Student Name:

South Sydney High School

(ACE TRIAL PAPER)

2017

YEAR 12

TRIAL EXAMINATION

Mathematics Extension 1

General Instructions

- Reading time - 5 minutes
- Working time - 2 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A reference sheet is provided at the back of this paper
- Show relevant mathematical reasoning and/or calculations in Questions 11-14

Total marks - 70

Section I

10 marks

- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section II

60 marks

- Attempt Questions 11-14
- Allow about 1 hour 45 minutes for this section

Section I

10 marks
Attempt Questions 1 - 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10

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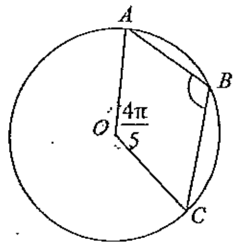
1 The points A, B and C lie on a circle with centre O, as shown in the diagram.

The size of $\angle AOC$ is $\frac{4\pi}{5}$ radians.



Not to scale

What is the size of $\angle ABC$ in radians?

(A) $\frac{3\pi}{10}$

(B) $\frac{\pi}{2}$

(C) $\frac{3\pi}{5}$

(D) $\frac{4\pi}{5}$

2 Which of the following is the exact value of $\int_{\frac{3}{\sqrt{2}}}^{3} \frac{4dx}{\sqrt{9-x^2}}$?

(A) $-\pi$

(B) $-\frac{\pi}{4}$

(C) $\frac{\pi}{4}$

(D) π

3 What are the coordinates of the point that divides the interval joining $P(2, 1)$ and $Q(2, 8)$ internally in the ratio 3: 4?

(A) (1, 7)

(B) (2, 4)

(C) (2, 7)

(D) (4, 2)

4 What is the exact value of the definite integral $\int_0^{\frac{\pi}{3}} \sin^2 x\,dx$?

(A) $\frac{\pi}{3}-\frac{1}{4}$

(B) $\frac{\pi}{3}-\frac{\sqrt{3}}{4}$

(C) $\frac{\pi}{6}-\frac{1}{8}$

(D) $\frac{\pi}{6}-\frac{\sqrt{3}}{8}$

5 How many ways can a football team of eleven be chosen from 15 players?

(A) 15

(B) 165

(C) 1365

(D) 5.44×10^{10}

6 Which integral is obtained when the substitution $u = 1+3x$ is applied to $\int x\sqrt{1+3x}\,dx$?

(A) $\frac{1}{9}\int (u-1)\sqrt{u}\,du$

(B) $\frac{1}{3}\int (u-1)\sqrt{u}\,du$

(C) $\int (u-1)\sqrt{u}\,du$

(D) $3\int (u-1)\sqrt{u}\,du$

7 A particle is moving under SHM in a straight line with an acceleration of $\ddot{x} = 25-5x$, where x is the displacement after t seconds. What is the centre of motion?

(A) $x=0$

(B) $x=5$

(C) $x=10$

(D) $x=15$

8 The function $f(x) = \sin x - \frac{2x}{3}$ has a real root close to $x = 1.5$.

Let $x = 1.5$ be a first approximation to the root.
What is the second approximation to the root using Newton's method?

(A) 1.495

(B) 1.496

(C) 1.503

(D) 1.504

9 Seven children are seated randomly around a circular table.
What is the probability that the two oldest children sit together?

(A) $\frac{5!2!}{6!}$

(B) $\frac{5!}{6!2!}$

(C) $\frac{5!2!}{7!}$

(D) $\frac{5!}{7!2!}$

10 A bottle of water has a temperature of 20°C and is placed in a refrigerator whose temperature is 2°C. The cooling rate of the bottle of water is proportional to the difference between the temperature of the refrigerator and the temperature T of the bottle of water. This is expressed by the equation $\frac{dT}{dt} = -k(T-2)$ where k is a constant of proportionality and t is the number of minutes after the bottle of water is placed in the refrigerator. After 20 minutes in the refrigerator the temperature of the bottle of water is 10°C. What is the value of k in the above equation?

