Name__________

ASCHAM SCHOOL

MATHEMATICS TRIAL EXAMINATION 2015

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using **black** pen.
- Board-approved calculators may be used.
- A table of standard integrals is provided at the back of this paper.
- Show all necessary working in Questions 11–16.

Section I — 10 marks

- Attempt Questions 1–10 using the Multiple Choice sheet.
  Detach the Multiple Choice sheet from the back of this booklet.
- Allow about 15 minutes for this section.

Section II — 90 marks

- Attempt Questions 11–16.
- Allow about 2 hours 45 minutes for this section.
- Do each question in a separate booklet.
- Write your name/number and your teacher's name on each booklet.

Clearly label the front of each booklet with the number of the question.

Collection

- Start each question of Section II in a new booklet.
- If you use a second booklet for a question, place it inside the first.
  Indicate on the outside of the first booklet that you have used two booklets for that question.
- Write your name/number, teacher's name and question number on each booklet.

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\ n \neq -1;\ x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x,\ x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax},\ a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax,\ a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax,\ a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax,\ a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax,\ a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\ a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a},\ a > 0,\ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right),\ x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE: $\ln x = \log_e x,\ x > 0$

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet at the back of this exam paper for Questions 1 – 10

1 The value of $\log_2 \sqrt{8}$ is:

(A) 2 (B) $\frac{2}{3}$ (C) 3 (D) $\frac{3}{2}$

2 The exact value of $\sin\frac{5\pi}{4}$ is:

(A) $-\frac{1}{\sqrt{2}}$ (B) $\frac{1}{\sqrt{2}}$ (C) $\frac{1}{2}$ (D) $225°$

3 Given $5^x = 4$, find the value of 5^{1-2x}

(A) -11 (B) $\frac{1}{20}$ (C) $\frac{4}{5}$ (D) $\frac{5}{16}$

4 Given $\log_a x = 0.528$ and $\log_a y = 0.176$, find the value of $\log_a\left(\frac{y}{x}\right)$.

(A) -0.352 (B) 0.352 (C) $\frac{1}{3}$ (D) 3

5. Which of the following define the domain and range of the function $f(x) = \log_e x$?

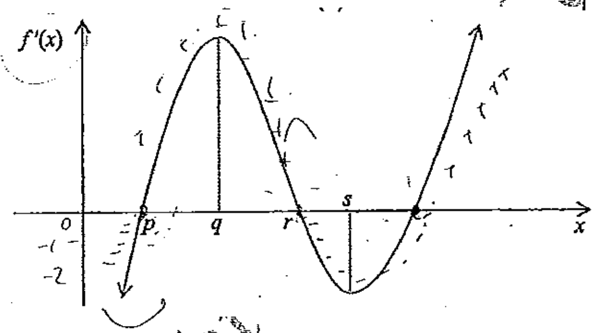
(A) Domain: all real x and Range: all real y.

(B) Domain: $x > 0$ and Range: $y > 0$.

(C) Domain: all real x and Range: $y > 0$.

(D) Domain: $x > 0$ and Range: all real y.

6. The diagram below shows the graph of the derivative $y = f'(x)$.



What is the x value of the minimum turning point on $y = f(x)$?

(A) p (B) q (C) r (D) s

7. What is the primitive of $\frac{3}{x} - \sin x$?

(A) $3\ln x + \cos x + C$ (B) $3\ln x - \cos x + C$

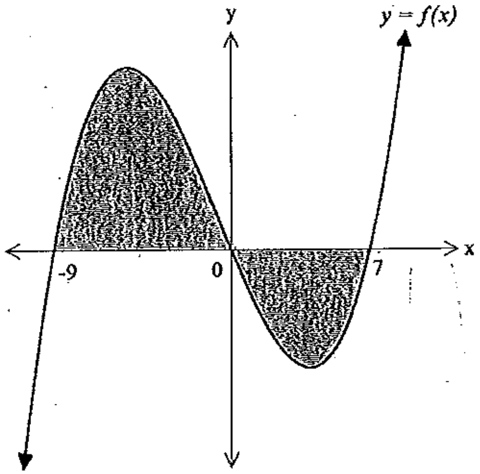
(C) $\frac{-3}{x^2} + \cos x + C$ (D) $\frac{-3}{x^2} - \cos x + C$

8. What is the limiting sum of this sequence?

$$45-15+5-1\frac{2}{3}+\ldots$$

(A) $33\frac{3}{4}$ (B) $34\frac{1}{6}$ (C) $66\frac{2}{3}$ (D) $67\frac{1}{2}$

9. Consider the diagram below.



Which of the following represents the shaded area?

