2(m-9)-4(m-9)$$

$$=(m-9)(m^2-4)$$

$$=(m-9)(m+2)(m-2)$$

11(e) 2 marks

Outcomes Assessed: P4, P5

Targeted Performance Bands: 3-4

Criteria	Marks
• Gives the correct answers for *b* **and** *c*	2
• Finds the value of *b* **or** *c*	1

$$\text{let } \alpha = \sqrt{3}+1 \quad \beta = \sqrt{3}-1$$

$$x^2 - (\alpha + \beta)x + \alpha\beta = x^2 + bx + c$$

$$b = -(\alpha + \beta) = -2\sqrt{3}$$

$$c = \alpha\beta = 2$$

11(f) 2 marks

Outcomes Assessed: H3, H5

Targeted Performance Bands: 4

Criteria	Marks
• Gives the correct equation	2
• Integrates correctly	1

Sample Answer

$$y = \int \frac{1}{2x-1}\,dx$$

$$= \frac{1}{2}\ln(2x-1) + C$$

substitute $(1,3)$

$$3 = \frac{1}{2}\ln 1 + C$$

$$C = 3$$

$$\therefore y = \frac{1}{2}\ln(2x-1) + 3$$

11(g) 3 marks

Outcomes Assessed: P4, H3

Targeted Performance Bands: 3-4

Criteria	Marks
• Gives the correct solution	3
• Gives simplifies expression $2^{3x+3} = 2^{2x-1}$	2
• Gives simplifies expression $2^{3x+3} = 2\left(2^{2x-2}\right)$	1

Sample Answer

$$\left(2^3\right)^{x+1}=2\left[\left(2^2\right)^{x-1}\right]$$
$$2^{3x+3}=2\left(2^{2x-2}\right)$$
$$2^{3x+3}=2^{2x-1}$$
$$\therefore 3x+3=2x-1$$
$$x=-4$$

Question 12 (15 marks)

(a) (i) 3 marks

Outcomes Assessed: P2, H5

Targeted Performance Bands: 4

Criteria	Marks
• Gives the correct solution with reasons	3
• Gives the correct proof for congruent triangles and states the test used	2
• Progress towards the proof for congruent triangles	1

Sample Answer

In $\triangle ABX$ and $\triangle CBY$

$BX = BY = r$ cm (given)

$AX = CY$ (given)

$AB = BC = 10$ cm (given)

$\therefore \triangle ABX \equiv \triangle CBY$ (SSS)

$\therefore \angle ABX = \angle CBY$ (corresponding angles in congruent triangles are equal)

12 (a) (ii) 1 mark

Outcomes Assessed: P3

Targeted Performance Bands: 3

Criteria	Marks
• Gives the correct solution for r	1

$\angle ABX = 20°$

$\therefore \cos 20° = \dfrac{10}{r}$

$r \cos 20° = 10$

$r = \dfrac{10}{\cos 20°} \approx 10.64 \, \text{cm}$

12 (a) (iii) 2 marks

Outcomes Assessed: H5

Targeted Performance Bands: 4

Criteria	Marks
• Gives the correct solution	2
• Substitutes correctly into the area of a sector	1

Sample Answer

$A = 100 - \left[\dfrac{1}{2} \times 10.64^2 \times \dfrac{50\pi}{180} \right]$

$\approx 51 \, \text{cm}^2$

12 (b) (i) 2 marks

Outcomes Assessed: P4, H4

Targeted Performance Bands: 4

Criteria	Marks
• Gives the correct solution	2
• Correct substitution into Pythagoras Theorem	1

Sample Answer
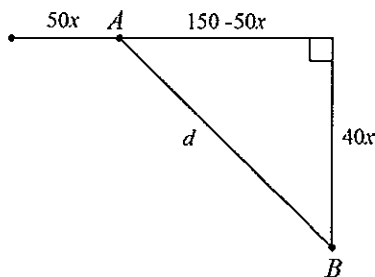
$d^2 = (150 - 50x)^2 + (40x)^2$

$= 22500 - 15000x + 2500x^2 + 1600x^2$

$d = \sqrt{4100x^2 - 15000x + 22500}$

12(b) (ii)

