Student Number:

NEWINGTON COLLEGE

IN FIDE SCIENTIAM

2015

YEAR 12

HSC Trial EXAMINATION

Mathematics

General Instructions

- Date of Task – Tuesday 18th August
- Reading time - 5 minutes
- Working time - 3 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- Show all necessary working in Questions 11-16

Total marks - 100

Section I

10 marks

- Attempt Questions 1-10 on answer sheet provided
- Allow about 15 minutes for this section

Section II

90 marks

- Attempt Questions 11-16
- Allow about 2 hour 45 minutes for this section

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\ n \neq -1;\ x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x,\ x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax},\ a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax,\ a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax,\ a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax,\ a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax,\ a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\ a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a},\ a > 0,\ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right),\ x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE: $\ln x = \log_e x,\ x > 0$

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

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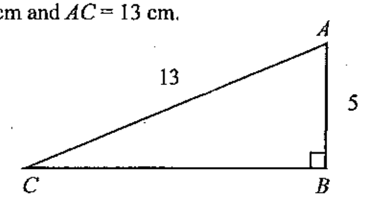
1. Which term of the series with nth term $T_n = 15 - 2n$ is equal to -37?

(A) -26
(B) 26
(C) -11
(D) 11

2. The diagram shows the right triangle ABC.
$\angle ABC = 90°$, $AB = 5$ cm and $AC = 13$ cm.



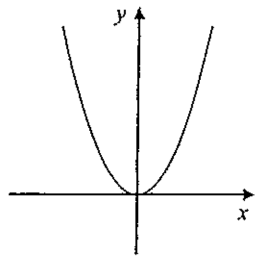
What is the value of $\tan \angle BAC$?

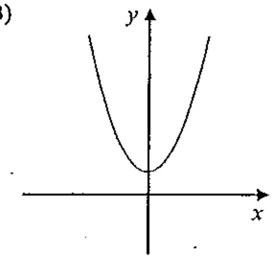
(A) $\frac{5}{12}$
(B) $\frac{5}{13}$
(C) $\frac{13}{5}$
(D) $\frac{12}{5}$

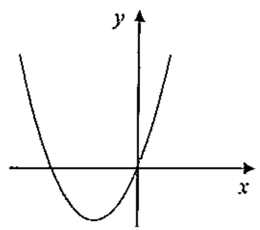
3. An infinite geometric series has a first term of 3 and a limiting sum of 1.8.
What is the common ratio?

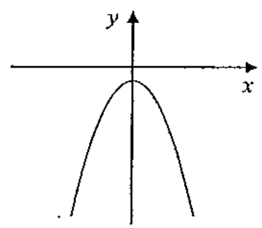
(A) $-0.\dot{3}$
(B) $-0.\dot{6}$
(C) -1.5
(D) -3.75

6. Which graph represents a quadratic equation with discriminant $\Delta = 0$?

(A)


(B)


(C)


(D)


7. What is the solution to the equation $2\cos^2 x - 1 = 0$ in the domain $0 \le x \le 2\pi$?

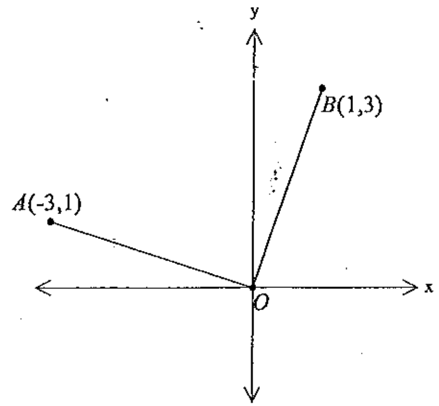
(A) $x = \frac{\pi}{6}, \frac{11\pi}{6}$
(B) $x = \frac{\pi}{4}, \frac{7\pi}{4}$
(C) $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$
(D) $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

8. Which expression is the gradient function of $(5x-4)^7$?

(A) $7(5x-4)^6$
(B) $\frac{5}{8}(5x-4)^8$
(C) $7(5x-4)^6$
(D) $35(5x-4)^6$

Question 12 (15 marks) **Start a new booklet** **Marks**

(a) Points $A(-3,1)$ and $B(1,3)$ are on a number plane.



Copy the diagram into your writing booklet.