(A) $k = -\frac{1}{20}\log_e \frac{9}{4}$

(B) $k = -\frac{1}{10}\log_e \frac{4}{9}$

(C) $k = \frac{1}{20}\log_e \frac{9}{4}$

(D) $k = \frac{1}{10}\log_e \frac{4}{9}$

Section II

60 marks
Attempt Questions 11 – 14
Allow about 1 hour and 45 minutes for this section

Answer each question in the appropriate writing booklet.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) **Marks**

(a) Find the size of the acute angle between the lines $x-y-4=0$ and $3x-y+4=0$. Answer to the nearest degree. **2**

(b) Solve the inequality $\frac{1}{|x-1|}<1$ **2**

(c) Newton's law of cooling states that when an object at temperature T°C is placed in an environment at temperature T_0°C, the rate of the temperature loss is given by the equation:

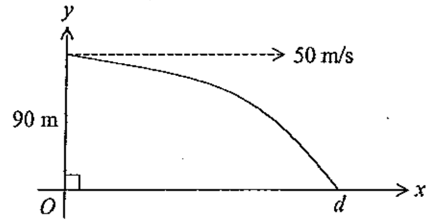
$$\frac{dT}{dt}=-k(T-T_0)$$

where t is the time in minutes and k is a positive constant.

(i) Show that $T=T_0+Ae^{-kt}$ satisfies the above equation. **1**

(ii) An object whose initial temperature is 60°C is placed in a room in which the internal temperature is maintained at 12°C. **3**
After 25 minutes, the temperature of the object is 30°C.
How long will it take for the object's temperature to reduce to 15°C?

(d) The diagram below shows the trajectory of a ball thrown horizontally, at a speed of 50 ms^{-1}, from the top of a tower 90 metres above ground level.



The ball strikes the ground d metres from the base of the tower.

(i) Show that the equations describing the trajectory of the ball are: **2**

$$x=50t \text{ and } y=90-\frac{1}{2}gt^2$$

where g is the acceleration due to gravity and t is the time in seconds.

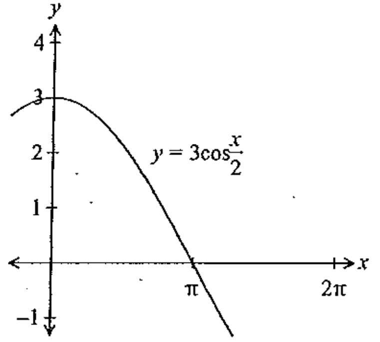
(ii) Prove that the ball strikes the ground at time $t=6\sqrt{\frac{5}{g}}$ seconds. **1**

(iii) How far from the base of the tower does the ball strike the ground? **1**

(e) Find the term independent of x in the binomial expansion of $\left(3x^8-\frac{2}{x^3}\right)^{11}$. **3**

Question 12 (15 marks) **Marks**

(a) The region bounded by the graph $y = 3\cos\frac{x}{2}$ and the x-axis between $x = 0$ and $x = \pi$ is rotated about the x-axis to form a solid. **3**



Find the exact volume of the solid.

(b) $P(2at, at^2)$ is any point on the parabola $x^2 = 4ay$. The line d is parallel to the tangent at P and passes through the focus S of the parabola.

(i) Find the equation of the line d. **3**

(ii) The line d intersects the x-axis at the point R. Find the coordinates of the midpoint, M, of the interval RS. **2**

(iii) Find the equation of the locus of M. **1**

(c) Find $\int \frac{1}{x^2 + 2x + 2}\,dx$ **2**

(d) A particle moves in a straight line so that its acceleration is given by $a = x + 1.5\ \text{ms}^{-2}$. Initially, the particle is 5 metres to the right of O and moving towards O with a velocity of $6\ \text{ms}^{-1}$.