Outcomes Assessed: P7, H5, H6

Targeted Performance Bands: 4

Criteria	Marks
• Gives the correct solution with reasons	3
• Gives the correct proof for congruent triangles and states the test used	2
• Progress towards the proof for congruent triangles	1

Sample Answer

$$d = \left(4100x^2 - 15000x + 22500\right)^{\frac{1}{2}}$$

$$d' = \frac{1}{2}\left(4100x^2 - 15000x + 22500\right)^{-\frac{1}{2}} \times \left(8200x - 15000\right)$$

$$d' = 0$$

$$\therefore \frac{4100x - 7500}{\sqrt{4100x^2 - 15000x + 22500}} = 0$$

$$4100x = 7500$$

$$x = 1.83 \text{ hours}$$

x	1.8	1.83	2
d'	-	0	+

$\therefore x = 1.83$ hours would give the minimum distance between the buses

12(c) (i) 2 marks

Outcomes Assessed: P4, P5

Targeted Performance Bands: 3-4

Criteria	Marks
• Gives the correct solution	2
• Gives the correct quadratic equation for point A in the form $ax^2 + bx + c = 0$	1

Sample Answer

$$(x-a)^2 = a - x^2$$
$$x^2 - 2ax + a^2 = a - x^2$$
$$2x^2 - 2ax + a^2 - a = 0$$
$$\Delta = 0$$
$$4a^2 - 4 \times 2(a^2 - a) = 0$$
$$4a^2 - 8a^2 + 8a = 0$$
$$-4a^2 + 8a = 0$$
$$-4a(a-2) = 0$$
$$a \neq 0 \quad a = 2$$

12 (c) (ii) 2 marks

Outcomes Assessed: P7, H5

Targeted Performance Bands: 4

Criteria	Marks
• Gives the correct solution	2
• Finds the correct coordinates of point A	1

Sample Answer

substitute $a = 2$ into $2x^2 - 2ax + a^2 - a = 0$
$$2x^2 - 4x + 4 - 2 = 0$$
$$x^2 - 2x + 1 = 0$$
$$(x-1)^2 = 0$$
$$x = 1$$
$$y = 1$$
$$\therefore A(1,1)$$
$$y = 2 - x^2$$
$$y' = -2x$$
when $x = 1$ $\quad m_{tangent} = -2$
$$y - 1 = -2(x-1)$$
$$y = -2x + 2 + 1$$
$$\quad = -2x + 3$$

Question 13 (15 marks)

(a) (i) 2 marks

Outcomes Assessed: P4, H2, H5

Targeted Performance Bands: 4

Criteria	Marks
• Correct proof giving reasons	2
• Finds the interior angles of a regular hexagon	1

Sample Answer

Each interior angle of a regular hexagon $= \dfrac{180°(6-2)}{6} = 120°$

$\therefore \angle ZUV = \angle UVW = 120°$

$\angle AUV = \angle AVU = 180° - 120°$ (angles on a straight line)

$\qquad\qquad\qquad = 60°$

$\therefore \angle UAV = 180° - 60° - 60°$ (angle sum of a triangle)

$\qquad\quad = 60°$

$\therefore \Delta AVU$ is an equilateral triangle since all angles are equal

13 (a) (ii) 2 marks

Outcomes Assessed: P4, H2, H5

Targeted Performance Bands: 4

Criteria	Marks
• Correct proof giving reasons	2
• Shows that $AZFW$ is a parallelogram	1

Sample Answer

$\angle ZAW = \angle WFZ = 60°$ (equilateral triangles)

$\angle FZA = \angle AWF = 120°$ (angles of a regular hexagon)

$\therefore ZAWF$ is a parallelogram (opposite angles are equal)

The regular hexagon and the equilateral triangles have common sides

$\therefore ZU = AU = AV = VW = WX = XF = FY = YZ$

and $ZA = AW = WF = FZ$

$\therefore ZAWF$ is a rhombus since all sides are equal.

