St George Girls High School

Trial Higher School Certificate Examination

2013

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using blue or black pen
- Begin each question in a new booklet
- Write your student number on each booklet.
- Board-approved calculators may be used.
- A table of standard integrals is provided at the back of this paper.
- Show all necessary working in Questions 11 – 16.
- Diagrams are not to scale.
- The mark allocated for each question is listed at the side of the question.

Total Marks – 100

Section I – Pages 2 – 4
10 marks

- Attempt Questions 1 – 10 using the answer sheet provided at the end of the paper
- Allow about 15 minutes for this section

Section II – Pages 5 – 10
90 marks

- Attempt Questions 11 – 16
- Allow about 2 hours 45 minutes for this section

Students are advised that this is a Trial Examination only and does not necessarily reflect the content or format of the Higher School Certificate Examination.

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

Section I

10 marks
Attempt Questions 1 to 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

1. If the line $4x - ky = 6$ passes through the point $(-1, -2)$. The value of k is:

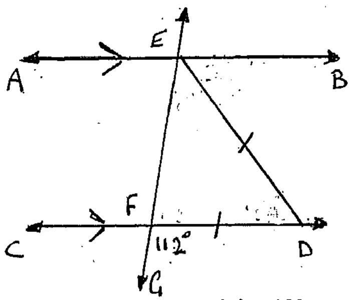
   (A) -1
   (B) 1
   (C) -5
   (D) 5

2. A parabola with equation $(x-2)^2 = 8(y+4)$ has its focus at the point:

   (A) $(2, -4)$
   (B) $(2, -2)$
   (C) $(4, -4)$
   (D) $(4, -2)$

3. From a block of clay exactly 10 statues can be made. If the linear dimensions of the statues are all halved then the number of smaller statues that can be made is:

   (A) 10
   (B) 20
   (C) 40
   (D) 80

4. 

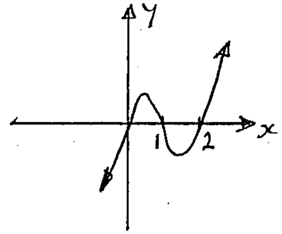
If $AB \| CD$, $ED = FD$ and $\angle DFG = 112°$ then $\angle BED =$

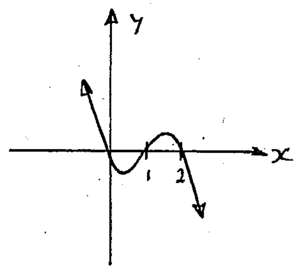
(A) 44° (B) 68° C. 24° D. 112°

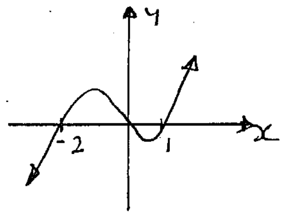
Section I (cont'd)

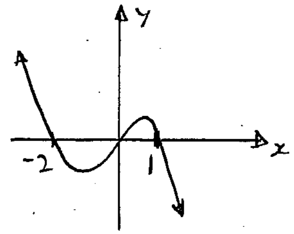
Marks

5. Which graph best illustrates $y = x(x-1)(2-x)$

   (A)


   (B)


   (C)


   (D)


6. $y = 4\sin\frac{1}{2}x$ has amplitude and period of

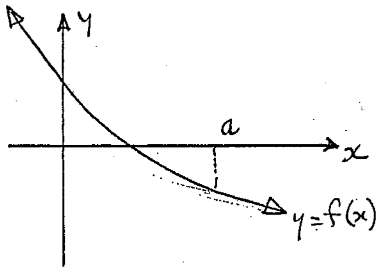
   (A) $4, \frac{1}{2}$
   (B) $4, 2\pi$
   (C) $\frac{1}{2}, 4$
   (D) $4, 4\pi$

7. A particle moves according to the rule $x = \frac{1}{2}t^2 - 4t + c$ where x is the displacement from the origin after t seconds. Initially the particle is 8 metres from the origin. When the particle is at rest its displacement from the origin is:

   (A) 0 metres
   (B) 4 metres
   (C) 8 metres
   (D) 16 metres

Section I (cont'd) Marks

8.