(A) $\int_{-9}^{7} f(x)\,dx$

(B) $2\int_{0}^{7} f(x)\,dx$

(C) $\int_{-9}^{0} f(x)\,dx - \int_{0}^{7} f(x)\,dx$ ✓

(D) $\int_{0}^{7} f(x)\,dx + \int_{-9}^{0} f(x)\,dx$

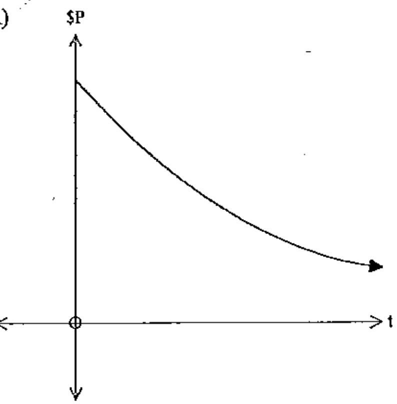
10. The price of one gram of gold, $\$P$, was observed over the period of t days.

Through the period of observation it was noted that:

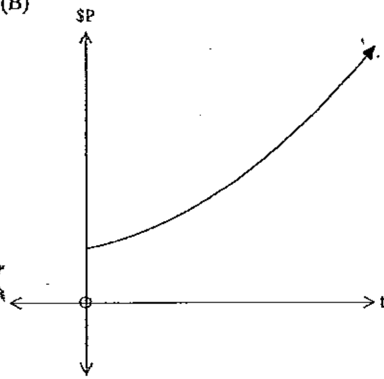
- $\frac{dP}{dt} > 0$
- The rate of change of the price of gold was decreasing.

Which of the following graphs could be used to represent this situation?

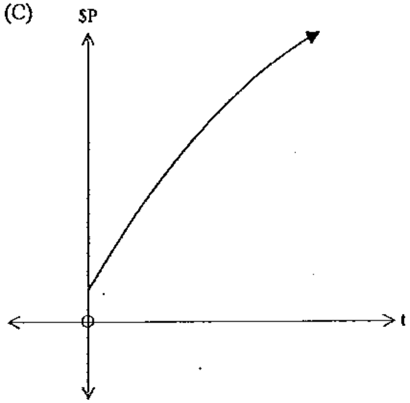
(A)



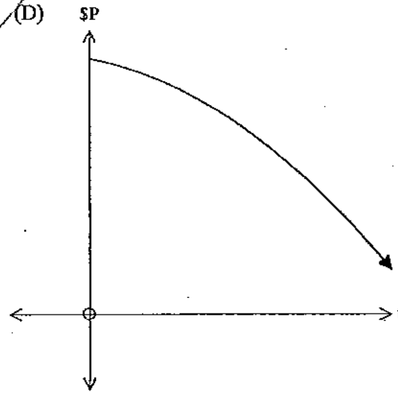
(B)



(C)



(D)



End of Multiple Choice.

Section II

90 marks
Attempt Questions 11 – 16
Allow about 2 hours and 45 minutes for this section.

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

In Questions 11 – 16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use the Question 11 Writing Booklet.

(a) Factorise fully: $3x^3 + 24$. **1**

(b) Rationalise the denominator and simplify: $\dfrac{3}{2-\sqrt{5}}$ **2**

(c) Consider the arithmetic series $5 + 12 + 19 + \ldots + 292$.

(i) How many terms are in the series? **2**

(ii) Find the sum of the series. **1**

(d) Differentiate y, with respect to x.

(i) $y = (5 - 3x^2)^7$ **2**

(ii) $y = 2x^2 \tan x$ **2**

(iii) $y = \log_e(3x^2 - 5x)$ **2**

(iv) $y = \dfrac{6x^2}{\sqrt{2x+1}}$ **3**

End of question 11.

Question 12 (15 marks) Start a new booklet.

(a) Evaluate $\sum_{n=3}^{5}(2n-1)^2$. **2**

(b) A sector with radius 4 cm has an arc length of 16 cm. Find the area of the sector. **2**

(c) The directrix of a parabola is the x-axis and the focus is the point $(1, 4)$.