13 (b) (i) 2 marks

Outcomes Assessed: P2.P4, H5

Targeted Performance Bands: 3-4

Criteria	Marks
• Correct solution	2
• Finds the gradient of AB or BC	1

Sample Answer

$$m_{AB} = \frac{2-0}{0+\frac{2}{3}} = 3$$

$$m_{BC} = \frac{5-2}{1-0} = 3$$

$\therefore A, B$ and C are collinear since $m_{AB} = m_{BC}$

13 (b) (ii) 2 marks

Outcomes Assessed: P2, P4, H5

Targeted Performance Bands: 4

Criteria	Marks
• Correct solution	2
• Finds the correct length of AC and CD	1

Sample Answer

$$d_{AC} = d_{CD}$$

$$\sqrt{\left(1 + \frac{2}{3}\right)^2 + (5-0)^2} = \sqrt{(x-1)^2 + (0-5)^2}$$

$$\sqrt{\frac{25}{9} + 25} = \sqrt{(x-1)^2 + 25}$$

$$\sqrt{\frac{250}{9}} = \sqrt{(x-1)^2 + 25}$$

$$\therefore (x-1)^2 + 25 = \frac{250}{9}$$

$$(x-1)^2 = \frac{25}{9}$$

$$x - 1 = \pm\frac{5}{3}$$

$$\therefore x = \frac{8}{3}, \text{ since } x > 0$$

13 (b) (iii) 2 marks

Outcomes Assessed: P2, P4, H5

Targeted Performance Bands: 3-4

Criteria	Marks
• Finds all 3 angles of the isosceles triangle correctly	2
• Uses $m = \tan\theta$ correctly to find $\angle CAD$	1

Sample Answer

let $\angle CAD = \theta$

$\tan\theta = m_{AC}$

$\tan\theta = 3$

$\therefore \theta \approx 71°34'$

$\angle CAD = \angle CDA = 71°34' \left(\text{base angles of an isosceles triangle}\right)$

$\therefore \angle ACD = 180° - 71°34' - 71°34'$

$\qquad\qquad = 36°52'$

13 (c) (i) 2 marks

Outcomes Assessed: H4, H5

Targeted Performance Bands: 3-4

Criteria	Marks
• Gives the correct solution	2
• Shows that $n > 16.6$	1

Sample Answer

$T_n = 200 + (n-1)50$

$\quad = 150 + 50n$

$150 + 50n > 980$

$50n > 830$

$n > 16.6$

$\therefore$ during the 17th year

13 (c) (ii) 2 marks

Outcomes Assessed: H4, H5

Targeted Performance Bands: 4-5

Criteria	Marks
• Gives the correct solution	2
• Finds an expression for the total maintenance fee cost	1

Sample Answer

$C = 3600 + \dfrac{n}{2}\left[400 + (n-1)50\right]$

$\quad = 3600 + 200n + 25n^2 - 25n$

$\quad = 3600 + 25n^2 + 175n$

13 (c) (iii) 1 mark

Outcomes Assessed: P4

Targeted Performance Bands: 3

Criteria	Marks
• Gives the correct solution	1

Sample Answer

$$25n^2 + 175n + 3600 = 8000$$
$$25n^2 + 175n - 4400 = 0$$
$$n^2 + 7n - 176 = 0$$
$$n = \frac{-7 \pm \sqrt{49 - 4 \times 1 \times -176}}{2}$$
$$\approx 10.22 \text{ years} \quad (n > 0)$$

Question 14 (15 marks)

(a) (i) 2 marks

Outcomes Assessed: H4, H5

Targeted Performance Bands: 4

Criteria	Marks
• Gives the correct answer	2
• Finds correctly $P(A\bar{B})$ or $P(\bar{A}B)$	1

Sample Answer

$$P(\text{exactly one printer is working}) = P(A\bar{B}) + P(\bar{A}B)$$
$$= (0.82 \times 0.15) + (0.18 \times 0.85)$$
$$= \frac{69}{250}$$
$$= 0.276$$

14 (a) (ii) 2 marks

Outcomes Assessed: H4, H5

Targeted Performance Bands: 4

Criteria	Marks
• Finds the correct answer	1

Sample Answer

$P\left(\text{at least one printer is working}\right) = 1 - P\left(\text{both printers malfunctioning}\right)$

$$0.83 = 1 - P\left(\overline{A}\,\overline{B}\right)$$

$\therefore P\left(\overline{A}\,\overline{B}\right) = 0.17 \text{ or } 17\%$

14 (b) (i) 2 marks

Outcomes Assessed: H5, H6

Targeted Performance Bands: 4-5

Criteria	Marks
• Gives the correct solution	2
• Finds the correct stationary point	1

