ALEXANDRIA PARK COMMUNITY SCHOOL

(WESTERN REGION EXAM)

Year 12 2013

HSC TRIAL EXAMINATION

Mathematics Extension 1

General Instructions

- Reading Time - 5 minutes
- Working Time - 2 hours
- Write using a blue or black pen. Black pen is preferred
- Board approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- Show all necessary working in Questions 11-16

Total marks (70)

Section I

Total marks (10)

- Attempt Questions 1-10
- Answer on the Multiple Choice answer sheet provided
- Allow about 15 minutes for this section

Section II

Total marks (60)

- Attempt questions 11 – 14
- Answer on the blank paper provided, unless otherwise instructed
- Start a new page for each question.
- All necessary working should be shown for every question
- Allow about 1 hour 45 minutes for this section

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\quad n\neq -1;\quad x\neq 0,\ \text{if } n<0$$

$$\int \frac{1}{x}\,dx = \ln x,\quad x>0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax},\quad a\neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax,\quad a\neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax,\quad a\neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax,\quad a\neq 0$$

$$\int \sec ax\tan ax\,dx = \frac{1}{a}\sec ax,\quad a\neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\quad a\neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a},\quad a>0,\quad -a<x<a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right),\quad x>a>0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE: $\ln x = \log_e x,\quad x>0$

Section I
10 marks
Attempt Questions 1-10
Allow about 15 minutes for this section

Use the multiple choice answer sheet for Questions 1 – 10.

1. The height of a giraffe has been modeled using the equation:

$H = 5.40 - 4.80e^{-kt}$

where H is the height in metres, t is the age in years and k is a positive constant. If a 6 year old giraffe has a height of 5.16 metres, find the value of k, correct to 2 significant figures.

(A) $k = 0.05$

(B) $k = 0.24$

(C) $k = 0.50$

(D) $k = 4.8$

2. $\displaystyle\int_5^6 \frac{dx}{\sqrt{x^2-16}} =$

(A) $ln\left(\frac{3+\sqrt{5}}{4}\right)$

(B) $ln\left(\frac{3+\sqrt{5}}{\sqrt{14}}\right)$

(C) $ln\left(\frac{6+2\sqrt{5}}{\sqrt{14}}\right)$

(D) $ln\left(-2+4\sqrt{5}\right)$

3. Find the solution to the inequality: $\dfrac{15}{2x-6} \leq 5$.

(A) $x \leq 3, x \geq 4.5$

(B) $x < 3, x \geq 4.5$

(C) $3 < x \leq 4.5$

(D) $x \leq 3, x > 4.5$

4. What is the acute angle between the lines $x - y + 2 = 0$ and $2x - y - 1 = 0$?

(A) $18^\circ 26'$

(B) $19^\circ 28'$

(C) $70^\circ 32'$

(D) $71^\circ 34'$

5. $\displaystyle\sum_{n=1}^{5} 3n^2 - 2n = ?$

(A) 64

(B) 65

(C) 134

(D) 135

6. Find the coordinates of the point which divides the interval joining A(2, 1) and B(2, 8) internally in the ratio 3: 4.

(A) (1, 7)

(B) (2, 4)

(C) (2, 7)

(D) (4, 2)

7. Find the volume generated when $y = sin\,x$ between $x = 0$ and $x = \frac{\pi}{3}$ is rotated around the x axis.

(A) $\frac{\pi^2}{6} - \frac{3}{8}$ cubic units.

(B) $\frac{\pi^2}{6} - \frac{\sqrt{3}\,\pi}{8}$ cubic units.

(C) $\frac{\pi^2}{3} - \frac{3\pi}{4}$ cubic units.

(D) $\frac{\pi^2}{6} - \frac{\sqrt{3}\,\pi}{4}$ cubic units.

8. What is the domain and range of the function $y = 6\cos^{-1}(3x)$?

(A) Domain $-\frac{1}{3} \le x \le \frac{1}{3}$; Range $0 \le y \le 6\pi$.

(B) Domain $-\frac{1}{3} \le x \le \frac{1}{3}$; Range $0 \le y \le 3\pi$.

(C) Domain $0 \le x \le 6\pi$; Range $-\frac{1}{3} \le y \le \frac{1}{3}$.

(D) Domain $0 \le x \le 3\pi$; Range $-\frac{1}{3} \le y \le \frac{1}{3}$.

9. Using $x = 2$ as an initial approximation to the root of $f(x) = \ln x - \sin x$ use one application of Newton's Method to find a better approximation.

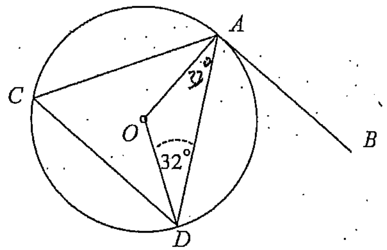
(A) $x = -2.238$

(B) $x = 1.764$

(C) $x = 2.236$

(D) $x = 6.238$

10. AB is a tangent to the circle, centre O.
$\angle ADO = 32^\circ$
Find the size of $\angle DAB$.



(A) 29°

(B) 32°

(C) 37°

(D) 58°

End of Section I

Section II

Total marks (60)
Attempt Questions 11-14
Allow about 1 hour 45 minutes for this section

Answer all questions, starting each question on a new sheet of paper with your name and the question number at the top of the page. Do not write on the back of sheets.