(i) Is the particle speeding up or slowing down? Give a reason. **1**

(ii) Show that $v^2 = x^2 + 3x - 4$. **2**

(iii) Where does the particle first change direction? **1**

Question 13 (15 marks) **Marks**

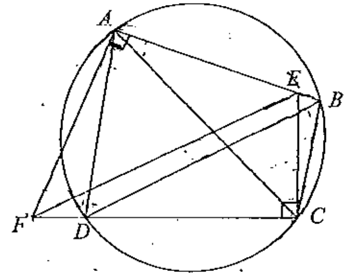
(a) (i) Prove that $\frac{\sec^2 x}{\tan x} = \operatorname{cosec} x \sec x$ **1**

(ii) Use the substitution $u = \tan x$ to find the exact value of this integral: **2**

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \operatorname{cosec} x \sec x\,dx$$

(b) Prove by mathematical induction that $5^n + 12n - 1$ is divisible by 16 for all positive integers n $(n \geq 1)$. **3**

(c) $ABCD$ is a cyclic quadrilateral with $\angle FAE = \angle ECD = 90^\circ$.



(i) Why is $AECF$ a cyclic quadrilateral? **1**

(ii) Hence show that EF is parallel to BD. **3**

(d) It is given that $P(x) = (x-a)^3 + (x-b)^2$ and the remainder when $P(x)$ is divided by $(x-b)$ is -8.

(i) What is the remainder when $P(x)$ is divided by $(x-a)$? **2**

(ii) Prove that $x = \frac{a+b}{2}$ is a zero of $P(x)$. **1**

(iii) Prove that $P(x)$ has no stationary points. **2**

Question 14 (15 marks) **Marks**

(a) A shooter hits the target 87% of the time. In a competition he will have fifty shots at the target.

(i) What is the probability he hits 40 targets? Answer correct to 4 decimal places. **1**

(ii) What is the probability he misses at most two times? Answer correct to 4 decimal places. **2**

(b) Consider the function $f(x) = \frac{x}{x+4}$.

(i) Show that $f'(x) > 0$ for all x in the domain. **1**

(ii) State the equation of the horizontal asymptote of $y = f(x)$. **1**

(iii) Without using any further calculus, sketch the graph of $y = f(x)$. **2**

(iv) Explain why $f(x)$ has an inverse function $f^{-1}(x)$. **1**

(v) Find an expression for the inverse function $f^{-1}(x)$. **1**

(c) A particle is moving in a straight line under SHM. At any time (t seconds) its displacement (x metres) from a fixed point O is given by:

$$x = A\cos\left(\frac{\pi}{4}t + \alpha\right) \text{ where } A > 0 \text{ and } 0 < \alpha < \frac{\pi}{2}$$

After 1 second the particle is 2 metres to the right of O and after 3 seconds the particle is 4 metres to the left of O.

(i) Show that $A\sin\alpha - A\cos\alpha = -2\sqrt{2}$ and $A\sin\alpha + A\cos\alpha = 4\sqrt{2}$ **2**

(ii) Show that $A = 2\sqrt{5}$ and $\alpha = \tan^{-1}\frac{1}{3}$ **2**

(ii) When does the particle first pass through O. **2**

End of paper

ACE Examination 2017
HSC Mathematics Extension 1 Yearly Examination
Worked solutions and marking guidelines

Section I		
	Solution	Criteria
1	Reflex $\angle AOC = 2\pi - \frac{4\pi}{5} = \frac{6\pi}{5}$ $\angle ABC = \frac{1}{2} \times \angle AOC = \frac{1}{2} \times \frac{6\pi}{5} = \frac{3\pi}{5}$ Angle at the centre is twice the angle at the circumference standing on the same arc.	1 Mark: C
2	$\int_{\frac{3}{\sqrt{2}}}^{3} \frac{4}{\sqrt{9-x^2}}dx = 4\times\int_{\frac{3}{\sqrt{2}}}^{3} \frac{1}{\sqrt{9-x^2}}dx = 4\left[\sin^{-1}\frac{x}{3}\right]_{\frac{3}{\sqrt{2}}}^{3}$ $= 4\left[\left(\sin^{-1}\frac{3}{3}\right)-\left(\sin^{-1}\frac{1}{\sqrt{2}}\right)\right]$ $= 4\left[\frac{\pi}{2}-\frac{\pi}{4}\right] = \pi$	1 Mark: D
3	$P(2, 1)$ and $Q(2, 8)$. Internally 3:4. $x = \frac{mx_2+nx_1}{m+n} = \frac{3\times2+4\times2}{3+4} = 2$ $y = \frac{my_2+ny_1}{m+n} = \frac{3\times8+4\times1}{3+4} = 4$ The coordinates are (2, 4).	1 Mark: B
4	$\int_0^{\frac{\pi}{3}} \sin^2 x dx = \int_0^{\frac{\pi}{3}} \frac{1}{2}(1-\cos 2x)dx$ $= \frac{1}{2}\left[x - \frac{1}{2}\sin 2x\right]_0^{\frac{\pi}{3}}$ $= \frac{1}{2}\left[\left(\frac{\pi}{3}-\frac{1}{2}\sin\frac{2\pi}{3}\right)-\left(0-\frac{1}{2}\sin 0\right)\right]$ $= \frac{1}{2}\left[\frac{\pi}{3}-\frac{1}{2}\times\frac{\sqrt{3}}{2}\right] = \frac{\pi}{6}-\frac{\sqrt{3}}{8}$	1 Mark: D
5	Unordered selection ${}^{15}C_{11} = 1365$	1 Mark: C

