2009 Trial Higher School Certificate

Trialmaths Enterprises

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
- Approved calculators may be used
- All necessary working should be shown in every question
- Answer each question in a SEPARATE Writing Booklet

Total marks –120

- Attempt Questions 1 –10
- All questions are of equal value

Care has been taken to ensure that this paper is free of errors and that it mirrors the format and style of past HSC papers. Moreover, some questions have been adapted from previous HSC examinations as well as from trial examinations from a variety of schools, in an attempt to provide students with exposure to a broad range of possible questions.

However, there is no guarantee whatsoever that the 2009 HSC examination will have similar content, style or format. This paper is intended only as a trial for the HSC examination or as revision leading up to the examination.

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, n \neq -1;\ x \neq 0, \text{ if } n<0$$

$$\int \frac{1}{x}\,dx = \ln x,\ x>0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax},\ a\neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax,\ a\neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax,\ a\neq 0$$

$$\int \sec^2 ax\,dx, = \frac{1}{a}\tan ax,\ a\neq 0$$

$$\int \sec ax\tan ax\,dx = \frac{1}{a}\sec ax,\ a\neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\ a\neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a},\ a>0,\ -a<x<a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln(x+\sqrt{x^2-a^2}),\ x>a>0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln(x+\sqrt{x^2+a^2})$$

NOTE : $\ln x = \log_e x, \quad x>0$

Question 1 (12 marks) Use a SEPARATE page/ booklet. **Marks**

(a) Find the value of $15^{-1.5}$. Write your answer to three decimal places. **2**

(b) Rationalise the denominator of $\frac{5}{3-\sqrt{7}}$ **2**

(c) Write the exact value of $\sin\frac{5\pi}{4}$ **2**

(d) Find the value of $\log_e e^2 + \log_e 1$ **2**

(e) Find $\int \sec^2 2x\,dx$ **2**

(f) Differentiate $e^{2x} - 5x$ with respect to x. **2**

Question 2 (12 marks) Use a SEPARATE page/ booklet. **Marks**

(a) Given the points A(1,2), B(3,1), C(-1,4)

(i) Find the equation of the line BC. **1**

(ii) Find the perpendicular distance from point A to the line BC. **2**

(iii) Hence, or otherwise, find the area of ΔABC. **2**

(b) Find the sum of the first 15 terms of the series

$1+3+3^2+3^3+3^4+\ldots\ldots$ **2**

(c) Find the maximum area a triangle can have if the sum of its base and height is 10cm. **3**

(d) The area of a sector AOB of a circle centre 0, radius 4 cm is 28 cm^2. Find the length of the minor arc AB. **2**

Question 3 (12 marks) Use a SEPARATE page/ booklet. **Marks**

(a) Differentiate with respect to x:

(i) $y = x^2 \ln x$ **2**

(ii) $y = \sin^2 2x$ **2**

(b) Find:

(i) $\int \cos 2x \, dx$ **2**

(ii) $\int_0^1 \frac{3}{x+1} dx$ **2**

(c) If $\frac{dy}{dx} = 6x - 1$ and the function passes through $(1, 2.5)$, find y as a function of x. **2**

(d) Use the trapezoidal rule with four function values, to find an approximate value of the area under the curve $y = 3^x$, bounded by the x axis and x = 1 and x = 4 **2**

Question 4 (12 marks) Use a SEPARATE page/ booklet. **Marks**

(a) Sketch the graph $y = 4\sin 2x$ in the domaim $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$ **2**

(b) Solve the equation $9^x - 10(3^x) + 9 = 0$ **2**

(c) The mass M in grams of a radioactive substance may be expressed as $M = Ae^{kt}$, where t is the time in years and k is a constant.

(i) At time t=0, M=10. Find A. **1**

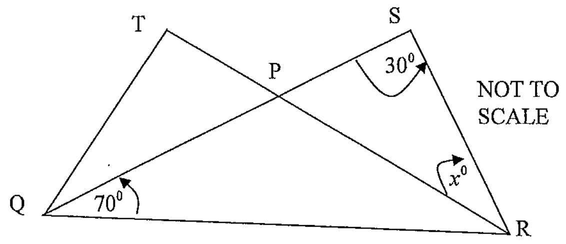
(ii) After 5 years the mass is 9 grams. Find the mass after 20 years. **3**

(d) (i) Show that $\frac{10}{4x^2 - 25} = \frac{1}{2x-5} - \frac{1}{2x+5}$ **1**

(ii) Hence evaluate $\int \frac{dx}{4x^2 - 25}$ **3**

Question 5 (12 marks) Use a SEPARATE page/ booklet. **Marks**

(a)



The equal sides QP and RP of the isosceles triangle QPR are produced to S and T respectively, such that PS=PT.