(i)	Find the gradient of line OA.	1
(ii)	Show that OA is perpendicular to OB.	1
(iii)	$OACB$ is a quadrilateral in which BC is parallel to OA. Show that the equation of BC is $x+3y-10=0$.	2
(iv)	The point C lies on the line $x=-2$. What are the coordinates of point C?	1
(v)	Show that the length of the line BC is $\sqrt{10}$.	1
(vi)	Find the area of $OACB$.	1

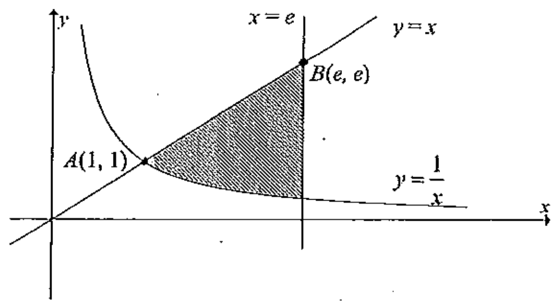
(b) The table shows the values of a function $f(x)$ for five values of x. **2**

x	1	1.5	2	2.5	3
$f(x)$	4	1.5	−2	2.5	8

Use Simpson's rule with these five values to estimate $\int_1^3 f(x)dx$.

Question 12 (Continued)

(c) The line $y=x$ and the hyperbola $y=\frac{1}{x}$ intersect at the point $A(1,1)$.
The line $y=x$ and the line $x=e$ intersect at the point $B(e,e)$



Calculate the area enclosed by the line $y=x$, the line $x=e$ and the hyperbola $y=\frac{1}{x}$. **3**

(d) Bag A contains 3 red cubes and 2 white cubes. Bag B contains 2 red cubes and 3 white cubes. A bag is selected at random and then a cube is selected at random from that bag.

(i) Draw a tree diagram to show the possible outcomes. Show the probability on each branch. **2**

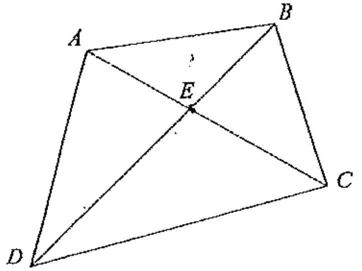
(ii) What is the probability that the cube selected is white? **1**

End of Question 12

Question 14 (15 marks) **Start a new booklet** **Marks**

(a) Differentiate $f(x) = x\cos x$ 1

(b) In quadrilateral $ABCD$ the diagonals AC and BD intersect at E.
Given $AE = 3$, $EC = 6$, $BE = 4$ and $ED = 8$.



Not to scale

(i) Show that $\Delta ABE \mid\mid\mid \Delta DEC$ 3

(ii) What type of quadrilateral is $ABCD$? Justify your answer. 2

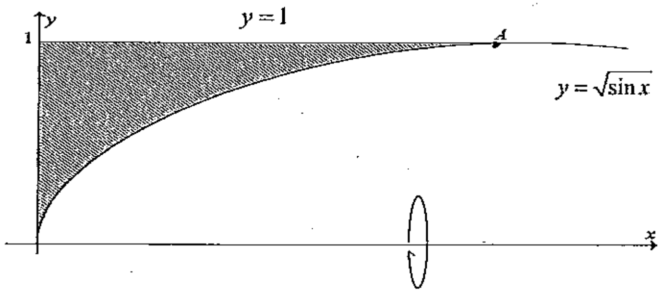
(c) Find the shortest distance between the point $(0,5)$ and the line $3x - y + 1 = 0$ 2

(d) The parabola $y = ax^2 + bx + c$ has a vertex at (3, 1) and passes through (0, 0).

(i) Find the other x-intercept of the parabola. 1

(ii) Find a, b and c. 2

(e) The region bounded by the curve $y = \sqrt{\sin x}$, the y-axis and the line $y = 1$ is rotated around the x-axis to form a solid.