(i) Write down the focal length of the parabola. **1**

(ii) Find the equation of the parabola. **2**

(d) Shade the region in the plane defined by $x^2 + y^2 \leq 4$ and $y > x - 1$.
Do not find the coordinates of the points of intersection. **3**

(e) Find the equation of the tangent to the curve $y = e^{2x}$ at the point where $x = 0$. **3**

(f) Solve the following equation for x: $3e^{3x} - e^{2x} = 0$. **2**

End of question 12.

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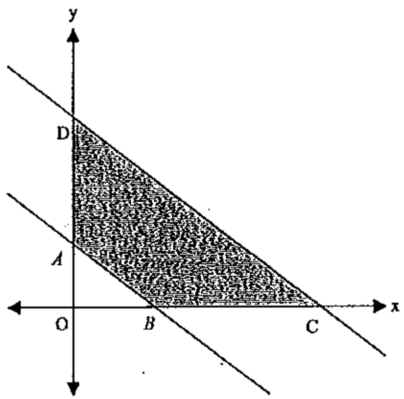
Question 13 **(15 marks)** Start a new booklet.

(a) The diagram shows the straight line AB with equation $x+2y=2$.



(i) Show that the equation of the line DC which is parallel to AB and passes through the point $(2,2)$ is $x+2y=6$. **1**

(ii) Calculate the distance between AB and DC. **2**

(iii) Calculate the area of the trapezium $ABCD$. **2**

(iv) Write down a set of inequalities that uniquely defines the region $ABCD$. **2**

(b) Consider the function $f(x)=x^3-x^2-x+1$.

(i) By using the grouping in pairs method, or otherwise, factorise x^3-x^2-x+1. **1**

(ii) Hence, state the x-intercepts of $f(x)$. **1**

(iii) Determine the coordinates and the nature of the stationary points on $y=f(x)$. **2**

(iv) Find the coordinates of the point(s) of inflexion of $y=f(x)$. **2**

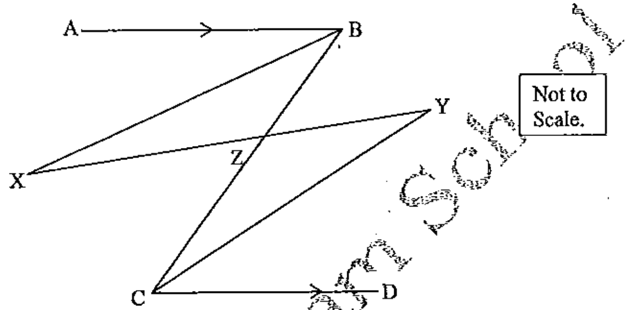
(v) Hence, sketch the graph of $y=f(x)$ on a number plane, showing all essential features. **2**

End of question 13.

Question 14 **(15 marks)** Start a new booklet.

(a) Solve $4\cos^2 x=3$ for $-\pi \le x \le \pi$. **2**

(b) In the diagram below, AB is parallel to CD.
XB bisects $\angle ABC$ and YC bisects $\angle BCD$. $BX=CY$.



Copy the diagram.
Prove that Z is the midpoint of BC. **3**

(c) Find $\int \frac{4x}{4-x^2}\,dx$. **2**

(d) Determine if $f(x)=x\sin x-\cos x$ is odd, even, or neither. **2**

(e) Use Simpson's rule, with 5 function values to approximate **3**

$$\int_0^2 \sqrt{x^2+4x}\,dx.$$

(f) For what values of k does the equation $3x^2-kx+3=0$ have real and unequal roots? **3**

End of question 14.

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Question 15 **(15 marks)** Start a new booklet.

(a) The rate of growth of a certain population of cockroaches is given by the equation $\frac{dP}{dt} = 200(0.4 - 0.08t)$ where P is the population after t months.
Initially there are 200 cockroaches.

(i) Show that the population after t months is $P = 200(1 + 0.4t - 0.04t^2)$. **2**

(ii) In how many months will the initial population double itself? **2**

(b) In a chemical reaction the amount (M kilograms) of undissolved solid after t hours is given by $M = Ae^{-kt}$. A chemical reaction starts with 20 kg of solid and 5 kg remains after 24 hours.

(i) Find the values of A and k. **2**

(ii) Find the amount of undissolved solid after 3 hours (give your answer to the nearest gram). **1**

(iii) Find the time taken to dissolve 19 kg of solid (give your answer correct to the nearest hour). **2**

Question 15 continues on page 11.

Question 15 – continued.