Question 11 (15 marks) Use a separate sheet of paper. **Marks**

a) For the function $f(x) = (x-1)^2$, find the equation for the inverse function $f^{-1}(x)$. Include a sketch of the inverse function $y = f^{-1}(x)$ and state the range of the inverse function. **3**

b) Roger and Jo agree to play 5 sets of tennis against one another in practice for the Austrian Open. Based on past experience, Roger has a 0.6 probability of winning any one set played between them. What is the probability that Roger will win at least three of the five sets? **2**

c) Use the method of mathematical induction to prove that $5^n > 4n + 12$ for all integers $n > 1$. **3**

d) Using the substitution $u = x^3 - 1$, or otherwise, evaluate $\int_1^2 x^2\sqrt{x^3 - 1}\, dx$. **3**

e) i) Express $5\sin x + 12\cos x$ in the form $A\sin(x + \alpha)$ where $0 \le \alpha \le \frac{\pi}{2}$. **2**
(Give the value of α in radians, correct to two decimal places.)

ii) Hence or otherwise, solve $5\sin x + 12\cos x = 8$ for $0 \le x \le \pi$. **2**
(Give the value, or values, of x in radians, correct to two decimal places.)

End of Question 11

Question 12 (15 marks) Use a separate sheet of paper **Marks**

a) i) Find the value of k if $x-4$ is a factor of $P(x) = x^3 - 3kx^2 + 32$. **2**

ii) Using this value of k, fully factorise $P(x) = x^3 - 3kx^2 + 32$. **2**

b) Show that $\sin 3A = 3\sin A - 4\sin^3 A$. **2**

c) Evaluate $\displaystyle\int_0^{\frac{\sqrt{3}}{2}} \frac{dx}{9+4x^2}$, giving your answer as an exact value. **3**

d) A circular metal plate is heated so that its diameter is increasing at a constant rate of 0.005 m/s. At what rate is the area of the circular surface of the plate increasing when its diameter is 6 metres? **3**
(Answer in m^2/s, correct to 2 decimal places.)

e) Tiarne has ten music videos saved in a folder on her laptop, which she can arrange to play in any order. Of these videos, three are by The Script, two are by Foster the People, and the other five are all by different artists.

i) How many arrangements are there of the ten videos? **1**

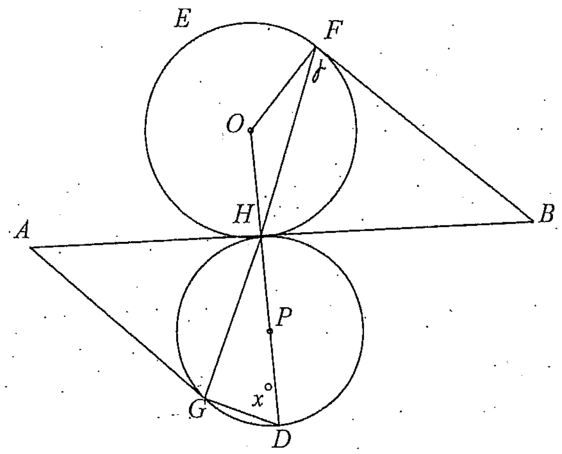
ii) Tiarne decides that when she plays the ten videos those that are by the same artist will play together, in any order. How many arrangements of ten videos are now possible? **1**

iii) Tiarne then decides that she only has time to play 5 videos. The five will start with one by The Script and end with one by Foster the People with three others by different artists between. How many arrangements of five videos are possible? **1**

End of Question 12

Question 13 (15 marks) Use a separate sheet of paper **Marks**

a) The diagram shows two circles with centres O and P respectively, which touch at the point H. OP is produced to meet the smaller circle at D. AB is a common tangent drawn through H. A secant is drawn through H meeting the respective circles at F and G. FB and AG are tangents to the respective circles. GD and OF are joined. $\angle GDH = x°$



i) Show that $\angle HFB = \angle GDH$. **2**

ii) Show that $\angle HAG = 2 \times \angle OFH$ **2**

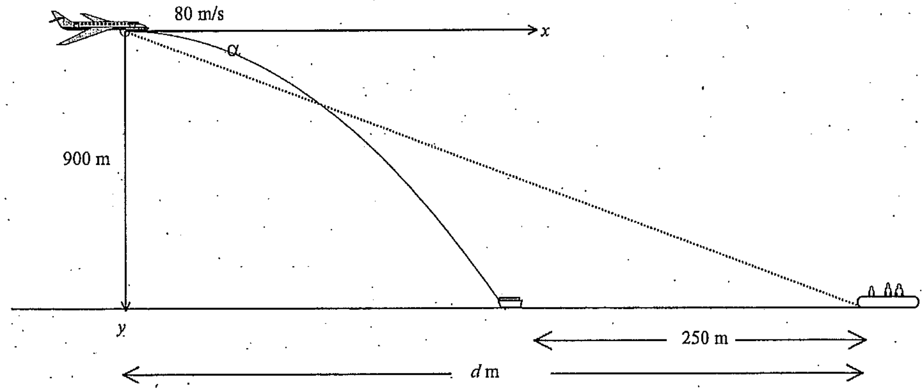
Question 13 continues over the page

Question 13 (continued) **Marks**

b) A rescue plane, flying at a height of 900 m and a speed of 80 m/s is approaching three sailors in a life raft. The plane is to drop a survival package to the raft. The package will be released to fall under gravity from the plane as it approaches the raft.

Taking the origin at the point at which the package is released, the equations of motion of the package as it falls (using gravity as 10 m/s^2) are:

$$x = 80t \quad \text{and} \quad y = -5t^2$$



i) Show that the equation of the trajectory of the package as it falls is $x^2 = -1\,280y$. **2**

ii) Find the time it takes for the package to hit the water. **1**

iii) The package will be released at a horizontal distance d, from the raft so as to hit the water 250 m in front of the raft as the plane approaches. What is the angle of depression (α) of the raft from the plane at the point where the package is to be released? **2**

Question 13 continues over the page

Question 13 (continued) **Marks**

c) A shade sail with corners A, B and C is shown in diagram 1, supported by three vertical posts. The posts at corners A and C are the same height, and the post at corner D is 2.4 m taller. Diagram 2 shows the sail in more detail. D is the point on the taller post horizontally level with the tops of the other two poles. **3**
$AD = 6.4$ m and $DC = 5.2$ m. $\angle ADC = 125°$.
Find the area of the shade sail.