Sample Answer

$$y' = \frac{2x}{x^2 + 2}$$

• for stationary points $y' = 0$

$\therefore \dfrac{2x}{x^2+2} = 0$

$x = 0$ and $y = \ln 2$

$\therefore \left(0, \ln 2\right)$

x	-1	0	1
y'	$-\dfrac{2}{3}$	0	$\dfrac{2}{3}$

$\searrow \quad - \quad \nearrow$

$\therefore \left(0, \ln 2\right)$ is a minimum turning point

- 14 -

14 (b) (ii) 3 marks

Outcomes Assessed: H5, H6

Targeted Performance Bands: 4-5

Criteria	Marks
• Gives the correct solution	3
• Finds the correct points	2
• Finds y''	1

Sample Answer

$$y' = \frac{2x}{x^2 + 2}$$

$$y'' = \frac{(x^2 + 2) \times 2 - 2x \times 2x}{(x^2 + 2)^2}$$

$$= \frac{4 - 2x^2}{(x^2 + 2)^2}$$

• for points of inflexion $y'' = 0$

$$\therefore \frac{4 - 2x^2}{(x^2 + 2)^2} = 0$$

$$x^2 = 2$$

$$x = \sqrt{2} \quad \text{or} \quad x = -\sqrt{2}$$

$$\left(\sqrt{2}, 2\ln 2\right) \quad \left(-\sqrt{2}, 2\ln 2\right)$$

x	1	$\sqrt{2}$	2
y''	$\frac{2}{9}$	0	$-\frac{1}{9}$

x	-2	$-\sqrt{2}$	1
y''	$-\frac{1}{9}$	0	$\frac{2}{9}$

$\therefore$ there is a change in concavity at $x = -\sqrt{2}$ and $x = \sqrt{2}$

Hence, $\left(\sqrt{2}, 2\ln 2\right)$ and $\left(-\sqrt{2}, 2\ln 2\right)$ are both points of inflexions

14 (b) (iii) 2 marks

Outcomes Assessed: H5, H6, H9

Targeted Performance Bands: 4-5

Criteria	Marks
• Draws the correct graph clearly labelled	2
• Shows the stationary point and the points of inflexion	1

Sample Answer
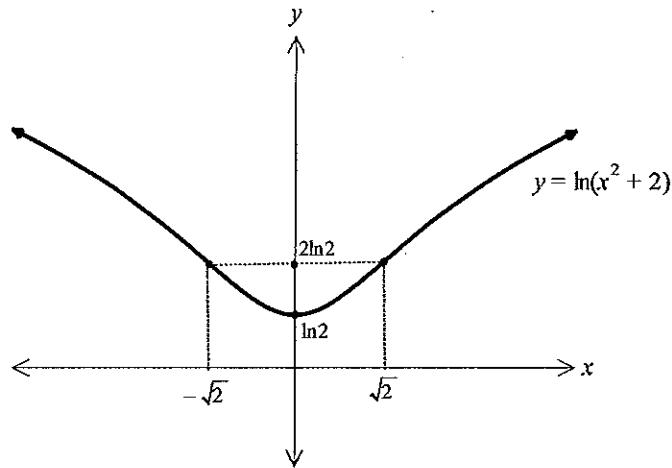
14 (c) (i) 1 mark

Outcomes Assessed: H3, H4

Targeted Performance Bands: 3

Criteria	Marks
• Gives the correct solution	1

Sample Answer

$t = 0$

$x = 70e^0 - 20(0)$

$= 70\,\text{m}$

14 (c) (ii) 2 marks

Outcomes Assessed: H3, H4, H5

Targeted Performance Bands: 4-5

Criteria	Marks
• Shows that $\dot{x} \neq 0$	2
• Finds the correct expression for velocity ($\dot{x}$)	1