6	$u = 1+3x$ or $x = \frac{1}{3}(u-1)$ $\frac{du}{dx} = 3$ or $dx = \frac{1}{3}du$ $\int x\sqrt{1+3x}dx = \int \frac{1}{3}(u-1)\sqrt{u}\frac{1}{3}du = \frac{1}{9}\int (u-1)\sqrt{u}\,du$	1 Mark: A
7	$\frac{d^2x}{dt^2} = 25-5x = -5(x-5)$ Centre of motion at $x = 5$ (SHM $\frac{d^2x}{dt^2} = -n^2(x-b)$ with centre of motion at $x = b$)	1 Mark: B
8	$f(x) = \sin x - \frac{2x}{3}$ $f'(x) = \cos x - \frac{2}{3}$ $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$ $= 1.5 - \frac{\sin 1.5 - \frac{2}{3}\times 1.5}{\cos 1.5 - \frac{2}{3}} = 1.49579... \approx 1.496$	1 Mark: B
9	No restrictions $= (7-1)! = 6!$ Arrangements $= 2\times(6-1)! = 2!5!$ $P(E) = \frac{5!2!}{6!}$	1 Mark: A
10	$T = 2 + Ae^{-kt}$ satisfies the equation $\frac{dT}{dt} = -k(T-2)$ Initially $t = 0$ and $T = 20$ $T = 2 + Ae^{-kt}$ $20 = 2 + Ae^{-k\times0}$ $A = 18$ Also $t = 20$ and $T = 10$ $T = 2 + 18e^{-kt}$ $10 = 2 + 18e^{-k\times20}$ $e^{-k\times20} = \frac{8}{18}$ $-20k = \log_e\frac{4}{9}$ $k = -\frac{1}{20}\log_e\frac{4}{9} = \frac{1}{20}\log_e\frac{9}{4}$	1 Mark: C

Section II		
11(a)	$x-y-4=0 \qquad 3x-y+4=0$ $y=x-4 \qquad y=3x+4$ $m_1=1 \qquad m_2=3$ $\tan\theta=\left\|\frac{m_1-m_2}{1+m_1m_2}\right\|=\left\|\frac{1-3}{1+1\times 3}\right\|=\frac{1}{2}$ $\theta=26.56505118...$ $\approx 27^\circ$	2 Marks: Correct answer. 1 Mark: Finds the gradient of the lines or shows some understanding.
11(b)	$\frac{1}{\|x-1\|}<1 \qquad x\neq 1$ $\|x-1\|>1$ $x-1>1$ or $x-1<-1$ $x>2 \qquad x<0$	2 Marks: Correct answer. 1 Mark: Finds one correct region or makes significant progress.
11(c) (i)	$T=T_0+Ae^{-kt}$ or $Ae^{-kt}=T-T_0$ $\frac{dT}{dt}=-kAe^{-kt}$ $=-k(T-T_0)$	1 Mark: Correct answer.
11(c) (ii)	Initially $t=0$ and $T=60$, $T_0=12$ $T=T_0+Ae^{-kt}$ $60=12+Ae^{-k\times 0}$ or $A=48$ Also $t=25$ and $T=30$ $30=12+48e^{-k\times 25}$ $e^{-25k}=\frac{18}{48}=\frac{3}{8}$ $-25k=\log_e\frac{3}{8}$ $k=-\frac{1}{25}\log_e\frac{3}{8}=\frac{1}{25}\log_e\frac{8}{3}$ We need to find t when $T=15$ $15=12+48e^{-kt}$ $e^{-kt}=\frac{3}{48}=\frac{1}{16}$ $-kt=\log_e\frac{1}{16}$ $t=\frac{1}{k}\log_e 16=25\frac{\log_e 16}{\log_e\frac{8}{3}}=70.66950..\approx 71$ minutes	3 Marks: Correct answer. 2 Marks: Finds the value of A and an expression for k. 1 Mark: Finds the value of A.