Which of the following is true at $x = a$

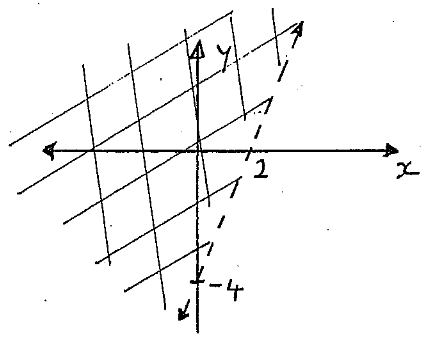
(A) $f'(a) > 0$ and $f''(a) < 0$

(B) $f'(a) > 0$ and $f''(a) > 0$

(C) $f'(a) < 0$ and $f''(a) < 0$

(D) $f'(a) < 0$ and $f''(a) > 0$

9.



The shaded region is best described by the inequality.

(A) $2x - y - 4 < 0$

(B) $2x - y - 4 > 0$

(C) $2x - y - 4 \leq 0$

(D) $2x - y - 4 \geq 0$

10. The perpendicular distance from the line $3x - y = 4$ and the point $(2, 1)$ is given by:

(A) $\frac{9}{\sqrt{5}}$ (B) $\frac{9}{\sqrt{10}}$ (C) $\frac{1}{\sqrt{5}}$ (D) $\frac{1}{\sqrt{10}}$

Section II

90 Marks
Attempt Questions 11 – 16
All about 2 hours and 45 minutes for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

In Questions 11 – 16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 – Start A New Booklet – (15 marks) Marks

a) Evaluate $\frac{1.9^3 - 18}{\sqrt{2.1^4 - 0.8^3}}$ to 2 decimal places. 2

b) Solve $|4 - 2x| \leq 6$ 2

c) Express as a single fraction with a rational denominator $\frac{1}{2-\sqrt{3}} + \frac{1}{2\sqrt{2}+3}$ 3

d) Differentiate $x^3 \ln x$ 2

e) Solve $\tan\theta = \sqrt{3}$ for $0 \leq \theta \leq 2\pi$ 2

f) Factorise $125 - 8p^3$ 2

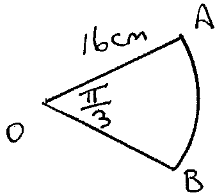
g) Find the primitive of $\frac{6-3x^2}{x^2}$ 2

Question 12 – Start A New Booklet – (15 marks)

Marks

a) Evaluate $e^{3.7}$ correct to 4 significant figures. 2

b)



AB is an arc of the circle centre O. 2
$OA = 16$ cm and $\angle AOB = \frac{\pi}{3}$.
Find the exact area of the sector AOB

c) If 2

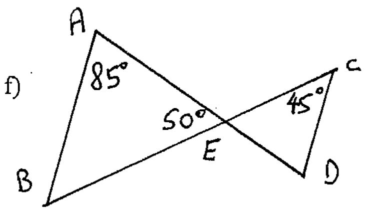
$$f(x) = \begin{cases} x+2 & \text{for } x \le -2 \\ 4 - x^2 & \text{for } -2 < x < 2 \\ 3x - 6 & \text{for } x \ge 2 \end{cases}$$

Evaluate $f(3) + f(0) - f(-2)$

d) Solve $3^{2x} - 6(3^x) - 27 = 0$ 2

e) The first term of an arithmetic series is 5 and the 10th term is 4 times the second. Find the common difference. 2

f)



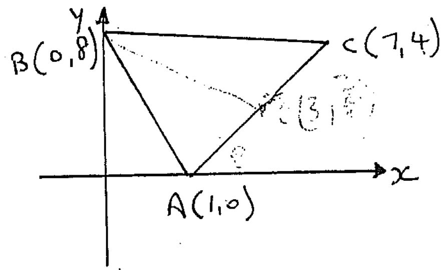
Prove $AB \| CD$, stating all reasons. 2

g) Find $\frac{d}{dx} \log_e(\cos x)$, and hence find $\int \tan x \, dx$ 3

Question 13 – Start A New Booklet – (15 marks)

Marks

a)



The points A, B and C have co-ordinates $(1,0)$, $(0,8)$ and $(7,4)$ as shown on the diagram. The angle between CA and the positive x-axis is $\theta°$.