(i) If $y = \sqrt{\sin x}$ and $y = 1$ and meet at the point A, show that the coordinates of A are $\left(\frac{\pi}{2}, 1\right)$. 1

(ii) Find the volume of the solid. 3

End of Question 14

Question 16 (15 marks) **Start a new booklet** **Marks**

(a) The third and seventh terms of a geometric series are 1.25 and 20 respectively. What is the first term? 3

(b) The displacement of a particle moving along the x-axis is given by

$$x = 5\sin\frac{\pi}{2}t,$$

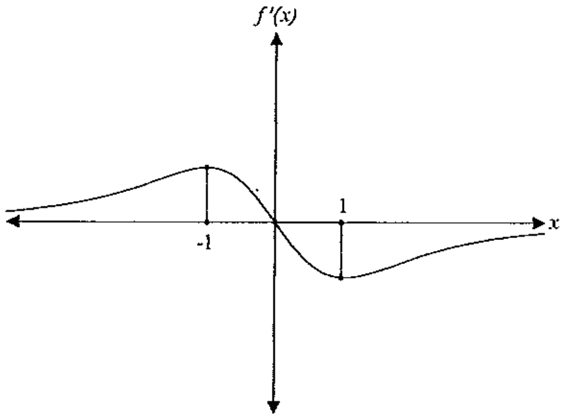
where x is the displacement from the origin in metres, t is the time in minutes and $t \geq 0$.

(i) What is the furthest distance the particle moves away from the origin. 1

(ii) When does the particle first return to its starting position? 1

(iii) Find the acceleration of the particle when $t = 3\text{min}$. 3

(c) The graph of $f'(x)$ shown in the diagram passes through the origin. As $x \to \pm\infty$ $f'(x) \to 0$ and $f(x) \to 0$. 3



Sketch the graph of $y = f(x)$, given $f(x) > 0$.

YR 12 MATHEMATICS TRIAL HSC 2015

SOLUTIONS

SECTION I (10 MARKS)

1. $15 - 2n = -37$
$-2n = -52$
$n = 26$ **B**

2. $CB = \sqrt{13^2 - 5^2}$
$= 12$
$\tan \angle BAC = \frac{12}{5}$
D

3. $1.8 = \frac{3}{1-r}$
$1.8 - 1.8r = 3$
$-1.8r = 1.2$
$r = -\frac{12}{18} = -\frac{2}{3}$
$= -0.6$ **B**

4. $\left[\frac{1}{3}e^{3x} + x\right]_0^1$
$= \left(\frac{1}{3}e^3 + 1\right) - \left(\frac{1}{3} + 0\right)$
$= \frac{1}{3}e^3 + 1 - \frac{1}{3}$
$= \frac{1}{3}e^3 + \frac{2}{3}$
$= \frac{1}{3}(e^2 + 2)$ **D**

5. $-(3\times3) + -(3\times3)$
$= -9 - 9$
$= -18$ **D**

6. **A**

7. $2\cos^2 x = 1$
$\cos^2 x = \frac{1}{2}$
$\cos x = \pm\frac{1}{\sqrt{2}}$
$\therefore x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$ **D**

8. $y = (5x-4)^7$
$\frac{dy}{dx} = 7(5x-4)^6 \cdot 5$
$= 35(5x-4)^6$ **D**

9. $x^2 - 2x = 6y + 11$
$x^2 - 2x + 1 = 6y + 12$
$(x-1)^2 = 6(y+2)$
Vertex $(1, -2)$
Focal length: $4a = 6$
$a = \frac{6}{4} = 1\frac{1}{2}$
$\therefore$ Focus $(1, -\frac{1}{2})$ **D**

10. $\frac{2\pi}{n} = \frac{3\pi}{4}$
$3n = 8$
$n = \frac{8}{3}$ **C**

QUESTION 11 (15 MARKS)

(a) $\frac{1}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1} - \left(\frac{1}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1}\right)$
$= \frac{\sqrt{2}+1}{1} - \left(\frac{\sqrt{2}-1}{1}\right)$ 1
$= \sqrt{2} + 1 - \sqrt{2} + 1$ 1
$= 2$

(b) $(2a)^3 - (y)^3$ 1
$= (2a - y)(4a^2 + 2ay + y^2)$ 1

(c) $(x^3 - 1)(x^3 + 1)$
$= x^6 - 1$ ✓
$\frac{d}{dx}(x^6 - 1) = 6x^5$ ✓

(d) $\left[3\sqrt{3}\tan\frac{x}{3}\right]_{\frac{\pi}{2}}^{\pi}$ 1
$= 3\sqrt{3}\left[\tan\frac{\pi}{3} - \tan\frac{\pi}{6}\right]$
$= 3\sqrt{3}\left[\sqrt{3} - \frac{1}{\sqrt{3}}\right]$ 1
$= 3\times3 - 3$
$= 6$ 1

(e) Find n
$a = 24 \quad d = 4 \quad T_n = 136$
$24 + 4(n-1) = 136$
$24 + 4n - 4 = 136$
$4n = 116$
$n = 29$ 1