(c) Kenneth borrows \$80 000 in order to buy an apartment. The interest rate is 6% per annum reducible and the loan is to be repaid in equal repayments of \$$M$ at the end of each month over 20 years with the interest compounded monthly.
Let \$$A_n$ be the amount owing after the nth repayment.

(i) Write down expressions for \$$A_1$ and \$$A_2$, the amounts owing after the first and second repayments respectively. **1**

(ii) Show that the amount of each monthly repayment is \$573.14 (correct to the nearest cent). **2**

(iii) After $2\frac{1}{2}$ years (i.e. 30 repayments) the interest rate rises to $7\frac{1}{2}\%$ p.a.
Find the new monthly repayment correct to the nearest cent.
(Assume that the total duration of the loan is still 20 years.) **3**

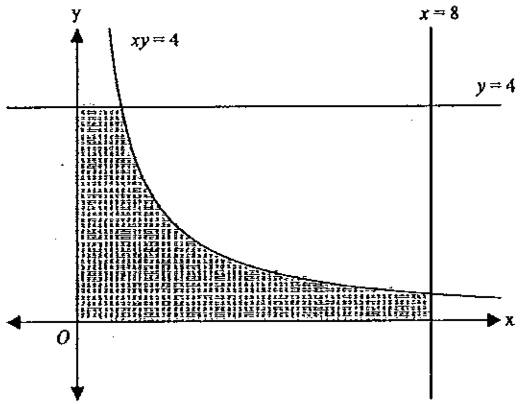
End of question 15.

Question 16 **(15 marks)** Start a new booklet

(a) (i) Differentiate $y = \cos^3 x$. **2**

(ii) Hence, evaluate $\int_0^{\frac{\pi}{4}} \sin x \cos^2 x \; dx$. **2**

(b) The area enclosed by the curve $xy = 4$, the x and y-axes and the lines $y = 4$ and $x = 8$, is rotated about the y-axis.



(i) Show that the volume of the solid of revolution obtained is given by **2**

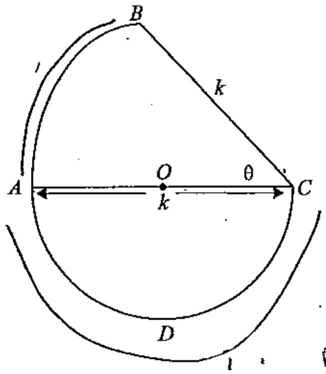
$$V = 32\pi + \pi\int_{\frac{1}{2}}^{4} \frac{16}{y^2}\, dy.$$

(ii) Hence find the volume of the solid. **2**

Question 16 continues on page 13.

Question 16 – continued.

(c) The shape drawn below consists of a semi-circle ADC with centre O and diameter k units and a sector ABC of radius k units, centre C and angle θ.



(i) Find the perimeter $ABCD$ of the shape in terms of k and θ. **1**

(ii) If the area of the of this shape is 1 square unit, show that the perimeter P is given by $P = \frac{2}{k} + k(1 + \frac{\pi}{4})$. **3**

(iii) Show that the least perimeter occurs when $k^2 = \dfrac{8}{\pi + 4}$. **3**

END OF EXAM.

Student Number: MASTER COPY – SOLUTIONS

Name:

SECTION I Mathematics Multiple Choice Answer Sheet **10 Marks**

This sheet must be handed in separately. Detach it from the question paper.

Shade the correct answer:

	A	B	C	D	
1.	A ○	B ○	C ○	D ●	✓
2.	A ●	B ○	C ○	D ○	✓
3.	A ○	B ○	C ○	D ●	✓
4.	A ●	B ○	C ○	D ○	✓
5.	A ○	B ○	C ○	D ●	✓
6.	A ●	B ○	C ○	~~D ●~~	X
7.	A ●	~~B ●~~	C ○	D ○	X
8.	A ●	B ○	C ○	D ○	✓
9.	A ○	B ○	C ●	~~D ●~~	X
10.	~~A ●~~	B ○	C ●	D ○	X