$\angle PQR = 70^0$, $\angle PSR = 30^0$ and $\angle PRS = x^0$

(i) Find the size of x. **2**

(ii) Prove there is another angle equal to x^0. **2**

(b) (i) If $y = e^{2x^3}$, find $\dfrac{dy}{dx}$ **2**

(ii) Hence, or otherwise, evaluate $\int_0^1 x^2 e^{2x^3}\,dx$ **2**

(c) Find the equations of the tangents to the parabola $y = x^2 - 2x - 3$ at the points where the line y = 5 cuts the parabola. **3**

(d) Is the following series an arithmetic or geometric progression? Justify your answer.

$\ln(x) + \ln(x^2) + \ln(x^3) + \ln(x^4) + \ldots\ldots$ **1**

Question 6 (12 marks) Use a SEPARATE page/ booklet. **Marks**

(a) Find the value(s) of m for which the equation $4x^2 - mx + 9 = 0$ has exactly one real root. **2**

(b) Madi is planning to buy a computer. She investigates the payment plans of two companies. Payments are made at the end of each month for both plans.

Pel Computers	**Hot Computers**
Pay a deposit of \$1510. The first monthly payment is \$120, and then each successive payment is \$20 more than the previous.	Pay a deposit of \$3160. The first monthly payment is \$500, and then each successive payment is \$10 less than the previous.

(i) Calculate in terms of n, where n = monthly repayment, the total amount paid, if Madi selects Pel computers. **2**

(ii) Calculate how many months it will be before the total amount paid into the Pel computer plan would be the same as the total paid into the Hot computer plan. **4**

(c) David and his father are playing with a wind-up toy, the velocity v cm/sec of the toy is given by $v = 60 - 3t^2$, where t is the time in seconds after the toy starts.

(i) After 10 seconds the toy is 15 cm from David. How far was the toy from David at the start. **2**

(ii) Determine the maximum velocity of the toy. **2**

Question 7 (12 marks) Use a SEPARATE page/ booklet. **Marks**

(a) The temperature in the kitchen of Nameeta's new home is controlled by a thermostat.

The temperature can be modelled by the equation: $T = 21.7 + \sin(\frac{t}{2})$ where T is the temperature in degrees Celsius and t is the time in minutes since Nameeta entered the kitchen.

How long will Nameeta have to wait before the temperature gets to 22.5^0 C. **2**

(b) (i) Sketch the curve $y = x^2$ from $x = 0$ to $x = 4$ and $y = -5x + 6$ on the same number plane, showing the point of intersection. **2**

(ii) Hence find the area enclosed between the two curves and the y axis. **3**

(c) Find the maximum and minimum turning points on the curve $y = \frac{x^3}{3} - 3x^2 + 8x + 4$ **3**

(d) The volume V cm^3 of a balloon is increasing such that its volume at any time t seconds is given by $V = \frac{\pi t^3}{3} - \frac{\pi t^2}{6} + \frac{1}{2}$. Find the rate at which the volume is increasing when t = 2 **2**

Question 8 (12 marks) Use a SEPARATE page/ booklet. **Marks**

(a) The curve $y = 4\ln x$, between $x = 1$ and $x = e$, is rotated about the y-axis.

(i) Show that the volume formed is given by

$$V = \pi \int_0^4 e^{0.5y}\, dy$$ **2**

(ii) Hence, find the exact volume. **2**

(b) (i) Prove $(\alpha + \beta)^3 = \alpha^3 + 3\alpha^2\beta + 3\alpha\beta^2 + \beta^3$ **1**

(ii) If α, β are the roots of $2x^2 - 3x - 1 = 0$, find the value of $\alpha^3 + \beta^3$. **2**

(c) Find the value of p, where x = p is a vertical line that divides the area between $y = \sqrt{x}$, $x = 16$ and the x-axis into two equal parts. **3**

(d) Given the sum of the first n terms of a series is given by $S_n = 3^n + 2n^2$, find the 13^{th} term. **2**

Question 9 (12 marks) Use a SEPARATE page/ booklet. **Marks**

(a) (i) Prove that the curve $y = x + 5 - \frac{4}{x-1}$, increases as x increases, ($x \neq 1$). **2**

(ii) Find the co-ordinates of the two points on the curve where the gradient is 2. **2**

(iii) If the tangents at these points meet the x-axis at P and Q, find the length of PQ. **1**

(b) The velocity, v metres per second, of a particle at time t seconds is given by $v = \frac{4t^3}{3} - 4t^2 + 6t + 1$. Show that the acceleration is never less than 2 metres per second per second. **3**

(c) Jason borrows \$50 000 from a bank at an interest rate 6% per annum compounded monthly. The loan will be repaid over ten years by equal monthly instalments of \$M. Let $R = 1 + \frac{0.06}{12}$.