Diagram 1

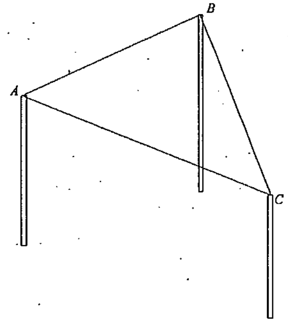
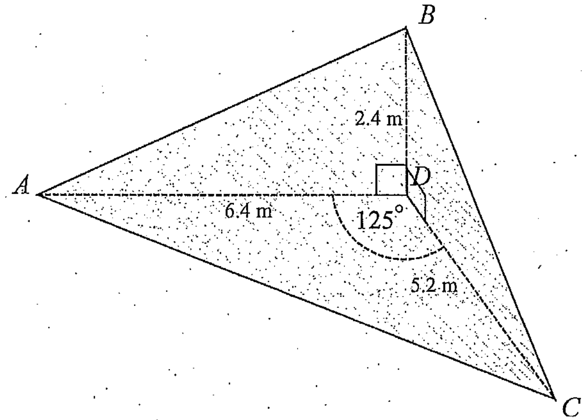


Diagram 2



d) A pack of 48 cards is used in a board game. They are evenly divided into four types which are coloured, red, blue, green and yellow.

i) In how many ways can six cards be selected without replacement from the pack, so that exactly two are blue and four are yellow? (The order in which they are selected is not important.) **1**

ii) In how many ways can six cards be selected without replacement from the pack, so that at least five of the cards are of the same colour? **2**

End of Question 13

Question 14 **(15 marks)** Use a separate sheet of paper **Marks**

a) i) Divide $2x^2$ by $x-3$ and express the result in the form **2**

$$\frac{2x^2}{x-3} = ax + b + \frac{c}{x-3}$$

ii) Without the use of calculus, sketch $y = \frac{2x^2}{x-3}$ showing the vertical and sloping asymptotes. **2**

b) i) Find the value of the coefficient of x^2 in the expansion of $(1+x)^3(1+x)^7$. **1**

ii) Using the expansion of $(1+x)^3(1+x)^7$, show that **2**

$$\binom{3}{0}\binom{7}{5} + \binom{3}{1}\binom{7}{4} + \binom{3}{2}\binom{7}{3} + \binom{3}{3}\binom{7}{2} = \binom{10}{5}$$

c) A particle is moving in a straight line according to the equation:

$$x = 3sin^2(4t)$$

where x is the displacement in metres and t is the time in seconds.

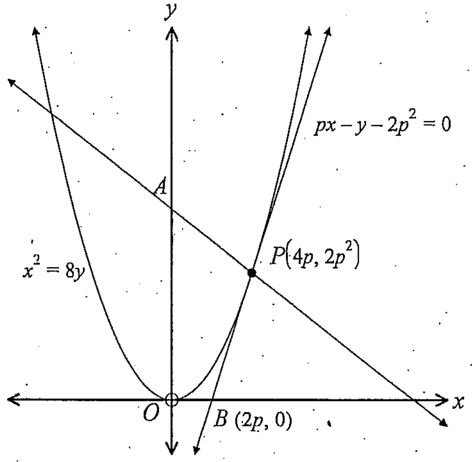
i) Prove that the particle is moving in simple harmonic motion by showing that x satisfies an equation of the form $\ddot{x} = -n^2(x-c)$. **2**

ii) The particle begins from rest at the origin. Find the next four times when it is at rest. **2**

Question 14 continues over the page

Question 14 (continued) **Marks**

d) The graph of the parabola $x^2 = 8y$ is shown below. The tangent at the point $P(4p, 2p^2)$ has equation $px - y - 2p^2 = 0$ and cuts the x axis at $B(2p, 0)$. The normal to the parabola at P, cuts the y axis at A.



i) Show that the equation of the normal at P is $x + py - 2p^3 - 4p = 0$. **1**

ii) Show that the coordinates of A are $(0, 2p^2 + 4)$. **1**

iii) Let C be the midpoint of AB. Find the Cartesian equation of the locus of C. **2**

End of Examination

Trial HSC Examination – Mathematics Extension 1 2013

Section I – Multiple Choice Answer Sheet

Name ______________________________

Allow about 15 minutes for this section

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ~~●~~ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.