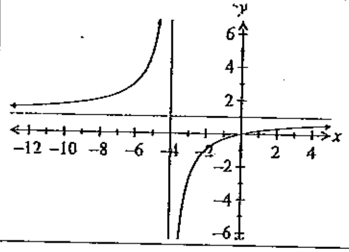
11(d) (i)	Horizontal $\ddot{x}=0$ $\dot{x}=50\cos 0^\circ=50$ $x=50t+c$ When $t=0$, $x=0$ implies $c=0$ $x=50t$ Vertical $\ddot{y}=-g$ $\dot{y}=-gt+50\sin 0^\circ=-gt$ $y=-\frac{1}{2}gt^2+c$ When $t=0$, $y=90$ implies $c=90$ $y=90-\frac{1}{2}gt^2$	2 Marks: Correct answer. 1 Mark: Finds horizontal or vertical parametric equations or shows some understanding of the problem.
11(d) (ii)	Ball strikes the ground $y=0$ $90-\frac{1}{2}gt^2=0$ $\frac{1}{2}gt^2=90$ $t^2=\frac{180}{g}$ $t=\sqrt{\frac{180}{g}}=6\sqrt{\frac{5}{g}}$ as $t>0$	1 Mark: Correct answer.
11(d) (iii)	Ball strikes the ground when $t=6\sqrt{\frac{5}{g}}$ seconds. Now $x=50t$ $d=50\times 6\sqrt{\frac{5}{g}}=300\sqrt{\frac{5}{g}}$ metres	1 Mark: Correct answer.
11(e)	$T_{k+1}={}^{11}C_k(3x^8)^{11-k}\left(-\frac{2}{x^3}\right)^k$ $={}^{11}C_k\times 3^{11-k}\times x^{88-8k}\times(-2)^k\times x^{-3k}$ $={}^{11}C_k(-2)^k\times 3^{11-k}\times x^{88-11k}$ The term independent of x: $88-11k=0$ $k=8$ Required term is ${}^{11}C_8(-2)^8\times 3^{11-8}=1{,}140{,}480$	3 Marks: Correct answer. 2 Marks: Finds the value of k or makes significant progress. 1 Mark: Uses the expression for the general term of a binomial expansion.
12(a)	$V=\pi\int_a^b y^2\,dx=\pi\int_0^\pi 9\cos^2\frac{x}{2}\,dx$ $=\frac{9\pi}{2}\int_0^\pi(1+\cos x)\,dx$ $=\frac{9\pi}{2}[x+\sin x]_0^\pi=\frac{9\pi^2}{2}$ cubic units	3 Marks: Correct answer. 2 Marks: Applies the double angle trig identity. 1 Mark: Sets up the integral for volume