$S_{29} = \frac{29}{2}(24 + 136)$
$= 2320$ 1

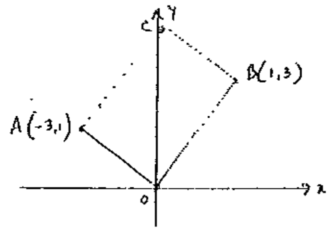
(f) $\frac{dy}{dx} = 3ax^2 - 1 = 0$ 1
at $x = 2$ $\quad 3a\times2^2 - 1 = 0$
$12a - 1 = 0$
$12a = 1$
$a = \frac{1}{12}$ 1

(g) $\log_6(9\times24)$
$= \log_6 216$ 1
$6^x = 216$
$\therefore x = 3$ 1

(OR just use change of base
$\frac{\log_{10} 216}{\log_{10} 6} = 3$ ✓

QUESTION 12 (15 MARKS)

(a)



(i) Grad OA $= -\frac{1}{3}$ 1

(ii) Grad OB $= \frac{3}{1} = 3$ 1

$3 \times -\frac{1}{3} = -1 \quad \therefore OA \perp OB$

(iii) grad $= -\frac{1}{3}$ pt $(1,3)$

$y - 3 = -\frac{1}{3}(x-1)$ 1

$3y - 9 = -x + 1$

$x + 3y - 10 = 0$ Eqn of BC 1

(iv) Sub $x = -2$ into $x + 3y - 10 = 0$

$-2 + 3y - 10 = 0$

$3y = 12$

$y = 4$

$\therefore$ Pt C is $(-2,4)$ 1

(v) $d_{BC} = \sqrt{(-2-1)^2 + (4-3)^2}$

$= \sqrt{10}$ units 1

(vi) $BC = \sqrt{10}$ units

$\therefore$ AREA OACB $= \sqrt{10} \times \sqrt{10}$ 1

$= 10$ sq. units

(b) $A = \frac{2-1}{6}\{f(1) + 4f(1.5) + f(2)\}$

$+ \frac{3-2}{6}\{f(2) + 4f(2.5) + f(3)\}$ 1

$= \frac{1}{6}[4 + 4 \times 1.5 + (-2) + -2 + 4 \times 2.5 + 8]$

$= \frac{1}{6}(8 + 16)$

$\doteqdot 4$ 1

(c) $A = \int_1^e x\,dx - \int_1^e \frac{1}{x}\,dx$ 1

$= \left[\frac{x^2}{2} - \ln x\right]_1^e$ 1

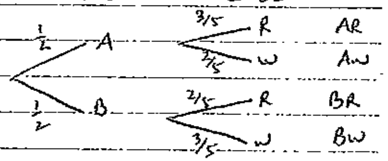
$= \left(\frac{e^2}{2} - 1\right) - \left(\frac{1}{2} - 0\right)$

$= \frac{e^2}{2} - 1 - \frac{1}{2}$

$= \frac{e^2}{2} - \frac{3}{2}$ 1

$= \frac{1}{2}(e^2 - 3)$ sq. units

(d) 2



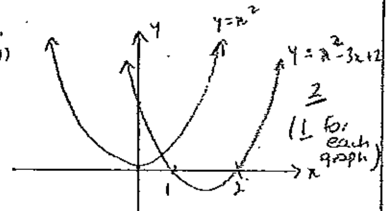
P(white) = P(AW) + P(BW)

$= \frac{1}{2} \times \frac{2}{5} + \frac{1}{2} \times \frac{3}{5}$

$= \frac{5}{10} = \frac{1}{2}$ 1

QUESTION 13 (15 MARKS)

(a) (i)



2 (1 for each graph)