A ~~●~~ B ~~●~~ correct C ○ D ○

Start here → 1. A ○ B ○ C ● D ○

2. A ● B ○ C ○ D ○

3. A ○ B ● C ○ D ○

4. A ● B ○ C ○ D ○

5. A ○ B ○ C ○ D ●

6. A ○ B ● C ○ D ○

7. A ○ B ● C ○ D ○

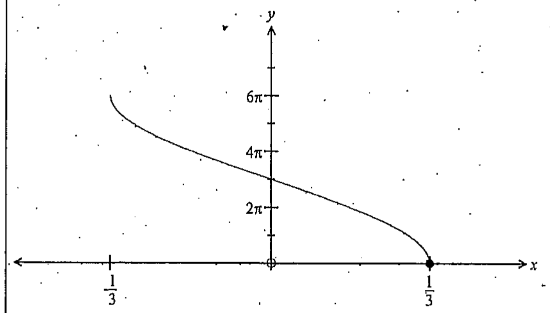
8. A ● B ○ C ○ D ○

9. A ○ B ○ C ● D ○

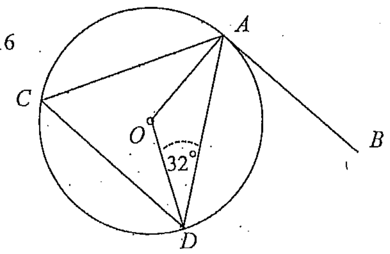
10. A ○ B ○ C ○ D ●

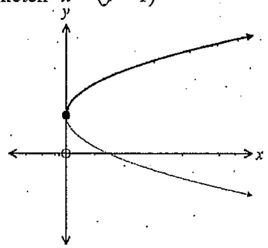
Questions 1 -10	HSC Trial Examination- Maths Ext 1 Worked Solutions to Multiple Choice Questions	2013
1.	$H = 5.40 - 4.80e^{-kt}$ When $t = 6, H = 5.16.$ $5.16 = 5.40 - 4.80 \times e^{-6k}$ $-0.24 = -4.80 \times e^{-6k}$ $0.05 = e^{-6k}$ $ln(0.05) = -6k$ $k = \frac{ln(0.05)}{-6}$ $k = 0.499288..$ $k = 0.50$ (2 sig figs)	**C**
2.	$\int_5^6 \frac{dx}{\sqrt{x^2-16}} = \left[ln\left(x+\sqrt{x^2-16}\right)\right]_5^6$ $= ln\left(6+\sqrt{20}\right) - ln\left(5+\sqrt{9}\right)$ $= ln\left(6+\sqrt{20}\right) - ln(8)$ $= ln\left(\frac{6+2\sqrt{5}}{8}\right)$ $= ln\left(\frac{3+\sqrt{5}}{4}\right)$ By Standard Integrals	**A**
3.	$\frac{15}{2x-6} \le 5$ $\frac{15(2x-6)^2}{2x-6} \le (2x-6)^2.5$ $15(2x-6) \le \left(4x^2-24x+36\right).\ 5$ $30x-90 \le 20x^2-120x+180$ $0 \le 20x^2-150x+270$ $0 \le 2x^2-15x+27$ $2x^2-6x-9x+27 \ge 0$ $2x(x-3)-9(x-3) \ge 0$ $(x-3)(2x-9) \ge 0$ By testing $x < 3,\ x \ge 4.5$	**B**

Questions 1 -10	HSC Trial Examination- Maths Ext 1	2013
	Worked Solutions to Multiple Choice Questions	
4.	Find gradients of both lines. $x-y+2=0$ $\quad$ $2x-y-1=0$? $y=x+2$ $\quad$ $y=2x-1$ $m_1=1$ $\quad$ $m_2=2$ $\tan\theta=\left\|\frac{m_1-m_2}{1+m_1m_2}\right\|$ $=\left\|\frac{1-2}{1+1\times2}\right\|$ $=\left\|\frac{-1}{3}\right\|$ $=\frac{1}{3}$ $\theta=\tan^{-1}\frac{1}{3}$ $=18^\circ26'$	**A**
5.	$\sum_{n=1}^{5} 3n^2-2n = 3(1^2)-2(1)+3(2^2)-2(2)+3(3^2)-2(3)+\ldots$ $=1+8+21+40+65$ $=135$	**D**
6.	$x=\frac{kx_2+lx_1}{k+l}$ $\quad$ $y=\frac{ky_2+ly_1}{k+l}$ $x=\frac{3\times2+4\times2}{3+4}$ $\quad$ $y=\frac{3\times8+4\times1}{3+4}$ $x=\frac{14}{7}$ $\quad$ $y=\frac{28}{7}$ Point is $(2, 4)$	**B**

Questions 1 -10	HSC Trial Examination- Maths Ext 1	2013
	Worked Solutions to Multiple Choice Questions	
7.	When $y=\sin x$ is rotated around the y axis, the volume is given by $V=\pi\int_0^{\frac{\pi}{3}}\sin^2x\,dx$ $=\frac{\pi}{2}\int_0^{\frac{\pi}{3}}1-\cos 2x\,dx$ $=\frac{\pi}{2}\left[x-\frac{1}{2}\sin 2x\right]_0^{\frac{\pi}{3}}$ $=\frac{\pi}{2}\left[\left(\frac{\pi}{3}-\frac{\sqrt{3}}{8}\right)-(0)\right]$ $=\frac{\pi^2}{6}-\frac{\sqrt{3}\,\pi}{8}$	**B**
8.	$y=6\cos^{-1}(3x)$  Domain $-\frac{1}{3}\le x\le\frac{1}{3}$ $\quad$ Range $0\le y\le 6\pi$	**A**
9.	Using $x=2$ as an initial approximation to a root of $f(x)=\ln x-\sin x$ so $f'(x)=\frac{1}{x}-\cos x$ $f(2)=\ln(2)-\sin(2)=-0.216$ (3 dp) and $f'(2)=\frac{1}{2}-\cos(2)=0.916$ (3 dp) $x_2=x_1-\frac{f(x_1)}{f'(x_1)}$ $=2-\frac{-0.216}{0.916}$ $=2+0.236$ $=2.236$	**C**

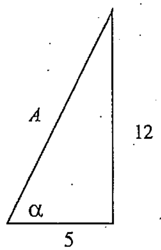
Questions 1 -10	HSC Trial Examination- Maths Ext 1	2013
	Worked Solutions to Multiple Choice Questions	

10.	$\angle OAD = 32^\circ$ $\angle DOA = 180 - 64 = 116$ $\angle DCA = \frac{116}{2} = 58^\circ$ $\angle DAB = \angle DCA = 58^\circ$  	**D**

Question 11	HSC Trial Examination- Maths Ext 1	2013	
Part	Solution	Marks	Comment
(a)	For $f(x) = y = (x-1)^2$ Inverse is obtained from $x = (y-1)^2$ $\pm\sqrt{x} = y - 1$ Sketch $x = (y-1)^2$  For a function take only the positive square root. $y = \sqrt{x} + 1$ Inverse function is $f^{-1}(x) = \sqrt{x} + 1$ Domain is $x \geq 0$ (Not Required for Answer) Range is $y \geq 1$	3	1 mark for inverse function 1 mark for the sketch 1 mark for range
(b)	Let R denote the number of sets that Roger wins. $P(R \geq 3) = P(R=3) + P(R=4) + P(R=5)$ $= \binom{5}{3} \times 0.6^3 . \times 0.4^2 + \binom{5}{4} \times 0.6^4 . \times 0.4^1$ $+ \binom{5}{5} \times 0.6^5 \times 0.4^0$ $= 0.3456 + 0.2592 + 0.07776$ $= 0.68256$ OR $P(R \geq 3) = 1 - [P(R=0) + P(R=1) + P(R=2)]$ $= 1 - \left[\binom{5}{0} \times 0.6^0 . \times 0.4^5 + \binom{5}{1} \times 0.6 . \times 0.4^4\right.$ $\left. + \binom{5}{2} \times 0.6^2 . \times 0.4^3\right]$ $= 1 - [1.01024 + 0.0768 + 0.2304]$ $= 1 - 0.31744$ $= 0.68256$	2	2 marks for correct answer. 1 mark for correct combination of binomial probabilities with a simple calculation error, or if missing or incorrect in one of the binomials.