12(b) (i)	To find the gradient of the tangent $y=\frac{1}{4a}x^2$ and $\frac{dy}{dx}=\frac{1}{2a}x$ At $P(2at,at^2)$ $\frac{dy}{dx}=\frac{1}{2a}\times 2at=t$ Line d has a gradient of t and passes through $S(0,a)$ $y-y_1=m(x-x_1)$ $y-a=t(x-0)$ $tx-y+a=0$	3 Marks: Correct answer. 2 Marks: Makes significant progress. 1 Mark: Finds or states the gradient of the tangent at P.
12(b) (ii)	To find the coordinates of R Substitute $y=0$ into $tx-y+a=0$ then $x=-\frac{a}{t}$ $R(-\frac{a}{t},0)$ To find the coordinates of M $x=\frac{x_1+x_2}{2}=\frac{-\frac{a}{t}+0}{2}=-\frac{a}{2t}$ $y=\frac{y_1+y_2}{2}=\frac{0+a}{2}=\frac{a}{2}$ $M(-\frac{a}{2t},\frac{a}{2})$	2 Marks: Correct answer. 1 Mark: Finds the coordinates of R.
12(b) (iii)	To find the equation of the locus eliminate t. However y is independent of t. $y=\frac{a}{2}$	1 Mark: Correct answer.
12(c)	$\int\frac{1}{x^2+2x+2}dx=\int\frac{1}{(x+1)^2+1}dx$ $=\tan^{-1}(x+1)+c$	2 Marks: Correct answer. 1 Mark: Completes the square.
12(d) (i)	Initially $x=5$ and $v=-6$ Acceleration $a=x+1.5=5+1.5=6.5$ Therefore $a>0$ and $v<0$ (different signs) The particle is slowing down.	1 Mark: Correct answer.
12(d) (ii)	$\frac{d}{dx}(\frac{1}{2}v^2)=x+1.5$ $\frac{1}{2}v^2=\frac{1}{2}x^2+1.5x+c_1$ $v^2=x^2+3x+c_2$ When $x=5$, $v=-6$ then $(-6)^2=5^2+3\times5+c_2$ or $c_2=-4$ Therefore $v^2=x^2+3x-4$	2 Marks: Correct answer. 1 Mark: Determines $\frac{1}{2}v^2=\frac{1}{2}x^2+1.5x+c_1$ or makes similar progress.

12(d) (iii)	Particle changes direction when $v=0$ $x^2+3x-4=0$ $(x+4)(x-1)=0$ Particle starts at $x=5$ and is moving to the left ($v=-6$). At $x=1$ the particle is at rest $v=0$ and $a=2.5>0$ It then changes direction and moves to the right ($v>0$) $\therefore x=1$ metres	1 Mark: Correct answer.
13(a) (i)	$\text{LHS}=\frac{\sec^2 x}{\tan x}$ $=\frac{1}{\cos^2 x}\div\tan x$ $=\frac{1}{\cos^2 x}\times\frac{\cos x}{\sin x}$ $=\frac{1}{\cos x}\times\frac{1}{\sin x}$ $=\text{cosec}x\sec x$ $=\text{RHS}$	1 Mark: Correct answer.
13(a) (ii)	$u=\tan x$ $u=\tan\frac{\pi}{3}=\sqrt{3}$ $u=\tan\frac{\pi}{4}=1$ $du=\sec^2 x dx$ $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}\text{cosec}x\sec x\,dx=\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}\frac{\sec^2 x}{\tan x}dx$ $=\int_1^{\sqrt{3}}\frac{1}{u}du$ $=[\log_e u]_1^{\sqrt{3}}$ $=\log_e\sqrt{3}-\log_e 1$ $=\log_e\sqrt{3}$	2 Marks: Correct answer. 1 Mark: Recognises the use of part (i) or makes progress in the substitution.
13(b)	Step 1: To prove the statement true for $n=1$ $5^1+12\times1-1=16$ (Divisible by 16) Result is true for $n=1$ Step 2: Assume the result true for $n=k$ $5^k+12k-1=16P$ where P is an integer (1)	3 Marks: Correct answer. 2 Marks: Proves the result true for $n=1$ and attempts to use the result of $n=k$ to prove the result for $n=k+1$.