Sample Answer

$$\dot{x} = -7e^{-\frac{t}{10}} - 20$$

let $\dot{x} = 0$

$$-7e^{-\frac{t}{10}} - 20 = 0$$

$$e^{-\frac{t}{10}} \neq -\frac{20}{7}$$

Since $e^{-\frac{t}{10}} > 0$

∴ Particle never comes to rest as $\dot{x} \neq 0$

14 (c) (iii) 1 mark

Outcomes Assessed: H4

Targeted Performance Bands: 4-5

Criteria	Marks
• Gives the correct solution	1

Sample Answer

$t = 0 \quad x = 70\,\text{m}$

$t = 3 \quad x = -8.14\,\text{m}$

∴ total distance travelled $= 78.14\,\text{m}$

14 (c) (iv) 1 mark

Outcomes Assessed: H3, H4

Targeted Performance Bands: 4-5

Criteria	Marks
• Gives the correct solution	1

Sample Answer

$\dot{x} = -7e^{-\frac{t}{10}} - 20$

$t \rightarrow \infty$

$\dot{x} \rightarrow -20 \, \text{m/s}$

$\therefore \text{speed} \rightarrow 20 \, \text{m/s}$

Question 15 (15 marks)

(a) (i) 1 mark

Outcomes Assessed: P3

Targeted Performance Bands: B3

Criteria	Marks
• Gives the correct answer	1

Sample Answer

$let \angle AOB = 2\alpha$

$\cos \alpha = \dfrac{1}{5}$

$\alpha = \cos^{-1}\left(\dfrac{1}{5}\right)$

$\therefore \angle AOB = 2\cos^{-1}\left(\dfrac{1}{5}\right)$

$\approx 2.74 \, \text{radians}$

15(a) (ii) 2 marks

Outcomes Assessed: H4

Targeted Performance Bands: 4

Criteria	Marks
• Gives the correct solution	2
• Progress towards the solution	1

Sample Answer

Volume = Area of segment cut off by AB × length of tub

$$=\frac{1}{2}(50)^2\left[2.74-\sin(2.74)\right]\times180$$

$$\approx 528550.92\,\text{cm}^3$$

$$\approx 529\,\text{litres}$$

15(b) (i) 2 marks

Outcomes Assessed: H4, H5

Targeted Performance Bands: 4-5

Criteria	Marks
• Gives the correct solution	2
• Finds the correct amount of caffeine remaining after 1 hour	1

Sample Answer

□ immediately after finishing cappuccino when $t = 0$

$C = 105\,\text{mg}$

□ $t = 1$

$C = 0.6\times105$

$\quad = 63\,\text{mg}$

$\therefore 63 = 105e^{-k}$

$$\frac{3}{5}=e^{-k}$$

$$\ln\left(\frac{3}{5}\right)=-k$$

$$k=-\ln\left(\frac{3}{5}\right)\ \text{or}\ k=-\ln(0.6)$$

15(b) (ii) 1 mark

Outcomes Assessed: H4, H5

Targeted Performance Bands: 4-5

Criteria	Marks
• Gives the correct solution	3

Sample Answer

$$10 = 105e^{\ln\left(\frac{3}{5}\right)t}$$

$$\frac{2}{21} = e^{\ln\left(\frac{3}{5}\right)t}$$

$$\ln\left(\frac{2}{21}\right) = \ln\left(\frac{3}{5}\right)t$$

$$t = \frac{\ln\left(\frac{2}{21}\right)}{\ln\left(\frac{3}{5}\right)}$$

$$\approx 4.6 \text{ hours}$$

15(c) (i) 1 mark

Outcomes Assessed: H3

Targeted Performance Bands: 4

Criteria	Marks
• Gives the correct expression	1

Sample Answer

$$x + 2 = e^y$$

$$x = e^y - 2$$

15 (c) (ii) 2 marks

Outcomes Assessed: H5, H8

Targeted Performance Bands: 5-6

Criteria	Marks
• Gives the correct solution	3
• Integrates correctly	2
• Gives the correct expression for calculating the volume: $V = \left[\pi(3)^2 \ln 5 \right] - \pi \int_{\ln 2}^{\ln 5} \left(e^y - 2 \right)^2 dy \quad or \quad V = \pi \int_0^{\ln 5} 3^2 dy - \pi \int_{\ln 2}^{\ln 5} \left(e^y - 2 \right)^2 dy$	1