Question 11	HSC Trial Examination- Maths Ext 1	2013	
Part	Solution	Marks	Comment
(c)	Let $n = 2$ (given $n > 1$) $5^2 > 4(2) + 12$ $25 > 20$ (True) $\therefore$ inequality is true for $n = 2$ Assume true for $n = k$ i.e. $5^k > 4k + 12$ Prove true for $n = k + 1$ Prove $5^{k+1} > 4(k+1) + 12$ $5^{k+1} > 4k + 16$ From assumption $5^k > 4k + 12$ $5(5^k) > 5(4k + 12)$ $5^{k+1} > 20k + 60$ Now for $n > 1$, $20k > 4k$ and $60 > 16$ $\therefore$ $20k + 60 > 4k + 16$ $5^{k+1} > 4k + 16$ $5^{k+1} > 4(k+1) + 12$ So true for $n = k + 1$ Since true for $n = 2$, by induction, it must be true for all integers greater than 2. Hence $5^n > 4n + 12$ for all integers $n > 1$.	3	1 mark for proof of case $n = 2$ 2 marks for proof of case $n = k+1$, based on assumption. Deduct a mark if no conclusion stated.

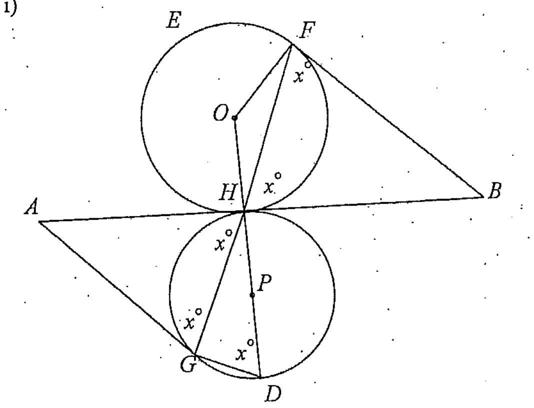
Question 11	HSC Trial Examination- Maths Ext 1	2013	
Part	Solution	Marks	Comment
(d)	$\int_1^2 x^2\sqrt{x^3 - 1}\,dx$ $u = x^3 - 1$ $\quad$ $x = 1, u = x^3 - 1 = 1^3 - 1 = 0$ $\frac{du}{dx} = 3x^2$ $\quad$ $x = 2, u = x^3 - 1 = 2^3 - 1 = 7$ $du = 3x^2 dx$ $\int_1^2 x^2\sqrt{x^3 - 1}\,dx = \frac{1}{3}\int_1^2 \sqrt{x^3 - 1}\,.\,3x^2 dx$ $= \frac{1}{3}\int_0^7 \sqrt{u}\,du$ $= \frac{1}{3}\int_0^7 u^{\frac{1}{2}}\,du$ $= \frac{1}{3}\left[\frac{2}{3}u^{\frac{3}{2}}\right]_0^7$ $= \frac{2}{9}\left[(7\sqrt{7}) - (0)\right]$ $= \frac{14\sqrt{7}}{9}$	3	1 mark for finding boundaries correctly by either method 1 mark for converting to an integral in u. 1 mark for evaluating the integral.

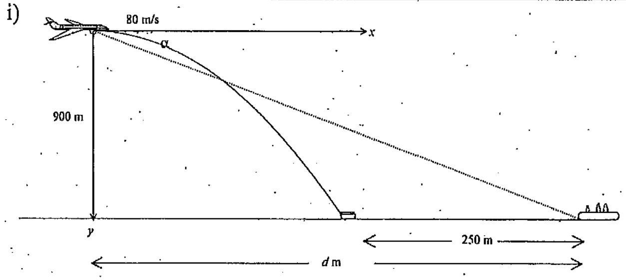
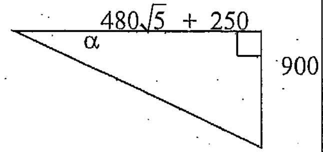
(e)	i) Express $5\sin x + 12\cos x$ in the form $A\sin(x+\alpha)$ $A\sin(x+\alpha) = A\sin x\cos\alpha + A\cos x\sin\alpha$ $= 5\sin x + 12\cos x$ $\therefore\ A\cos\alpha = 5$ and $A\sin\alpha = 12$ $\frac{A\sin\alpha}{A\cos\alpha} = \frac{12}{5}$ $\tan\alpha = \frac{12}{5}$ $\alpha = \tan^{-1}\left(\frac{12}{5}\right)$ $\alpha = 1.18^c$  $A = \sqrt{5^2 + 12^2}$ $= 13$ $5\sin x + 12\cos x = 13\sin(x + 1.18^c)$	2	2 marks for correct result. 1 mark if only α is found correctly 1 mark if only A is found correctly
	ii) $5\sin x + 12\cos x = 8$ $13\sin(x + 1.18^c) = 8$ $\sin(x + 1.18^c) = \frac{8}{13}$ $x + 1.18^c = 0.66,\ 2.48,\ 6.946\ldots$ $x = -0.513,\ 1.303,\ 5.766\ldots$ $x = 1.303$ for $0 \le x \le \pi$	2	Award 2 marks for the correct answers in radians. Award 2 marks if answers are calculated correctly from a wrong answer in i) Award 1 mark if a mistake is made in an otherwise correct solution.