	Step 3: To prove the result is true for $n=k+1$ $5^{k+1}+12(k+1)-1=16Q$ where Q is an integer. $\text{LHS}=5^{k+1}+12(k+1)-1$ $=5^{k+1}+12k+11$ $=5(5^k+12k-1)-48k+16$ $=5(5^k+12k-1)+16(1-3k)$ $=5(16P)+16(1-3k)$ from (1) $=16(5P+1-3k)$ $=16Q$ $=\text{RHS}$ Q is an integer as P and k are integers. Result is true for $n=k+1$ if true for $n=k$ Step 4: Result true by principle of mathematical induction.	1 Mark: Proves the result true for $n=1$.
13(c) (i)	$\angle FAE=\angle ECF=90^\circ$ (given) $\therefore AECF$ is a cyclic quadrilateral (if two opposite angles of a quadrilateral are supplementary then the quadrilateral is cyclic)	1 Mark: Correct answer.
13(c) (ii)	$\angle BDC=\angle BAC$ (Angles in the same segment of a circle are equal) $\angle EAC=\angle EFC$ (Angles in the same segment of a circle are equal) $\angle BAC=\angle EAC$ (same angle) $\therefore \angle BDC=\angle EFC$ (corresponding angles are equal if and only if $EF \parallel BD$) Therefore EF is parallel to BD	3 Marks: Correct answer. 2 Marks: Makes some progress towards the solution. 1 Mark: States one relevant statement and circle theorem.
13(d) (i)	Given the remainder of -8 when $P(x)$ is divided by $(x-b)$ $P(b)=(b-a)^3+(b-b)^2$ $=(b-a)^3$ $(b-a)^3=-8$ $b-a=-2$ $a=b+2$ To find the remainder when $P(x)$ is divided by $(x-a)$ $P(a)=(a-a)^3+(a-b)^2$ $=(a-b)^2$ $=(b+2-b)^2$ $=4$ Therefore the remainder is 4.	2 Marks: Correct answer. 1 Mark: Applies the remainder theorem.

13(d) (ii)	If $x=\frac{a+b}{2}$ is a zero of $P(x)$ then the remainder is 0. $P(\frac{a+b}{2})=(\frac{a+b}{2}-a)^3+(\frac{a+b}{2}-b)^2$ $=(\frac{a+b-2a}{2})^3+(\frac{a+b-2b}{2})^2$ $=(\frac{b-a}{2})^3+(\frac{a-b}{2})^2$ $=-1^3(\frac{a-b}{2})^3+(\frac{a-b}{2})^2$ $=-(\frac{a-b}{2})^2\left(\frac{a-b}{2}-1\right)$ $=-(\frac{b+2-b}{2})^2\left(\frac{b+2-b}{2}-1\right)$ $=-(1)(0)=0$	1 Mark: Correct answer.
13(d) (iii)	$P(x)=(x-a)^3+(x-b)^2$ $P'(x)=3(x-a)^2+2(x-b)$ Stationary points occur when $P'(x)=0$ $3(x-a)^2+2(x-b)=0$ $3x^2-6ax+3a^2+2x-2b=0$ $3x^2+(2-6a)x+(3a^2-2b)=0$ $\Delta=b^2-4ac$ $=(2-6a)^2-4\times3\times(3a^2-2b)$ $=4-24a+36a^2-36a^2+24b$ $=4-24a+24b$ $=4-24\times(b+2)+24b$ $=4-24b-48+24b=-44<0$ $P(x)$ has no stationary points.	2 Marks: Correct answer. 1 Mark: Finds the derivative and uses the discriminate.
14(a) (i)	Let p be the probability of hitting the target ($p=0.87$) Let q be the probability of not hitting the target ($q=0.13$) $P(k\text{ successes})={}^nC_k(0.87)^k(0.13)^{n-k}$ $P(40\text{ targets})={}^{50}C_{40}(0.87)^{40}(0.13)^{10}$ ≈ 0.0539	1 Mark: Correct answer.
14(a) (ii)	Misses at most 2 targets then $k=48$, 49 and 50 $P(\text{At most 2 misses})$ $={}^{50}C_{48}0.87^{48}0.13^2+{}^{50}C_{49}0.87^{49}0.13^1+{}^{50}C_{50}0.87^{50}$ $\approx 0.0.339$	2 Marks: Correct answer. 1 Mark: Makes some progress.