Sample Answer

$$V = \left[\pi(3)^2 \ln 5 \right] - \pi \int_{\ln 2}^{\ln 5} \left(e^y - 2 \right)^2 dy$$

$$= 9\pi \ln 5 - \pi \int_{\ln 2}^{\ln 5} \left(e^{2y} - 4e^y + 4 \right) dy$$

$$= 9\pi \ln 5 - \pi \left[\frac{1}{2} e^{2y} - 4e^y + 4y \right]_{\ln 2}^{\ln 5}$$

$$= 9\pi \ln 5 - \pi \left[\left(\frac{1}{2} e^{2\ln 5} - 4e^{\ln 5} + 4 \ln 5 \right) - \left(\frac{1}{2} e^{2\ln 2} - 4e^{\ln 2} + 4 \ln 2 \right) \right]$$

$$\approx 38.7 \, \text{units}^3$$

15(d) (i)

(α) 1 mark

Outcomes Assessed: H4, H5

Targeted Performance Bands: 4

Criteria	Marks
• Gives the correct solution	1

Sample Answer

$0.975P, \, (0.975)^2 P, \, (0.975)^3 P, \ldots\ldots\ldots$

$\therefore T_n = 0.975P \times (0.975)^{n-1}$

$\qquad = (0.975)^n P$

15 (d) (i)

(β) 1 mark

Outcomes Assessed: H4

Targeted Performance Bands: 3

Criteria	Marks
• Gives the correct solution	1

Sample Answer

$(0.975)^{18} \, P = 158497.75$

$P = \$250\,000$

15 (d) (ii) 3 marks

Outcomes Assessed: H3, H4, H5

Targeted Performance Bands: 5-6

Criteria	Marks
• Gives the correct solution	3
• Uses the log laws to make *n* the subject of the formula	2
• Simplifies expression and takes the log of both sides	1

Sample Answer

$$250\,000(0.975)^n = 128875(1.031)^{n-1}$$

$$\frac{2000}{1031}(0.975)^n = (1.031)^{n-1}$$

$$\frac{2000}{1031} = \frac{(1.031)^{n-1}}{(0.975)^n}$$

$$\ln\left(\frac{2000}{1031}\right) = \ln\left[\frac{(1.031)^{n-1}}{(0.975)^n}\right]$$

$$= \ln(1.031)^{n-1} - \ln(0.975)^n$$

$$= (n-1)\ln(1.031) - n\ln(0.975)$$

$$= n\ln(1.031) - \ln(1.031) - n\ln(0.975)$$

$$\ln\left(\frac{2000}{1031}\right) + \ln(1.031) = n\left[\ln(1.031) - \ln(0.975)\right]$$

$$\therefore n = \frac{\ln\left(\frac{2000}{1031}\right) + \ln(1.031)}{\ln(1.031) - \ln(0.975)}$$

$$\approx 12.41 \text{ years}$$

Question 16 (15 marks)

(a)

(i) 1 mark

Outcomes Assessed: H4

Targeted Performance Bands: 3

Criteria	Marks
• Gives the correct answer using the units metres per year	1

Sample Answer

$$\frac{dh}{dt} = \frac{110}{81} \text{ metres per year}$$

$$\approx 1.36 \text{ metres per year } (2\text{dp})$$

16 (a) (ii) 3 marks

Outcomes Assessed: H5, H8

Targeted Performance Bands: 5-6

Criteria	Marks
• Gives the correct solution	3
• Substitutes $t = 5$ $t = 0$ correctly into h	2
• Integrates correctly	1

Sample Answer

$$h = \int_0^5 \frac{110}{(t+4)^2} dt + 0.5$$

$$= 110 \int_0^5 (t+4)^{-2} dt + 0.5$$

$$= 110 \left[\frac{(t+4)^{-1}}{-1 \times 1} \right]_0^5 + 0.5$$

$$= 110 \left[\frac{-1}{9} + \frac{1}{4} \right] + 0.5$$

$$= \frac{142}{9} \text{m}$$

$$\approx 15.8 \text{m}$$

16 (b) (ii) 3 marks

Outcomes Assessed: H5, H5

Targeted Performance Bands: 5-6

Criteria	Marks
• Gives the correct solution	3
• Integrates correctly	2
• Gives the correct expression for the shaded area i.e. $\int_0^9 \left[8 - 8\sin\left(\frac{\pi}{12}(x-3) \right) \right] dx$	1