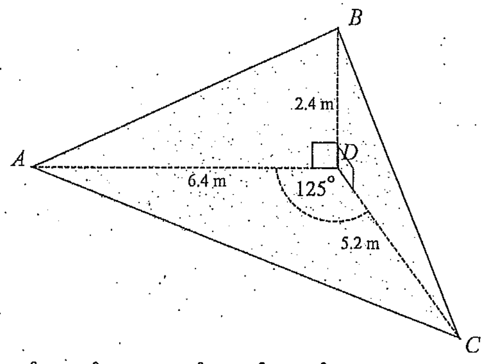
Question 12	HSC Trial Examination- Maths Ext 1		2013
Part	Solution	Marks	Comment
(a)	i) If $x-4$ is a factor of $P(x) = x^3 - 3kx^2 + 32$, then $P(4) = 0$ i.e. $P(4) = 4^3 - 3k \times 4^2 + 32 = 0$ $64 - 48k + 32 = 0$ $48k = 96$ $k = 2$	2	1 mark for substitution 1 mark for finding k
	ii) Want factors of $P(x) = x^3 - 3\times 2\times x^2 + 32 = x^3 - 6x^2 + 32$ $$\begin{array}{r|l} & x^2 - 2x - 8 \\ \hline x-4 & x^3 - 6x^2 \quad\quad + 32 \\ & x^3 - 4x^2 \\ \hline & -2x^2 \\ & -2x^2 + 8x \\ \hline & -8x + 32 \\ & -8x + 32 \\ \hline & 0 \end{array}$$ Factors of $x^2 - 2x - 8$ $x^2 - 2x - 8 = (x-4)(x+2)$ $\therefore P(x) = (x-4)^2(x+2)$	2	Factors can also be found by testing values using the factor theorem, but the double root could be a problem 2 marks for correct factors using either method 1 mark for significant progress, eg finding that $(x+2)$ is a factor.
(b)	$\sin(A+B) = \sin A\cos B + \cos A\sin B$ $\sin(A+A) = \sin A\cos A + \cos A\sin A$ $\sin 2A = 2\sin A\cos A$ $\cos(A+B) = \cos A\cos B - \sin A\sin B$ $\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$ $\sin(A+2A) = \sin A\cos 2A + \cos A\sin 2A$ $= \sin A(1 - 2\sin^2 A) + \cos A(2\sin A\cos A)$ $= \sin A - 2\sin^3 A + 2\sin A\cos^2 A$ $= \sin A - 2\sin^3 A + 2\sin A(1 - \sin^2 A)$ $= \sin A - 2\sin^3 A + 2\sin A - 2\sin^3 A$ $= 3\sin A - 4\sin^3 A$	2	2 marks for a correct derivation. (there will be variations on that given here) 1 mark for significant progress, e.g. correctly using the $\sin(A + 2A)$ and the double angle results, but making an error in algebra.

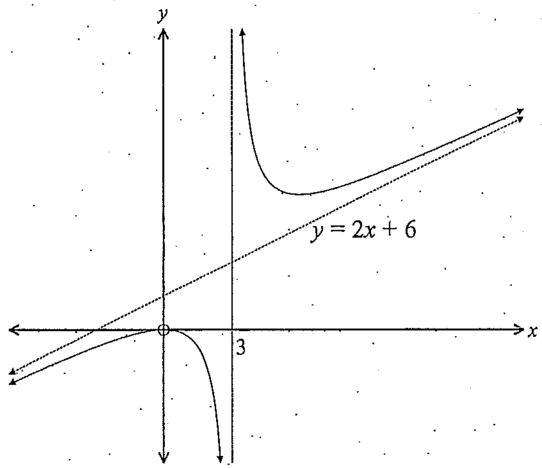
Question 12	HSC Trial Examination- Maths Ext 1	2013	
Part	Solution	Marks	Comment
(c)	$\int_0^{\frac{\sqrt{3}}{2}} \frac{dx}{9+4x^2} = \frac{1}{4}\int_0^{\frac{\sqrt{3}}{2}} \frac{dx}{\frac{9}{4}+x^2}$ $= \frac{1}{4}\left[\frac{2}{3}\tan^{-1}\left(\frac{2x}{3}\right)\right]_0^{\frac{\sqrt{3}}{2}}$ $= \frac{1}{4}\left[\left(\frac{2}{3}\tan^{-1}\left(\frac{2}{3}\times\frac{\sqrt{3}}{2}\right)\right)-\left(\frac{2}{3}\tan^{-1}(0)\right)\right]$ $= \frac{1}{4}\left[\left(\frac{2}{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)\right)-\left(\frac{2}{3}\tan^{-1}(0)\right)\right]$ $= \frac{1}{4}\left[\left(\frac{2}{3}\times\frac{\pi}{6}\right)-\left(\frac{2}{3}\times 0\right)\right]$ $= \frac{\pi}{36}$	3	3 marks for correct answer in exact form. 2 marks for significant progress, with only a minor error not giving the correct answer, or if not given as an exact value. 1 mark if some knowledge is demonstrated of manipulation of the integral to form an integral involving inverse trig
(d)	$A =$ Area of plate $D =$ Diameter of the plate $A = \pi\left(\frac{D}{2}\right)^2$ $A = \frac{\pi D^2}{4}$ $\frac{dA}{dD} = \frac{\pi D}{2}$ $\frac{dD}{dt} = 0.005$ $\frac{dA}{dt} = \frac{dA}{dD}\cdot\frac{dD}{dt}$ $= \frac{\pi D}{2}\times 0.005$ $= 0.0025\pi D$ When D = 6 $\frac{dA}{dt} = 0.0025\pi \times 6$ $= 0.015\pi$ $= 0.047\ m^2/s$	3	3 marks for correct answer rounded appropriately (no marks for rounding) 2 marks for obtaining the correct rates, but not combining them correctly or a simple error. 1 mark if $\frac{dA}{dD}$ is found correctly and attempt made to use it, or some knowledge of related rates shown, but incorrect rates are used.