(ii) $x^2 > x^2 - 3x + 2$

$0 > -3x + 2$

$3x > 2$

$\therefore x > \frac{2}{3}$ 1

(b) $\left[\frac{x^3}{3} - \frac{1}{2}\cos 2x\right]_0^{\frac{\pi}{6}}$

$= \left(\frac{\pi^3}{648} - \frac{1}{2}\cos\frac{\pi}{3}\right)$

$- \left(0 - \frac{1}{2}\right)$

$= \frac{\pi^3}{648} - \frac{1}{4} + \frac{1}{2}$

$= \frac{\pi^3}{648} + \frac{1}{4}$ $(0.297\ldots)$

(c)(i) $y = \frac{x^2-2}{x^2+2}$ u v

$\frac{dy}{dx} = \frac{(x^2+2).2x - (x^2-2).2x}{(x^2+2)^2}$ 1

$= \frac{2x^3 + 4x - 2x^3 + 4x}{(x^2+2)^2}$

$= \frac{8x}{(x^2+2)^2}$ 1

(c) (ii) $\int_2^4 \frac{x}{(x^2+2)^2}\,dx$

$= \left[\frac{1}{8}\left(\frac{x^2-2}{x^2+2}\right)\right]_2^4$ 1

$= \frac{1}{8}\left(\frac{4^2-2}{4^2+2} - \frac{2^2-2}{2^2+2}\right)$ 1

$= \frac{1}{8}\left(\frac{14}{18} - \frac{2}{6}\right)$

$= \frac{1}{8} \times \frac{4}{9} = \frac{1}{18}$ 1

(d) (i) $\angle IPA = 104° - 39°$

$= 65°$ 1

$AD^2 = 2650^2 + 2120^2 - 2.2650.2120.\cos 65°$

$= 6768361.211\ldots$

$AD = 2601.607\ldots$ 1

$\doteqdot 2600$ km (nearest 10 km)

(ii) $\frac{\sin\angle PDA}{2120} = \frac{\sin 65°}{2600}$

$\sin\angle PDA = \frac{\sin 65°}{2600} \times 2120$ 1

$= 0.738989426\ldots$

$\angle DAP = 48°$ (nearest degree)

$\angle N_1DP = 180° - 39° = 141°$ 1

$\therefore \angle N_1DA = 360° - 141° - 48°$

$= 171°$

$\therefore$ Bearing of Darwin from Adelaide

$= 360° - (180° - 171°) = 351°$ 1

QUESTION 14 (15 MARKS)

(a) $f(x) = x\cos x$

$f'(x) = x \cdot -\sin x + \cos x \cdot 1$

$= \cos x - x\sin x$ 1

(b)(i) In $\triangle ABE$ and $\triangle DEC$,

$\angle AEB = \angle DEC$ (vert opp $\angle$s are equal) 1

$\frac{AE}{EC} = \frac{3}{6} = \frac{1}{2}$ $\frac{BE}{ED} = \frac{4}{8} = \frac{1}{2}$ 1

$\therefore \triangle ABE ||| \triangle DEC$

(two pairs of corresponding sides are in proportion and the included angles are equal) 1

(ii) $\angle BAE = \angle DCE$ (matching $\angle$s in similar triangles are equal)

$\therefore \angle BAE$ and $\angle DCE$ are alternate angles and equal.

$\therefore AB \parallel CD$ (alternate angles are only equal if the lines $\parallel$)

$\therefore ABCD$ is a trapezium (one pair opposite sides parallel)

(c) $(0, 5)$ $3x - y + 1 = 0$

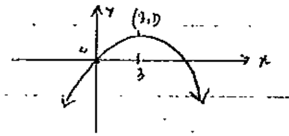
$a = 3$ $b = -1$ $c = 1$

$d = \frac{|0 \cdot 3 + 5 \cdot -1 + 1|}{\sqrt{3^2 + (-1)^2}}$ 1

$= \frac{|-4|}{\sqrt{10}} = \frac{4}{\sqrt{10}} \times \frac{\sqrt{10}}{\sqrt{10}}$

$= \frac{4\sqrt{10}}{10} = \frac{2\sqrt{10}}{5}$ 1

(d) (i) Parabola is symmetrical about the vertex $(3, 1)$



1

$\therefore$ other x-intercept is $(6, 0)$

(ii) All three points $(0,0)$, $(3,1)$, $(6,0)$ satisfy $y = ax^2 + bx + c$

So, sub. points into $y = ax^2 + bx + c$ and create 3 equations 1

$(0,0)$ $0 = 0 + 0 + c$

$\therefore c = 0$

$(3,1)$ $1 = 9a + 3b$ — ①

$(6,0)$ $0 = 36a + 6b$ — ②

Solve ① and ② simultaneously

① × 2 $2 = 18a + 6b$ — ③

$0 = 36a + 6b$ — ②

② − ③ $-2 = 18a$

$a = -\frac{1}{9}$

Sub into ① $1 = 9 \times -\frac{1}{9} + 3b$

$1 = -1 + 3b$

$b = \frac{2}{3}$

$\therefore a = -\frac{1}{9}$, $b = \frac{2}{3}$ and $c = 0$ 1

(e) PTO

14 continued

(e) (i) $\sqrt{\sin x} = 1$

$\sin x = 1$

$\therefore x = \frac{\pi}{2}$

Coordinates are $\left(\frac{\pi}{2}, 1\right)$ 1

(ii) $V = \pi \int_1^{\frac{\pi}{2}} y^2 \, dx$

$= \pi \int_0^{\frac{\pi}{2}} 1 \, dx - \pi \int_0^{\frac{\pi}{2}} \sin x \, dx$ 1