Sample Answer

$$Area = \int_0^9 \left[8 - 8\sin\left(\frac{\pi}{12}(x-3)\right) \right] dx$$

$$= \left[8x + \frac{96}{\pi}\cos\left(\frac{\pi}{12}(x-3)\right) \right]_0^9$$

$$= \left[\left(72 + \frac{96}{\pi}\cos\left(\frac{\pi}{12} \times 6\right) \right) - \left(0 + \frac{96}{\pi}\cos\left(\frac{\pi}{12} \times -3\right) \right) \right]$$

$$= 72 - \frac{96}{\pi} \times \frac{\sqrt{2}}{2}$$

$$= 72 - \frac{48\sqrt{2}}{\pi} \text{ units}^2$$

16 (c) (i) 1 mark

Outcomes Assessed: H5

Targeted Performance Bands: 4-5

Criteria	Marks
• Gives the correct explanation	1

Sample Answer

The south facing section of the sun shade has more curves than the north facing section. The Simpson's Rule will give a better approximation for the area as it uses an arc of a parabola between the points rather than a straight line like the Trapezoidal Rule uses.

16 (c) (ii) 2 marks

Outcomes Assessed: H5, H8

Targeted Performance Bands: 4-5

Criteria	Marks
• Gives the correct solution	2
• Substitutes correctly into the Trapezoidal or Simpson's Rule	1

Sample Answer

$$A \approx \frac{5}{2}\left[0+2\left(6+7+6.8+7.2+5\right)+0\right]+\frac{5}{3}\left[0+4\left(7.5+9+8.5\right)+2\left(5.5+5.5\right)+0\right]$$

$$\approx \frac{1090}{3}\,\text{m}^2$$

$$\approx 363\,\text{m}^2 \text{ (to the nearest square metre)}$$

16 (d) (i) 3 marks

Outcomes Assessed: H4, H5

Targeted Performance Bands: 5-6

Criteria	Marks
• Gives the correct solution	3
• Progress towards the correct expression for the amount owing after two fortnights A_2	2
• Converts the interest rate correctly to per fortnight and converts the time to fortnights.	1

Sample Answer

Amount borrowed $=\$1\,500\,000$

$$A_1 = 1\,500\,000\left(1+\frac{0.052}{26}\right)-M$$

$$A_2 = A_1\left(1.002\right)-M$$

$$=\left[1\,500\,000\left(1.002\right)-M\right]\left(1.002\right)-M$$

$$=1\,500\,000\left(1.002\right)^2-\left(1.002\right)M-M$$

$$=1\,500\,000\left(1.002\right)^2-M\left[1+1.002\right]$$

$$\therefore A_3 = 1\,500\,000\left(1.002\right)^3-M\left[1+1.002+1.002^2\right]$$

$$\therefore A_{780}=1\,500\,000\left(1.002\right)^{780}-M\left[1+1.002+1.002^2+\ldots\ldots\ldots\ldots+1.002^{779}\right]$$

$$=1\,500\,000\left(1.002\right)^{780}-M\left[\frac{1.002^{780}-1}{0.002}\right]$$

$$A_{780}=0$$

$$M \approx \$3799.70$$

16 (d) (ii)

(α) 1 mark

Outcomes Assessed: H4, H5

Targeted Performance Bands: 4

Criteria	Marks
• Gives the correct solution	1

Sample Answer

$$A_{390} = 1500\,000\,(1.002)^{390} - 3799.70\left(\frac{1.002^{390}-1}{0.002}\right)$$

$$\approx \$1\,028\,267.51 \text{ still owing}$$

(β) 1 mark

Outcomes Assessed: H4, H5

Targeted Performance Bands: 5-6

Criteria	Marks
• Gives the correct equity using the previous answer in part 1.	1

Sample Answer

$$A = 0.85\,(6\,500\,000 - 1\,028\,267.51)$$

$$= \$4\,650\,972.62$$