Question 12	HSC Trial Examination- Maths Ext 1	2013	
Part	Solution	Marks	Comment
(e)	i) There are ten videos which can be arranged in $10! = 3\,628\,800$ ways	1	Okay to leave answers in factorial notation, or permutation notation.
	ii) Consider the Script and Foster as single units, so there are 7 units to arrange in 7!ways. But the two units can be arranged internally in 3! and 2! ways, so the arrangements are $7!\times 3!\times 2! = 60\,480$ ways	1	Okay to leave answers in factorial notation, or permutation notation.
	iii) Choosing 1 Script from 3, 1 Foster from 2 and three others from 5. Number of arrangements $= {}^3P_1 \times {}^2P_1 \times {}^5P_3 = 3\times 2\times 60$ $= 360$ ways	1	Okay to leave answers in factorial notation, or permutation notation.

Question 13	HSC Trial Examination- Maths Ext 1	2013	
Part	Solution	Marks	Comment
(a)	i)  $\angle AGH = \angle GDH = x^\circ$ angle between tangent and chord equals the angle in alternate segment $AH = AG$ tangents from an external point are equal $\angle AHG = \angle AGH = x^\circ$ base angles of isosceles triangle $\angle FHB = \angle AHG = x^\circ$ vertically opposite angles $HB = FB$ tangents from an external point are equal $\angle FHB = \angle HFB = x^\circ$ base angles of isosceles triangle $\therefore \angle GDH = \angle HFB$	2	2 marks for the full proof with reasons. Other alternatives are possible. 1 mark for a start on the proof which includes at least two correct deduced facts with reasons.
	ii) $\angle AHD = 90^\circ$ angle between an tangent and radius $\angle AHG = x^\circ$ from i) $\angle GHD = 90 - x^\circ$ complementary adjacent angles $\angle OHF = \angle GHD = 90 - x^\circ$ vertically opposite angles $OH = OF$ equal radii $\angle OFH = \angle OHF = 90 - x^\circ$ base angles of isos Δ $\angle AHG = \angle AGH = x^\circ$ from i) $\angle HAG = 180^\circ - (\angle AGH + \angle AHG)$ angle sum of isosceles ΔAGH $= 180^\circ - 2x^\circ$ $= 2(90 - x)^\circ$ $= 2 \times \angle OFH$	2	2 marks for the full proof with reasons. Other alternatives are possible. 1 mark for a start on the proof which includes at least two correct deduced facts with reasons.

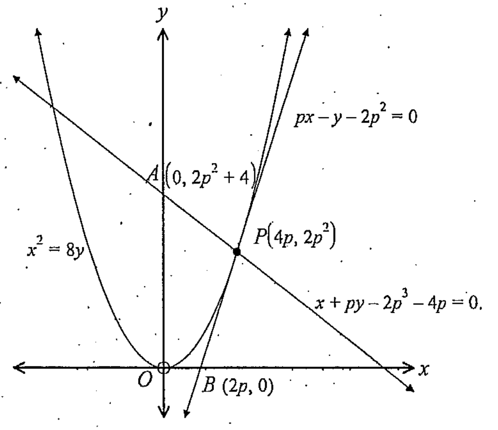
Question 13	HSC Trial Examination- Maths Ext 1	2013	
Part	Solution	Marks	Comment
(b)	i)  $t = \frac{x}{80}$ $y = -5\left(\frac{x}{80}\right)^2$ $6400y = -5x^2$ $x^2 = -1\,280y$	2	2 marks for correct answer. 1 mark if progress made but not achieving final result.
	ii) $y = -5t^2$ $-900 = -5t^2$ $t^2 = 180$ $t = 13.4\ s$ (nearest 10th of second) (or $6\sqrt{5}$ s as exact value)	1	1 mark for correct answer.
	iii) $x^2 = -1\,280y$ $y = -900$ $x^2 = -1\,280(-900)$ $x^2 = 1\,152\,000$ $x = 480\sqrt{5}$ $d = 480\sqrt{5} + 250$ $d = 1\,323\ m$ (nearest m)  Angle of depression $\alpha = tan^{-1}\left(\frac{900}{480\sqrt{5} + 250}\right)$ $\alpha = 34^\circ\,13'$	2	2 marks for correct answer. 1 mark if progress made but not achieving final result, or if an attempt made using equation for y.

Question 13	HSC Trial Examination- Maths Ext 1	2013	
Part	**Solution**	**Marks**	**Comment**
(c)	 $AB^2 = 6.4^2 + 2.4^2$ $\quad BC^2 = 5.2^2 + 2.4^2$ $= 46.72$ $\quad = 32.8$ $AB = 6.8$ $\quad BC = 5.7$ $AC^2 = 6.4^2 + 5.2^2 - 2 \times 6.4 \times 5.2 \times \cos 125°$ $= 106.2$ $AC = 10.3\ m$ $\cos \angle ABC = \dfrac{6.8^2 + 5.7^2 - 10.3^2}{2 \times 6.8 \times 5.7}$ ≈ -0.35 $\angle ABC = 110°40'$ (nearest minute) Area $\Delta ABC = \frac{1}{2} \times 6.8 \times 5.7 \times \sin 110°40'$ $= 18.1\ m^2$ (to 1 dec place)	3	3 marks for correct answer. 2 marks for a substantial attempt which -is not quite complete -involves a minor error 1 mark for an attempt which correctly calculates at least two of the required distances or angle. Or for an attempt which follows the correct reasoning, but involves several calculation errors.
(d)	i) Combinations of 2 blue and 4 yellow $= {}^{12}C_2 \times {}^{12}C_4$ $= 66 \times 495$ $= 32\ 670$	1	1 mark for correct answer
	ii) Combinations with six the same colour $= 4 \times {}^{12}C_6$ $= 4 \times 924$ $= 3\ 696$ Combinations with five the same colour $= 4 \times {}^{12}C_5 \times 3 \times {}^{12}C_1$ $= 4 \times 792 \times 3 \times 12$ $= 114\ 048$ Combinations with at least five the same colour $= 3\ 696 + 114\ 048$ $= 117\ 744$	2	2 marks for correct answer. 1 mark if only found one of the parts, or made an error in finding them.