(i) Find the gradient of CA 1

(ii) Calculate the size of θ, to the nearest degree. 1

(iii) Find the equation of CA 1

(iv) Find the coordinates of D, the midpoint of CB. 1

(v) Show $CA \perp BD$ 2

(vi) Calculate the area of ΔABC 2

b) Prove $2\cos^2\theta + 1 = 3 - 2\sin^2\theta$ 2

c) Differentiate with respect to x

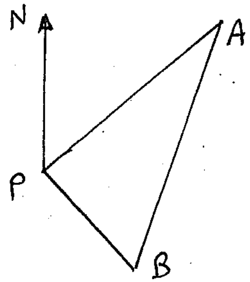
(i) $(2e^x - 3)^6$ 2

(ii) $\frac{x^2}{\sin x}$ 2

d) Integrate xe^{x^2} 1

Question 14 – Start A New Booklet – (15 marks) **Marks**

a)



Ship A sails 20 nautical miles from Port P on a bearing of 035°. Ship B is 36 nautical miles from Port P on a bearing of 110°.

(i) Copy the diagram into your answer booklet and mark on it all the given information. **1**

(ii) Show $\angle APB = 75°$ **1**

(iii) Use the cosine rule to determine the distance between the two ships, to the nearest nautical mile. **2**

b) State the domain and range of $y = \sqrt{1-x}$ **2**

c) The gradient function of a curve is given by $f'(x) = 2(x-1)(x+4)$ and the curve passes through the point $(0, 8)$

(i) Find the equation of $f(x)$ **2**

(ii) Sketch the curve clearly labelling turning points and the y-intercept. **2**

(iii) For what values of x is the curve concave up? **1**

d) Consider the function

$$y = \ln(x-3) \quad x > 3$$

(i) Sketch the function, showing its essential features. **2**

(ii) Use Simpson's Rule with 3 function values to find an approximation to **2**

$$\int_4^6 \ln(x-3)\, dx$$

Question 15 – Start A New Booklet – (15 marks) **Marks**

a) The first three terms of a GP are 0.1, 0.12, 0.144

(i) Find the 40th term, correct to 1 decimal place. **2**

(ii) Calculate the sum of the first 40 terms, correct to 1 decimal place. **2**

b) A particle moves in a straight line so that its displacement, in metres, is given by:

$$x = \frac{t-4}{t+1}$$

where t is measured in seconds.

(i) What is the displacement when $t = 0$ **1**

(ii) Show that $x = 1 - \frac{5}{t+1}$, and hence find expressions for the velocity and acceleration in terms of t **3**

(iii) Is the particle ever at rest? Give a reason for your answer. **1**

c) The population of a certain insect is growing exponentially according to

$$N = 400e^{kt}$$

where t is the time in weeks after the insects are first counted. At the end of five weeks the insect population has doubled.

(i) Calculate the exact value of k. **2**

(ii) How many insects will there be after 8 weeks? **1**

(iii) At what rate is the population increasing after 5 weeks. **1**

d) (i) Write down the discriminant of $2x^2 + 4x + k$ **1**

(ii) For what values of k does $2x^2 + 4x + k = 0$ have real roots. **1**

Question 16 – Start A New Booklet – (15 marks) Marks

a) (i) Find the equations of the tangent and normal to the curve with equation

$$y = 4x^2(1-x)$$

at the point $(1,0)$. 3

(ii) The tangent and normal cut the y-axis at A and B respectively. If the point of intersection of the tangent and the normal is C find the area of ΔABC. 2

b) (i) Sketch the graphs of $y = e^{-x}$, $y = x + 1$ and $x = 2$ on the same set of axes. 1

(ii) Find, by integration, the area bounded by $y = e^{-x}$, $y = x + 1$ and $x = 2$ (leave your answer in exact form) 2

c) Find:

(i) $\lim_{x \to 3} \dfrac{x^2 - 9}{x - 3}$ 1

(ii) $\lim_{x \to 0} \dfrac{\sin x}{x}$ 1

d) A store offers a special deal where it will charge no interest on loans on purchases for the first year, and charge 1% per month on the balance owing each month thereafter. However, normal repayments must be made at the end of each month. Emily decides to buy a \$4000 television using the special deal.

She agrees to repay the loan over 24 equal monthly repayments of $\$M$. Let $\$A_n$ be the amount owing at the end of the nth month.