$= \pi \left[x\right]_0^{\frac{\pi}{2}} + \pi \left[\cos x\right]_0^{\frac{\pi}{2}}$

$= \pi\left(\frac{\pi}{2} - 0\right) + \pi\left(\cos\frac{\pi}{2} - \cos 0\right)$ 1

$= \frac{\pi^2}{2} + \pi(0 - 1)$

$= \frac{\pi^2}{2} - \pi$ cu. units

or $\left(\pi\left(\frac{\pi}{2} - 1\right) \text{ cu units}\right)$ 1

QUESTION 15 (15 MARKS)

(a) $f(x) = x^2(3-x)$

x-intercepts are $x=0$, $x=3$

$f(x) = 3x^2 - x^3$

$f'(x) = 6x - 3x^2$

$f''(x) = 6 - 6x$ 1

(i)

St. pt occur when $f'(x)=0$

$6x - 3x^2 = 0$

$3x(2-x) = 0$

$x=0$ $x=2$

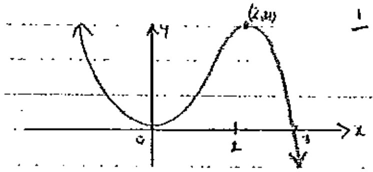
$(0,0)$ $(2,4)$ 1

Test:

$f''(0) = 6 > 0$ $f''(2) = 6-12 = -6 < 0$

$\therefore$ Min turn. pt at $(0,0)$ $\therefore$ MAX at $(2,4)$

(ii) 1



(b) $\alpha+\beta = \frac{-(-3)}{2} = 1\frac{1}{2}$

$\alpha\beta = \frac{6}{2} = 3$ 1

$(\alpha+\beta)^2 = \alpha^2 + 2\alpha\beta + \beta^2$

$\therefore \alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= (\frac{3}{2})^2 - 2\times 3$

$= \frac{9}{4} - 6$ 1

$= -\frac{15}{4} = -3\frac{3}{4}$

(c) For first 15 years

$P = \$1000$ $r = 6\%$ p.a $n = 15\times 12$

$= 0.005$ per month $= 180$ months

$A = 1000(1.005)^{180} + 1000(1.005)^{179} + \dots + 1000(1.005)^2 + 1000(1.005)$

$= 1000[1.005 + 1.005^2 + \dots + 1.005^{180}]$

$= 1000\left[\frac{1.005(1.005^{180}-1)}{1.005-1}\right]$ ✱

$a = 1.005$ $r = 1.005$ $n = 180$

For next 20 years

$P = \$2000$ $r = 7.5\%$ p.a $n = 20\times 12$

$= 0.00625$ per month $= 240$ months

$A = 2000(1.00625)^{240} + 2000(1.00625)^{239} + \dots + 2000(1.00625)^2 + 2000(1.00625)^1$