Question 14	HSC Trial Examination- Maths Ext 1	2013	
Part	**Solution**	**Marks**	**Comment**
(a)	i) $y = \dfrac{2x^2}{x-3}$ $$\begin{array}{r} 2x \;+6 \\ x-3\;\overline{)\,2x^2 \qquad\quad} \\ \underline{2x^2 - 6x} \\ 6x \qquad \\ \underline{6x - 18} \\ 18 \end{array}$$ $y = \dfrac{2x^2}{x-3} = 2x + 6 + \dfrac{18}{x-3}$	2	2 marks for the division correctly completed and the equation written in the form given. 1 mark if a mistake made in the division, or if the equation not written.
	ii) Vertical Asymptote is the line $x = 3$. Sloping asymptote is the line $y = 2x + 6$ (from above). With one discontinuity the graph will have two branches and two turning points, with the graph approaching the sloping asymptote. When $x > 3$ the graph will be above the asymptote and when $x < 3$ the graph is below the asymptote.  	2	2 marks for the asymptotes drawn correctly and the curve shown correctly as two branches around the asymptotes. Exact placement of turning points is not important. 1 mark if either asymptote not shown correctly or if curve not placed correctly relative to asymptotes.

Question 14	HSC Trial Examination- Maths Ext 1	2013	
Part	Solution	Marks	Comment
(b)	i) $(1+x)^3.(1+x)^7$ $\left(\binom{3}{0}+\binom{3}{1}x+\binom{3}{2}x^2+\binom{3}{3}x^3\right)\times\left(\binom{7}{0}+\binom{7}{1}x+\binom{7}{2}x^2+\dots\binom{7}{6}x^6+\binom{7}{7}x^7\right)$ Coefficients of x^3. $\binom{3}{0}\binom{7}{2}+\binom{3}{1}\binom{7}{1}+\binom{3}{2}\binom{7}{0}=1\times 21+3\times 7+3\times 1$ $=45$ Or any alternate method.	1	1 mark for correct answer
	ii) $(1+x)^3.(1+x)^7=(1+x)^{10}$ $\left(\binom{3}{0}+\binom{3}{1}x+\binom{3}{2}x^2+\binom{3}{3}x^3\right)\left(\binom{7}{0}+\binom{7}{1}x+\binom{7}{2}x^2+\dots\binom{7}{6}x^6+\binom{7}{7}x^7\right)$ $=\left(\binom{10}{0}+\binom{10}{1}x+\binom{10}{2}x^2+\dots\dots+\binom{10}{9}x^9+\binom{10}{10}x^{10}\right)$ Equating coefficients of x^5. $\binom{3}{0}\binom{7}{5}+\binom{3}{1}\binom{7}{4}+\binom{3}{2}\binom{7}{3}+\binom{3}{3}\binom{7}{2}=\binom{10}{5}$	2	2 marks for equating coefficients correctly to get result. 1 mark for substantial attempt that uses products of coefficients

(c)	i) $x=3sin^2(4t)$ $\dot{x}=6sin(4t).\ 4cos(4t)$ $\dot{x}=24sin(4t).cos(4t)$ $\ddot{x}=24[(sin(4t)(-4sin(4t))+cos(4t).\ 4cos(4t))]$ $\ddot{x}=24\left(-4sin^2(4t)+4cos^2(4t)\right)$ $\ddot{x}=96\left(-sin^2(4t)+1-sin^2(4t)\right)$ $\ddot{x}=96\left(1-2sin^2(4t)\right)$ $\ddot{x}=96-192sin^2(4t))$ $\ddot{x}=-64\left(3sin^2(4t)-1.5\right)$ $\ddot{x}=-8^2(x-1.5)$ Which is of the form $\ddot{x}=-n^2(x-c)$ So the particle is in simple harmonic motion.		
	ii) The particle has velocity zero when $\dot{x}=24sin(4t).cos(4t)=0$ $\dot{x}=0$ when $sin(4t)=0$ $4t=0,\pi,2\pi,3\pi.$ $t=0,\frac{\pi}{4},\frac{\pi}{2},\frac{3\pi}{4},\dots.$ and when $cos(4t)=0$ $4t=\frac{\pi}{2},\frac{3\pi}{2},\frac{5\pi}{2},\frac{7\pi}{2}\dots.$ $t=\frac{\pi}{8},\frac{3\pi}{8},\frac{5\pi}{8},\frac{7\pi}{8}\dots.$ The first4 times when the particle comes to rest are when: $t=\frac{\pi}{8},\frac{\pi}{4},\frac{3\pi}{8},\frac{\pi}{2}$	2	

(d)			
	i) Equation of normal through $(4p, 2p^2)$. Gradient of tangent = p $\therefore$ Gradient of normal $= -\frac{1}{p}$ Equation of normal is given by $y - 2p^2 = -\frac{1}{p}(x - 4p)$ $y - 2p^2 = -\frac{x}{p} + 4$ $py - 2p^3 = -x + 4p$ $x + py - 2p^3 - 4p = 0$	1	1 mark for correct answer
	ii) Substitute $x = 0$ into $x + py - 2p^3 - 4p = 0$. $py - 2p^3 - 4p = 0$ $py = 2p^3 + 4p$ $y = 2p^2 + 4$ $A(0, 2p^2 + 4)$	1	1 mark for correct answer
	iii) $C\ (x, y)$ is the midpoint of AB $x = \frac{2p+0}{2} = p$ $y = \frac{0 + 2p^2 + 4}{2} = p^2 + 2$ $\therefore$ $x = p$ and $y = p^2 + 2$ Substitute $x = p$ into y $y = x^2 + 2$	2	2 marks for correct answer. 1 mark for obtaining the midpoint correctly, or for substitution of an incorrect midpoint correctly.