(Q11)

a. $3(x^3+8)$

$3(x+2)(x^2-2x+4)$ ✓

b. $\frac{3}{2-\sqrt5} \times \frac{2+\sqrt5}{2+\sqrt5} = \frac{3(2+\sqrt5)}{4-5} = -3(2+\sqrt5)$ ✓

c. Terms $= a+(n-1)d$

$292 = 5+(n-1)7$ ✓

$41 = n-1$

$n = 42$ ✓

(ii) $S_n = \frac{n}{2}(2a+(n-1)d)$

$= \frac{42}{2}(10+(42-1)7)$ ✓

$= 21(10+(41)7)$

$= 6237$ ✓

d. $y=(5-3x^2)^7$

$y' = 7(5-3x^2)^6 \cdot -6x$ ✓

$= -42x(5-3x^2)^6$ ✓

(ii) $y = 2x^2\tan x$

$y' = \tan x \cdot 4x + 2x^2\sec^2 x$ ✓

$= 4x\tan x + 2x^2\sec^2 x$ ✓

(iii) $y = \log_e(3x^2-5x)$

$y' = \frac{6x-5}{3x^2-5x}$

(iv) $y = \frac{6x^2}{\sqrt{2x+1}}$

$y' = \frac{\sqrt{2x+1}\cdot 12x - 6x^2(2x+1)^{-\frac12}}{2x+1}$

$= \frac{12x\sqrt{2x+1} - 6x^2\cdot\frac{1}{\sqrt{2x+1}}}{2x+1}$ ✓

$= \frac{12x(2x+1)-6x^2}{(2x+1)\sqrt{2x+1}} = \frac{24x^2+12x-6x^2}{(2x+1)\sqrt{2x+1}} = \frac{6x(3x+2)}{(2x+1)\sqrt{2x+1}}$ ✓

Q 12

a. $n = 3 \quad (5)^2$

$n = 4 \quad (7)^2$

$n = 5 \quad (9)^2 \; +$

$25 + 49 + 91 = 165$

b. $r = 4$ cm

$l = 16$ cm

$l = r\theta$

$16 = 4 \cdot \theta$

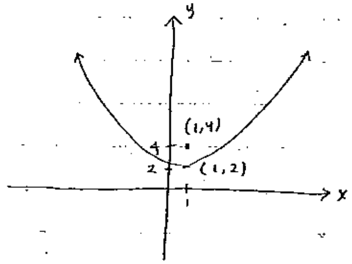
$\theta = 4$

$\text{area} = \frac{1}{2} r^2 \theta$

$= 16 \cdot 4 \times \frac{1}{2}$

$= \frac{64}{2}$ cm^2 $= 32$ cm^2

c.



focal length $= 4 \cdot 2 = 8$

$x^2 = 4ay$

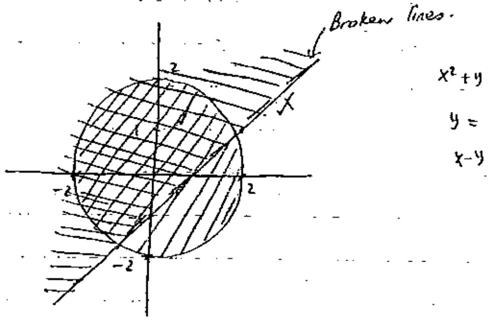
equation of parabola

$(x-h)^2 = 4a(y-k)^2$

$(x-1)^2 = 4a(y-2)$

$(x-1)^2 = 8(y-2)$

d.



$x^2 + y^2 = r^2$

$y = x - 1$

$x - y = 1$

$x^2 + y^2 \leq 4$

$y > x - 1$

e. $y = e^{2x}$

$\frac{dy}{dx} = 2e^{2x}$

at $x = 0$

$m = 2$

$x = 0, \; y = e^0$

$= 1$

$(y - y_1) = m(x - x_1)$

$(y - 1) = 2(x - 0)$

$(y - 1) = 2x$

$y = 2x + 1$

f. Solve

$3e^{3x} - e^{2x} = 0$

$3e^{3x} = e^{2x}$

$\frac{1}{3} = \frac{e^{3x}}{e^{2x}}$

$\frac{1}{3} = e^{3x - 2x}$

$\frac{1}{3} = e^{x}$

$\ln \frac{1}{3} = x$

$x = -1.1$

Q13. $AB = x + 2y = 2$

$DC = x + 2y = 6$ passes through $(2,2)$

$AB \; M = -\frac{a}{b}$

$= -\frac{1}{2}$

$DC \; M = -\frac{a}{b}$

$= -\frac{1}{2}$

$\therefore$ parallel.

passes through $(2,2)$

$DC = x + 2y = 6$

$= 2 + 2(2) = 6$

$\therefore$ passes through.