$= 2000[1.00625 + 1.00625^2 + \dots + 1.00625^{240}]$

$= 2000\left[\frac{1.00625(1.00625^{240}-1)}{1.00625-1}\right]$ ✱

$a = 1.00625$ $r = 1.00625$ $n = 240$

So, total the two amounts

Total = ✱ + ✱

$= \$292272.806 + \$1114383.084$

$= 1406655.89$

$= \$1406656$ 3

15 continued

(d) (i) Initially $t=0$, $R_0 = 8000$ 1

$R = R_0 e^{-kt}$

$R = 8000e^{-kt}$

Also, when $t=1$, $R = 7000$

$7000 = 8000e^{-k\times 1}$

$7000 = 8000e^{-k}$

$e^{-k} = \frac{7}{8}$

$\ln e^{-k} = \ln \frac{7}{8}$ 1

$-k = \ln \frac{7}{8}$

$\therefore k = -\ln(\frac{7}{8}) = 0.13353139\ldots$

(ii) when $t = 10$

$R = 8000e^{-0.13353\ldots\times 10}$

$= 2104.604\ldots$

$\doteq 2105$ kg 1

(iii) Find t when $R = 50$

$50 = 8000e^{-0.13353\ldots\times t}$

$e^{-0.13353\ldots\times t} = \frac{5}{800}$ 1

$-0.13353\ldots\times t = \ln\left(\frac{1}{160}\right)$

$t = \ln\left(\frac{1}{160}\right) \div -0.13353\ldots$

$= 38.0073458\ldots$

$\doteq 38$ years 1

QUESTION 16 (15 MARKS)

(a) $ar^6 = 20$ — ①

$ar^2 = 1.25$ — ② 1

① ÷ ②

$r^4 = 16$

$r = \pm 2$ 1

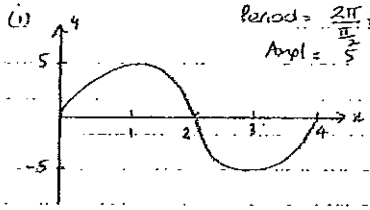
Sub into ② $a \times (2)^2 = 1.25$

$a = \frac{5}{16}$ 1

b) (i)

Period $= \frac{2\pi}{\frac{\pi}{2}} = 4$

Ampl $= 5$



Furthest distance $= 5$ m 1

(ii) $t = 2$ minutes 1

(iii) $v = \dot{x} = \frac{5\pi}{2}\cos\frac{\pi}{2}t$ 1

$a = \ddot{x} = -\frac{5\pi^2}{4}\sin\frac{\pi}{2}t$ 1

when $t = 3$

acceleration

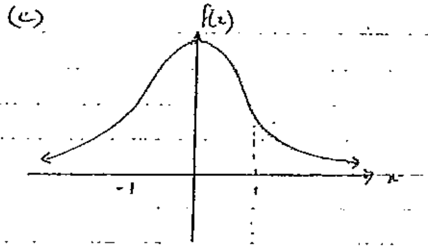
$= -\frac{5\pi^2}{4} \times \sin\frac{3\pi}{2}$

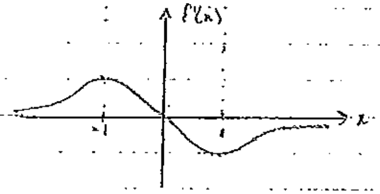
1

$= \frac{5\pi^2}{4}$ m / min^2

(12.337.... m/min^2)

(c)





1 Shows max turning pt at $x = 0$

1 Correct shape

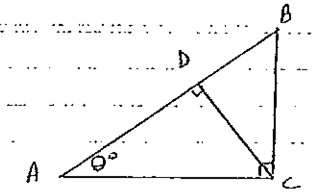
1 Correct positions of pts of inflexion at $x = \pm 1$

(d) P.T.O

(9)

16 (c)

(i)



$\cos\theta = \frac{AD}{AC}$ $\tan\theta = \frac{BC}{AC}$ $\cos\theta = \frac{AC}{AB}$ 1

$AD = AC\cos\theta$ $BC = AC\tan\theta$ $AB = AC \times \frac{1}{\cos\theta}$

$= AC\sec\theta$

Now $8AD + 2BC = 7AB$ 1

$8AC\cos\theta + 2AC\tan\theta = 7AC\sec\theta$

$\div AC$ $\therefore 8\cos\theta + 2\tan\theta = 7\sec\theta$

(ii) $8\cos\theta + 2\tan\theta = 7\sec\theta$

$8\cos\theta + \frac{2\sin\theta}{\cos\theta} = \frac{7}{\cos\theta}$

$8\cos^2\theta + 2\sin\theta = 7$

$8(1 - \sin^2\theta) + 2\sin\theta = 7$ 1

$8\sin^2\theta - 2\sin\theta - 1 = 0$

Let $u = \sin\theta$ $8u^2 - 2u - 1 = 0$

$(2u - 1)(4u + 1) = 0$

$u = \frac{1}{2}$ or $u = -\frac{1}{4}$

$\sin\theta = \frac{1}{2}$ $\sin\theta = -\frac{1}{4}$

$\theta = 30°$ $\theta = 165° 31'$

Since $0 \le \theta \le 90°$ (θ is in a right angled triangle) 1

$\therefore \theta = 30°$

(10)