(i) Find an expression for A_1 and A_{12} 2

(ii) Show $A_{15} = (4000 - 12M)(1.01)^3 - M(1 + 1.01 + 1.01^2)$ 1

(iii) Find the value of M 2

End of Paper

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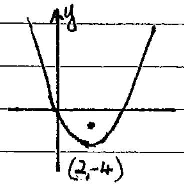
Section I Multiple Choice

1. D
2. B
3. D
4. A
5. B
6. D
7. A
8. D
9. A
10. D

1. $4x - 1 - kx - 2 = 6$
$-4 + 2k = 6$
$2k = 10$
$k = 5$

2.


$4a = 8$
$a = 2$
$F(2, -2)$

3. $V = \left(\frac{1}{2}\right)^3$ of original
$= \frac{1}{8}$ of original
$\therefore$ # of statues $= 8 \times 10 = 80$

4. $\angle DFE = 180^\circ - 112^\circ$
$= 68^\circ$
$\angle DEF = 68^\circ$
$\therefore x + 68 = 112$
$x = 44$

5. $y = 0$ when $x = 0, 1, 2$
When $x < 0$ $y > 0$
$\therefore$ B (not A)

6. Amplitude 4
Period $= \frac{2\pi}{\frac{1}{2}}$
$= 4\pi$

7. When $t = 0$ $x = 8$
$8 = 0 - 0 + c$
$x = \frac{1}{2}t^2 - 4t + 8$
$\dot{x} = t - 4$
At rest when $t = 4$
$x = \frac{1}{2} \times 4^2 - 4 \times 4 + 8$
$= 0$

8. Negative gradient $f'(a) < 0$
Concave up $f''(a) > 0$

9. Test $(0,0) \Rightarrow A$

10. $d = \frac{|3 \times 2 - 1 - 4|}{\sqrt{3^2 + (-1)^2}}$
$= \frac{|6 - 1 - 4|}{\sqrt{10}}$
$= \frac{1}{\sqrt{10}}$

Question 11

(a) $-2.5602295\ldots$
$= -2.56$ (2 dp)

(b) $|4 - 2x| \leq 6$
$-6 \leq 4 - 2x \leq 6$
$-10 \leq -2x \leq 2$
$5 \geq x \geq -1$
$-1 \leq x \leq 5$

(c) $\frac{1}{2 - \sqrt{3}} + \frac{1}{2\sqrt{2} + 3}$
$= \frac{1}{2 - \sqrt{3}} \times \frac{2 + \sqrt{3}}{2 + \sqrt{3}} + \frac{1}{2\sqrt{2} + 3} \times \frac{2\sqrt{2} - 3}{2\sqrt{2} - 3}$
$= \frac{2 + \sqrt{3}}{4 - 3} + \frac{2\sqrt{2} - 3}{8 - 9}$
$= \frac{2 + \sqrt{3}}{1} + \frac{2\sqrt{2} - 3}{-1}$
$= 2 + \sqrt{3} - 2\sqrt{2} + 3$
$= 5 + \sqrt{3} - 2\sqrt{2}$

(d) $\frac{d}{dx}(x^3 \ln x)$
$= 3x^2 . \ln x + x^3 . \frac{1}{x}$
$= 3x^2 \ln x + x^2$

(e) $\tan\theta = \sqrt{3}$ $0 \leq \theta \leq 2\pi$
$\theta_{acute} = \frac{\pi}{3}$
$\theta = \frac{\pi}{3}, \pi + \frac{\pi}{3}$
$= \frac{\pi}{3}, \frac{4\pi}{3}$

(f) $125 - 8p^3$
$= (5 - 2p)(25 + 10p + 4p^2)$

(g) Primitive of $\frac{6 - 3x^2}{x^2}$
$= \int 6x^{-2} - 3 \, dx$
$= \frac{6x^{-1}}{-1} - 3x + c$
$= -\frac{6}{x} - 3x + c$

Question 12

(a) $e^{3.7}$

$= 40.447304\ldots$

$= 40.45$ (4 sig figs)

(b) $A = \frac{1}{2}r^2\theta$

$= \frac{1}{2} \times 16^2 \times \frac{\pi}{3}$

$= \frac{128\pi}{3}$

Area is $\frac{128\pi}{3}$ cm^2

(c) $f(3) + f(0) - f(-2)$

$= 3\times3-6 + (4-0^2) - (-2+2)$

$= 3 + 4 - 0$

$= 7$

(d) $3^{2x} - 6(3^x) - 27 = 0$

Let $m = 3^x$

$m^2 - 6m - 27 = 0$

$(m-9)(m+3) = 0$

$m = 9, -3$

$3^x = 9$ or $3^x = -3$

$x = 2$ no solution since $3^x > 0$ for all x.