(ii) Distance AB & DC

$A(0,y) \quad B(x,0)$

$A(0,1) \quad B(2,0)$

$D(0,y) \quad C(x,0)$

$D(0,3) \quad C(6,0)$

distance perpendicular

$d = \left| \frac{ax + by + c}{\sqrt{a^2 + b^2}} \right|$

$= \left| \frac{x + 2y - 2}{\sqrt{5}} \right| = \left| \frac{2 + (2(2)) - 2}{\sqrt{5}} \right|$

$= \frac{4}{\sqrt{5}} = \frac{4\sqrt{5}}{5}$

(iii) $dAB = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$= \sqrt{(0-2)^2 + (1)^2}$

$= \sqrt{5}$

$dDC = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$= \sqrt{(-6)^2 + (9)}$

$= \sqrt{45} = 3\sqrt{5}$

Area $= \frac{\sqrt{5}+3\sqrt{5}}{2} \cdot \frac{4}{\sqrt{5}}$ ✓

$= 8\text{ unit}^2$ ✓

(iv) $x+2y \geqslant 2$ ✓

$x+2y \leqslant 6$ ✓

$x \geqslant 0$ ✓ , $y \geqslant 0$ ✓

b. (i) $x^3 - x^2 - x + 1$.

$x^3 - x - x^2 + 1$

$x(x^2-1) - (x^2-1)$ ✓

$(x-1)(x^2-1)$

$(x-1)(x-1)(x+1)$ $\quad (x-1)^2(x+1)$ ✓

(ii) intercept of $f(x)$

$x = 1 \ / -1$ ✓

(iii) stationary points.

$f'(x) = 3x^2 - 2x - 1$

$0 = 3x \quad 1$

$x \quad -1$

$= (3x+1)(x-1)$ ✓

$x = -\frac{1}{3} \ / \ x = 1$ $\quad \left(-\frac{1}{3}, \frac{32}{27}\right)$ ✓, $(1, 0)$ ✓

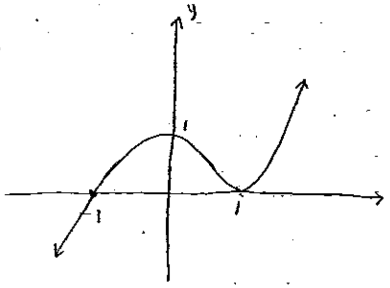
(iv) points of inflexion.

$f''(x) = 6x - 2$.

$0 = 6x - 2$ ✓

$x = \frac{1}{3}$ $\quad \left(\frac{1}{3}, \frac{16}{27}\right)$ ✓

(v)



a. $4\cos^2 x = 3$ for $-\pi \leqslant x \leqslant \pi$

$\cos^2 x = \frac{3}{4}$ $\Rightarrow$ 2U approach $\cos x = \pm\frac{\sqrt{3}}{2}$ $\Rightarrow$ $x = \frac{\pi}{3}, \frac{2\pi}{3}, -\frac{\pi}{3}, -\frac{2\pi}{3}$.

$\frac{1}{2}(\cos 2x + 1) = \frac{3}{4}$

$\cos 2x + 1 = \frac{6}{4}$

$\cos 2x = \frac{1}{2}$

$2x = \cos^{-1}\left(\frac{1}{2}\right)$

$2x = 60^\circ, -60^\circ$

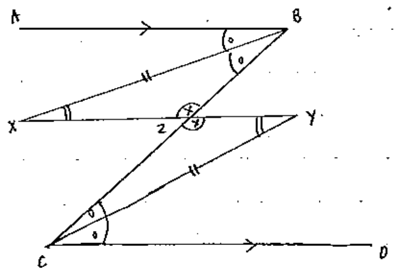
$x = 30^\circ, -30^\circ$

$x = \frac{\pi}{6}, -\frac{\pi}{6}$

sin	+
tan	cos

π ; $0, 2\pi$

b.



$\angle ABC = \angle BCD$ (alternate angle) ✓

$\angle BZX = \angle YZC$ (vertically opposite angle).

~~$\angle BXY = \angle XYC$~~ $BX = CY$ (Given)

$\triangle XBZ \equiv \triangle ZCY$ congruent triangle. AAS.

$\therefore XZ = ZY$ (Matching sides, congruent Δs)

then Z is midpoint;

c. $\int \frac{4x}{4-x^2} = -2\int \frac{-2x}{4-x^2}\,dx$ ✓

$= -2\ln(4-x^2) + c$ ✓

d. $f(x) = x\sin x - \cos x$.