14(b) (i)	$f(x)=\dfrac{x}{x+4}$ is defined for all $x\neq -4$ $f'(x)=\dfrac{(x+4)\times 1-x\times 1}{(x+4)^2}=\dfrac{4}{(x+4)^2}>0$ for all $x\neq -4$	1 Mark: Correct answer.
14(b) (ii)	$f(x)=\dfrac{x+4-4}{x+4}$ $=1-\dfrac{4}{x+4}$ As $x\to\pm\infty$ $\dfrac{4}{x+4}\to 0$ Horizontal asymptote is $y=1$	1 Mark: Correct answer.
14(b) (iii)		2 Marks: Correct answer. 1 Mark: Shows asymptotes or basic shape of the curve.
14(b) (iv)	The graph of $y=f(x)$ indicates a one-to-one increasing function (it satisfies the horizontal line test)	1 Mark: Correct answer.
14(b) (v)	The inverse function is $x=\dfrac{y}{y+4}$ $xy+4x=y$ $(1-x)y=4x$ $y=\dfrac{4x}{1-x}$ $f^{-1}(x)=\dfrac{4x}{1-x}$	1 Mark: Correct answer.
14(c) (i)	When $t=1$ then $x=2$ $2=A\cos\left(\dfrac{\pi}{4}\times 1+\alpha\right)$ $=A\left(\cos\dfrac{\pi}{4}\cos\alpha-\sin\dfrac{\pi}{4}\sin\alpha\right)$ $=A\left(\dfrac{1}{\sqrt{2}}\cos\alpha-\dfrac{1}{\sqrt{2}}\sin\alpha\right)$ $2\sqrt{2}=A\cos\alpha-A\sin\alpha$ $A\sin\alpha-A\cos\alpha=-2\sqrt{2}$	2 Marks: Correct answer. 1 Mark: Finds one of the equations or uses the compound angle formula with the given information.

	When $t=3$ then $x=-4$ $-4=A\cos\left(\dfrac{\pi}{4}\times 3+\alpha\right)$ $=A\left(\cos\dfrac{3\pi}{4}\cos\alpha-\sin\dfrac{3\pi}{4}\sin\alpha\right)$ $=A\left(-\dfrac{1}{\sqrt{2}}\cos\alpha-\dfrac{1}{\sqrt{2}}\sin\alpha\right)$ $-4\sqrt{2}=-A\cos\alpha-A\sin\alpha$ $A\sin\alpha+A\cos\alpha=4\sqrt{2}$	
14(c) (ii)	$A\sin\alpha-A\cos\alpha=-2\sqrt{2}$ (1) $A\sin\alpha+A\cos\alpha=4\sqrt{2}$ (2) Adding equations (1) and (2) then $2A\sin\alpha=2\sqrt{2}$ Subtracting equation (1) from (2) then $2A\cos\alpha=6\sqrt{2}$ $(2A\sin\alpha)^2+(2A\cos\alpha)^2=(2\sqrt{2})^2+(6\sqrt{2})^2$ $4A^2(\sin^2\alpha+\cos^2\alpha)=8+72$ $A^2=20$ or $A=2\sqrt{5}$ $\dfrac{2A\sin\alpha}{2A\cos\alpha}=\dfrac{2\sqrt{2}}{6\sqrt{2}}$ $\tan\alpha=\dfrac{1}{3}$ or $\alpha=\tan^{-1}\dfrac{1}{3}$	2 Marks: Correct answer. 1 Mark: Finds A or α. Alternatively shows some understanding of the problem.
14(c) (iii)	Particle passes through O when $x=0$ $A\cos\left(\dfrac{\pi}{4}t+\alpha\right)=0$ $\dfrac{\pi}{4}t+\alpha=\dfrac{\pi}{2},\dfrac{3\pi}{2},\dfrac{5\pi}{2}$ First passes through O $\dfrac{\pi}{4}t+\alpha=\dfrac{\pi}{2}$ $\dfrac{\pi}{4}t+\tan^{-1}\dfrac{1}{3}=\dfrac{\pi}{2}$ $\dfrac{\pi}{4}t=\dfrac{\pi}{2}-\tan^{-1}\dfrac{1}{3}$ $\dfrac{\pi}{4}t=\tan^{-1}3$ $t=\dfrac{4}{\pi}\tan^{-1}3$ Accept $t=2-\dfrac{4}{\pi}\tan^{-1}\dfrac{1}{3}$ or 1.59 seconds	2 Marks: Correct answer. 1 Mark: Finds $\dfrac{\pi}{4}t+\alpha=\dfrac{\pi}{2}$ or shows some understanding.