$\therefore x = 2$

(e) Let d be the common diff

$t_2 = a + d$

$= 5 + d$

$t_{10} = a + 9d$

$= 5 + 9d$

$\therefore 5 + 9d = 4(5+d)$

$= 20 + 4d$

$5d = 15$

$d = 3$

(f) $\angle ABE = 180^\circ - (85^\circ + 50^\circ)$ (angle sum of a triangle)

$= 45^\circ$

$\therefore \angle ABE = \angle ECD$

$= 45^\circ$

$\therefore AB \parallel CD$ (alternate angles are equal)

(g) $\frac{d}{dx}\log_e(\cos x) = \frac{1}{\cos x} \cdot -\sin x$

$= -\tan x$

$\therefore \int \tan x\, dx = -\log_e(\cos x) + c$

Question 13

(a)(i) Grad CA $= \frac{4-0}{7-1}$

$= \frac{4}{6}$

$= \frac{2}{3}$

(ii) $\tan\theta = \frac{2}{3}$

$\theta = 34^\circ$ (nearest degree)

(iii) Equation of CA

$y - 0 = \frac{2}{3}(x-1)$

$y = \frac{2}{3}x - \frac{2}{3}$

(iv) Midpoint CA $= \left(\frac{1+7}{2}, \frac{0+4}{2}\right)$

$D = (4, 2)$

(v) Grad BD $= \frac{8-2}{0-4}$

$= \frac{6}{-4}$

$= -\frac{3}{2}$

Grad BD $\times$ Grad CA $= \frac{2}{3} \times -\frac{3}{2}$

$= -1$

$\therefore BD \perp CA$

(vi) $AC = \sqrt{(7-1)^2 + (4-0)^2}$

$= \sqrt{52}$

$BD = \sqrt{(0-4)^2 + (8-2)^2}$

$= \sqrt{52}$

Area $\triangle ABC = \frac{1}{2} \times \sqrt{52} \times \sqrt{52}$

$= 26$ units^2

(b) LHS $= 2\cos^2\theta + 1$

$= 2(1 - \sin^2\theta) + 1$

$= 2 - 2\sin^2\theta + 1$

$= 3 - 2\sin^2\theta$

$=$ RHS.

(c)(i) $\frac{d}{dx}(2e^x - 3)^6$

$= 6(2e^x - 3)^5 \cdot 2e^x$

$= 12e^x(2e^x - 3)^5$

(ii) $\frac{d}{dx}\left(\frac{x^2}{\sin x}\right) = \frac{2x\sin x - x^2 \cdot \cos x}{(\sin x)^2}$

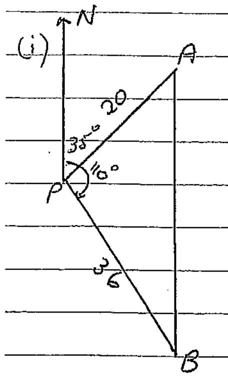
$= \frac{2x\sin x - x^2\cos x}{\sin^2 x}$

(d) $\int x e^{x^2} dx = \frac{1}{2}\int 2x\, e^{x^2} dx$

$= \frac{1}{2}e^{x^2} + c$

Question 14

(i)



(ii) $\angle APB = 110° - 35°$

$= 75°$

(iii) $AB^2 = 20^2 + 36^2 - 2 \times 20 \times 36 \cos 75°$

$= 1323.30\ldots$

$AB = 36.3771\ldots$

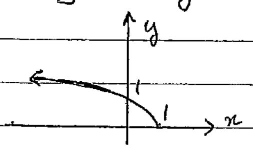
$\therefore$ Ships are 36 nmiles (nearest nm) apart.