$f(-x) = -x\sin(-x) - \cos(-x)$ $\qquad -f(x) = -(x\sin x - \cos x)$

$f(-x) = x\sin(x) - \cos x$ ✓ $\qquad = -x\sin x + \cos x$.

ODD $= f(-x) = -f(x)$

EVEN $= f(x) = f(-x)$ ✓

$\therefore$ it's even function. ✓

e. 5 function value.

	1	2	3	4	5
	0	0.5	1	1.5	2
	0	1.5	2.24	2.87	3.5

$$\int_0^2 \sqrt{x^2+4x}\,dx$$

$$\doteq \frac{h}{3}\left[f(1st) + f(last) + 4(f(even)) + 2f(odd)\right]$$

$$\doteq \frac{0.5}{3}\left[0 + 3.5 + 4(1.5 + 2.87) + 2(2.24)\right]$$

$$\doteq 4.24$$

f. real and unequal roots.

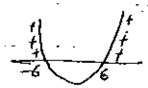
$b^2 - 4ac > 0$.

$3x^2 - kx + 3 = 0$.

$k^2 - 4(3)(3) > 0$.

$k^2 - 36 > 0$.

$k^2 > 36 \Rightarrow k^2 - 36 > 0$



$k > 6$ $k < -6$.

Q15.

$$\frac{dP}{dt} = 200(0.4 - 0.08t)$$

$t = 0$ $P = 200$

(i) $\frac{dP}{dt} = 200(0.4 - 0.08t)$

$\frac{dP}{dt} = 80 - 16t$

$P = 80t - 8t^2 + c$

$t = 0$, $P = 200$

$200 = c$.

$P = 80t - 8t^2 + 200$

$= 200(1 + 0.4t - 0.04t^2)$

(ii) $P = 400$.

$400 = 200(1 + 0.4t - 0.04t^2)$

$1 = 0.4t - 0.04t^2$

$100 = 40t - 4t^2$

$4t^2 - 40t + 100$

$t^2 - 10t + 25$

$t = 5$ month

b. $M = Ae^{-kt}$ t (hours)

Starts 20 kg $t = 0$ $t = 24$ $M = 5$

(i) Value of A, K

$20 = A$

$M = 20e^{-kt}$

$5 = 20e^{-24k}$

$\frac{\ln\frac{1}{4}}{-24} = k \approx 0.058$

(ii) Undissolved solid after 3 hours. nearest gram

$M = 20e^{\frac{\ln\frac{1}{4}}{24}\cdot t}$

$M = 20e^{\frac{\ln\frac{1}{4}}{24}\cdot 3}$

$M = 16.818$ kg

(iii) $1 = 20e^{\frac{\ln\frac{1}{4}}{24}\cdot t}$

$\ln\frac{1}{20} = \frac{\ln\frac{1}{4}}{24}\cdot t$

$\frac{\ln\frac{1}{20} \times 24}{\ln\frac{1}{4}} = t$

$t = 52$ hours.

c. 6% per annum

0.5% per month ✓

M at the end of each month over 20 years

(i) end $A_1 = \$80\,000\,(1.005) - M$ ✓

$A_2 = \$80\,000\,(1.005)^2 - M(1.005) - M$ ✓

$A_3 = \$80\,000\,(1.005)^3 - M(1.005)^2 - M(1.005) - M$

$\vdots$

(ii) $A_{240} = \$80\,000\,(1.005)^{240} - M\underbrace{(1.005^{239} + 1.005^{238} + \dots + 1)}_{GP}$ ✓

$A_{240} = \$80\,000\,(1.005)^{240} - M$

$A_{240} = \$80\,000\,(1.005)^{240} - M\left(\frac{1(1.005^{240}-1)}{1.005-1}\right)$

$80000\,(1.005)^{240} = M\left(\frac{1(1.005^{240}-1)}{1.005-1}\right)$ ✓

$80000\,(1.005)^{240} = \frac{M\,(1.005^{240}-1)}{0.005}$

$1324.08 = M\ 2.31$ ✓

$\$573.14 = M$

(iii) after $A_{30} = 80,000 - (573.14 \times 30)$

$= 62\,805.80 = P$

$A_n = PR^n - \frac{M\cdot(R^n-1)}{R-1}$ where $R = 1.00625$

$n = 210$

$0 = 62\,805.80\,(1.00625)^{210} - \frac{M(1.00625^{210}-1)}{0.00625}$

$M = \underline{\underline{\$537.90}}$

a. (i) $y = \cos^3 x$

$y' = 3\cos^2 x \cdot -\sin x$ ✓

$= -3\sin x\cos^2 x$

(ii) $\int_0^{\frac{\pi}{4}} \sin x \cos^2 x \, dx$

$-\frac{1}{3}\int_0^{\frac{\pi}{4}} -3\sin x\cos^2 x\, dx$

$-\frac{1}{3}\left[\cos^3 x\right]_0^{\frac{\pi}{4}}$ ✓

$-\frac{1}{3}\left[\cos^3\frac{\pi}{4} - \cos^3 0\right]$

$-\frac{1}{3}\left[\left(\frac{1}{2}\sqrt{2}\right)^3 - 1\right]$ ✓

$-\frac{1}{3}\left[\frac{2\sqrt{2}}{8} - 1\right]$

$-\frac{1}{3}\left[\frac{2\sqrt{2}-8}{8}\right]$

$\frac{8-2\sqrt{2}}{24} = \frac{4-\sqrt{2}}{12}$ ✓

b. Show that volume $V = 32\pi + \pi\int_{\frac{1}{2}}^{4} \frac{16}{y^2}\, dy$

When $x = 8$ $y = \frac{1}{2}$

$V = \int \pi x^2\, dy \quad + \quad \pi r^2 h$ ✓

$V = \int_{\frac{1}{2}}^{4} \pi\left(\frac{4}{y}\right)^2 dy \quad + \quad \pi\ 64\ \frac{1}{2}$

$V = \pi\int_{\frac{1}{2}}^{4} \frac{16}{y^2}\, dy \quad + \quad 32\pi$ ✓

$V = 32\pi + \pi\int_{\frac{1}{2}}^{4}\frac{16}{y^2}\, dy$ ✓

b. $\text{Vol} = 32\pi + \pi\int_{\frac{1}{2}}^{4} 16y^{-2}\, dy$ ✓

$= 32\pi + \pi\left[-16y^{-1}\right]_{\frac{1}{2}}^{4}$

$= 32\pi + \pi\left[-4 - (-32)\right]$

$= 32\pi + \pi\left[-4 + 32\right]$ ✓

$= 60\pi$

c. perimeter $= \frac{1}{2}\pi d + \ell\theta + k$

$= \frac{1}{2}\pi k + k\theta + k$

(ii) area of the shape 1 unit^2

$$A = \frac{1}{2}\pi r^2 + r^2\theta$$

$$1 = \frac{1}{2}\pi \frac{k^2}{4} + \frac{1}{2}k^2\theta$$

$$\theta = \frac{\frac{1}{2}\pi\frac{k^2}{4} - 1}{-\frac{1}{2}k^2}$$

$$= \frac{\left(\frac{1}{2}\pi\frac{k^2}{4} - 1\right) \cdot -2}{k^2}$$

$$= \frac{-\pi\frac{k^2}{4} + 2}{k^2}$$

$$\theta = \frac{2 - \pi\frac{k^2}{4}}{k^2}$$

$$P = \frac{1}{2}\pi k + k\left(\frac{2 - \pi\frac{k^2}{4}}{k^2}\right) + k$$

$$= \frac{\pi k}{2} + \frac{2 - \pi\frac{k^2}{4}}{k} + k$$

$$= \frac{\pi k^2 + 4 - \frac{2\pi k^2}{4} + 2k^2}{2k}$$

$$= \frac{\frac{4\pi k^2 + 16 - 2\pi k^2 + 8k^2}{4}}{2k}$$

$$= \frac{2\pi k^2 + 16 + 8k^2}{8k}$$

$$= \frac{\pi k}{4} + \frac{2}{k} + k$$

$$= \frac{2}{k} + k\left(1 + \frac{\pi}{4}\right)$$

(iii) least $P = k^2 = \frac{8}{\pi + 4}$

$$P = \frac{2}{k} + k\left(1 + \frac{\pi}{4}\right)$$

$$P = 2k^{-1} + k\left(1 + \frac{\pi}{4}\right)$$

$$P' = -2k^{-2} + \left(1 + \frac{\pi}{4}\right)$$

$$0 = -2k^{-2} + \left(1 + \frac{\pi}{4}\right)$$

$$2k^{-2} = 1 + \frac{\pi}{4}$$

$$k^{-2} = \frac{\pi + 4}{8}$$

$$k^2 = \frac{8}{\pi + 4}$$