(b) $y = \sqrt{1-x}$

Domain: $1 - x \geq 0$

$x \leq 1$

Range: $y \geq 0$



(c) $f'(x) = 2(x-1)(x+4)$

$= 2(x^2 + 3x - 4)$

$= 2x^2 + 6x - 8$

$f(x) = \frac{2x^3}{3} + \frac{6x^2}{2} - 8x + c$

$f(0) = 8 \quad \therefore \quad c = 8$

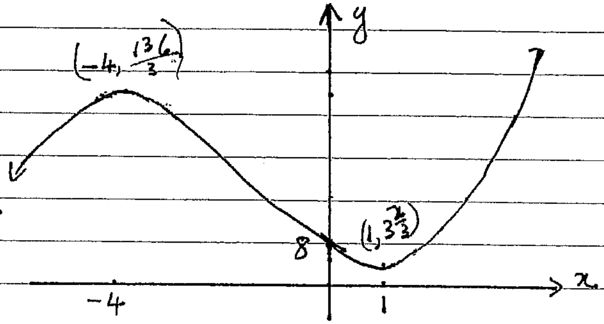
$f(x) = \frac{2}{3}x^3 + 3x^2 - 8x + 8$

(ii) Stationary points occur when $f'(x) = 0$

ie $x = 1$ or $x = -4$

$y = 3\frac{2}{3}$ $\quad y = \frac{136}{3} = 45\frac{1}{3}$

$f(0) = 8$ (y intercept)



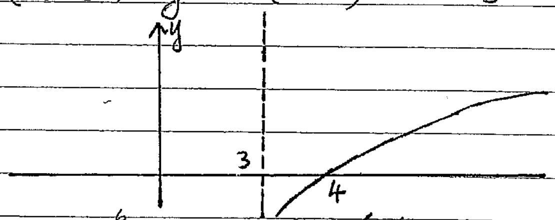
(iii) $f''(x) = 4x + 6$

Curve is concave up when $f''(x) > 0$

$4x + 6 > 0$

$x > -\frac{3}{2}$

(d) (i) $y = \ln(x-3) \quad x > 3$



(ii) $\int_4^6 \ln(x-3)\,dx \doteqdot \frac{h}{3}(y_1 + 4y_2 + y_3)$

$h = 1$

$y_1 = \ln(4-3) = 0$

$y_2 = \ln(5-3) = \ln 2$

$y_3 = \ln(6-3) = \ln 3$

$= \frac{1}{3}(0 + 4\ln 2 + \ln 3)$

$= \frac{1}{3}\ln 48$

Question 15

(a) $a = 0.1 \quad r = 1.2$

(i) $t_{40} = ar^{39}$

$= 0.1 \times 1.2^{39}$

$= 122.480\ldots$

$= 122.5$ (1dp)

(ii) $S_{40} = \frac{a(r^{40} - 1)}{r - 1}$

$= \frac{0.1(1.2^{40} - 1)}{1.2 - 1}$

$= 734.3875\ldots$

$= 734.4$ (1dp)

(b) (i) $x = \frac{t-4}{t+1}$

When $t = 0$ $x = \frac{-4}{1}$

Displacement is -4m

(ii) $1 - \frac{5}{t+1} = \frac{t+1-5}{t+1}$

$= \frac{t-4}{t+1}$

$\therefore x = 1 - \frac{5}{t+1}$

OR $\frac{t-4}{t+1} = \frac{t+1-5}{(t+1)}$

$= \frac{t+1}{t+1} - \frac{5}{t+1}$

$= 1 - \frac{5}{t+1}$

$v = \dot{x} = \frac{d}{dt}(1 - 5(t+1)^{-1})$

$= +5(t+1)^{-2} \times 1$

$= \frac{5}{(t+1)^2}$

$a = \ddot{x} = \frac{d}{dt} 5(t+1)^{-2}$

$= -10(t+1)^{-3} \cdot 1$

$= \frac{-10}{(t+1)^3}$

(iii) $\frac{5}{(t+1)^2} > 0$ for all values of t and hence $v \neq 0$, ie particle is never at rest.

(c) $N = 400e^{kt}$

(i) When $t = 0$ $N = 400e^0 = 400$

When $t = 5$ $N = 2 \times 400 = 800$

$\therefore 800 = 400e^{5k}$

$e^{5k} = 2$

$5k = \log_e 2$

$k = \frac{\log_e 2}{5}$

(ii) When $t = 8$ $N = 400e^{8k}$

$= 400e^{\frac{8\log_e 2}{5}}$

$= 1212.57\ldots$

$\therefore$ There will be 1213 insects after 8 weeks

(iii) $\frac{dN}{dt} = 400ke^{kt}$

$= kN$

When $t = 5$ $N = 800$

$\frac{dN}{dt} = \frac{\log_e 2}{5} \times 800$

$= 110.903\ldots$

Population is increasing at a rate of 111 insects/week.

(d) (i) $\Delta = 4^2 - 4 \times 2 \times k = 16 - 8k$

(ii) $2x^2 + 4x + k = 0$ has real roots when $\Delta \geq 0$

$16 - 8k \geq 0$

$8k \leq 16$

$k \leq 2$

Question 16

(a) (i) $y = 4x^2(1-x)$

$= 4x^2 - 4x^3$

$\frac{dy}{dx} = 8x - 12x^2$

When $x = 1$

$\frac{dy}{dx} = 8 - 12$

$= -4$

$\therefore$ Grad of tangent at $(1,0)$ $= -4$

Grad of normal at $(1,0)$ $= \frac{1}{4}$

Eq^n of tangent is

$y - 0 = -4(x-1)$

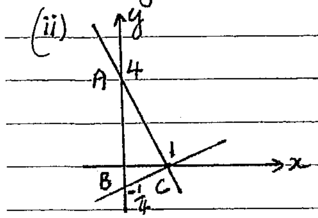
$y = -4x + 4$

Eq^n of normal is

$y - 0 = \frac{1}{4}(x-1)$

$y = \frac{1}{4}x - \frac{1}{4}$

(ii)



$AB = 4\frac{1}{4} = \frac{17}{4}$

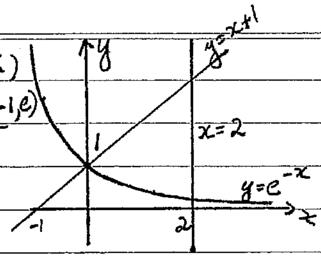
$OC = 1$

Area $\Delta ABC = \frac{1}{2} \times OC \times AB$

$= \frac{1}{2} \times 1 \times \frac{17}{4}$

$= \frac{17}{8}$ units^2

(b) (i) $(-1, e)$



(ii) $A = \int_0^2 x + 1 - e^{-x}\, dx$

$= \left[\frac{x^2}{2} + x + e^{-x}\right]_0^2$

$= \left(\frac{4}{2} + 2 + e^{-2}\right) - (0 + 0 + e^0)$

$= 3 + e^{-2}$

Area $= 3 + \frac{1}{e^2}$ units^2

(c) (i) $\lim_{x\to 3} \frac{x^2-9}{x-3} = \lim_{x\to 3} \frac{(x+3)(x-3)}{(x-3)}$

$= \lim_{x\to 3} (x+3)$

$= 6$

(ii) $\lim_{x\to 0} \frac{\sin x}{x} = 1$

(d) (i) $A_1 = 4000 - M$

$A_2 = A_1 - M = 4000 - 2M$

$A_3 = A_2 - M = 4000 - 3M$

By the same pattern

$A_{12} = 4000 - 12M$

(ii) $A_{13} = A_{12} + \text{interest} - M$

$= A_{12} \times 1.01 - M$

$= 4000 \times 1.01 - 12M \times 1.01 - M$

$A_{14} = A_{13} \times 1.01 - M$

$= 4000 \times 1.01^2 - 12M \times 1.01^2 - M \times 1.01 - M$

$A_{15} = A_{14} \times 1.01 - M$

$= 4000 \times 1.01^3 - 12M \times 1.01^3 - M \times 1.01^2 - M \times 1.01 - M$

(iii) By the same pattern

$A_{24} = (4000 - 12M) \times 1.01^{12} - M \times 1.01^{11} - M \times 1.01^{10} - \ldots - M$

$= (4000 - 12M) \times 1.01^{12} - M(1 + 1.01 + 1.01^2 + \ldots + 1.01^{11})$

$= (4000 - 12M) \times 1.01^{12} - \frac{M.1(1.01^{12} - 1)}{1.01 - 1}$

If loan repaid after 24 months then $A_{24} = 0$

$0 = (4000 - 12M) \times 1.01^{12} - \frac{M(1.01^{12} - 1)}{0.01}$

$100M(1.01^{12} - 1) = (4000 - 12M) \times 1.01^{12}$

$100M(1.01^{12} - 1) + 12M \times 1.01^{12} = 4000 \times 1.01^{12}$

$M[100(1.01^{12} - 1) + 12 \times 1.01^{12}] = 4000 \times 1.01^{12}$

$M = \frac{4000 \times 1.01^{12}}{112 \times 1.01^{12} - 100}$

$= 172.00544\ldots$

Repayments are \$172 per month