(i) Show that the total amount owing, A, after n months is given by:

$$A = 50000R^n - M(1 + R + R^2 + \dots\dots R^{n-1})$$ **2**

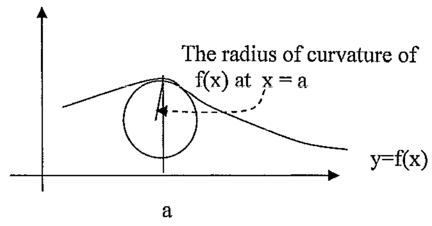
(ii) From this expression, calculate the monthly repayments for the loan to be repaid after 12 years. **2**

Question 10 (12 marks) Use a SEPARATE page/ booklet. **Marks**

(a) Sonia is in a dark room selecting socks from her drawer. She has only six socks in her drawer, a mixture of black and white. If she chooses two socks, the chances that she draws out a white pair is $\frac{2}{3}$.
What are the chances of drawing out a black pair? **3**

(b) Solve $\log_2 x - \log_2(x-2) = \frac{2}{3}\log_2 27$ **2**

(c) The radius of curvature of function y = f(x) is the radius of the circle that most closely approximates a curve at a given point. It is one measure of how sharply a curve is bending.



(i) Given: radius of curvature $= \dfrac{\left(1 + (\frac{dy}{dx})^2\right)^{3/2}}{\left|\frac{d^2y}{dx^2}\right|}$ use this formula to find the radius of curvature of the parabola $y = x^2$ at $x = 1$. **3**

(ii) Use the above formula, to show that the radius of curvature of the circle $y = \sqrt{r^2 - x^2}$ is r at every point. **4**

END OF PAPER

Trialmaths Enterprises

Mathematics

- **Solutions including marking scale**

These suggested solutions and marking schemes are issued as a guide only. Individual teachers may find many acceptable responses and employ different marking schemes.

ANSWERS QUESTION 1 and 2

Criteria
Q1 (a) One for approximation and one for rounding. (b) One multiplying by the conjugate and one for simplification (c) One for $\frac{1}{\sqrt{2}}$ and one for neg sign. (d) One for ln 1= 0, one for simplification. (e) One for $\tan 2x$ one for dividing by 2 (f) One each for derivative of e^{2x} and $5x$ **Q2** (a) (i) One for the correct answer.(a) (ii) One for substitution into formula, one for simplification. (iii) One for distance of bc, one for simplification. (b) One for substitution into formula, one for simplification. (c) One for finding A in terms of b, one for stationary point and one for maximun area. (d) One for θ and one for the length.

1(a) $15^{-1.5} = 0.01721... = 0.017(3\text{dp})$

(b) $\frac{5}{3-\sqrt{7}} = \frac{5}{3-\sqrt{7}} \times \frac{3+\sqrt{7}}{3+\sqrt{7}} = \frac{5(3+\sqrt{7})}{2}$

(c) $\sin\frac{5\pi}{4} = -\sin\frac{\pi}{4} = -\frac{1}{\sqrt{2}}$

(d) $\log_e e^2 + \log_e 1 = 2 + 0 = 2$

(e) $\int \sec^2 2x\, dx = \frac{1}{2}\tan 2x + c$

(f) $\frac{d}{dx}\left(e^{2x} - 5x\right) = 2e^{2x} - 5$

2(a)(i) $m = \frac{4-1}{-1-3} = \frac{3}{-4} = \frac{-3}{4}$

$y - 1 = \frac{-3}{4}(x-3)$

$4y - 4 = -3x + 9$

$3x + 4y - 13 = 0$

(ii) $p = \left|\frac{3(1)+4(2)-13}{\sqrt{3^2+4^2}}\right| = \left|\frac{-2}{5}\right| = \frac{2}{5}$

(iii) $d_{bc} = \sqrt{(3+1)^2 + (1-4)^2} = 5$

$\text{Area} = \frac{1}{2} \times 5 \times \frac{2}{5} = 1 \text{ sq unit}$

(b) $1 + 3 + 3^2 + 3^3 + 3^4 + ...$

$S_n = \frac{a\left(r^n - 1\right)}{r-1}$

$S_{15} = \frac{3^{15}-1}{3-1} = 7174453$

(c)
Let b be the base and h the height

$b + h = 10 \therefore h = 10 - b$

$A = \frac{1}{2}bh = \frac{1}{2} \times b \times (10-b)$

$A = 5b - \frac{1}{2}b^2$

$\frac{dA}{db} = 5 - b$

$0 = 5 - b \therefore b = 5$

$\frac{d^2A}{db^2} = -1 < 0 \therefore$ max area $= \frac{1}{2} \times 5 \times 5 = 12.5\text{cm}^2$

(d) $A = \frac{1}{2}r^2\theta$

$28 = \frac{1}{2}(4)^2\theta$

$\theta = \frac{7^c}{2}$

$L = r\theta$

$= 4 \times \frac{7}{2}$

$= 14 \text{ cm}$

ANSWERS QUESTION 3 and 4

Criteria

Q3 (a) (i) One for product rule, one for simplification. (ii) One for derivative of sin 2x and one for simplification (b) (i) One for sin 2x and one for a $\frac{1}{2}$ (ii) One for integration and one for simplification. (c) One for integration and one for simplification. (d) One for substitution into trapezoidal rule and one for simplification.
Q4 (a) One for shape of curve and period, one for x intercepts and amplitude (b) One for quadratic equation, one for simplification. (c) (i) One for correct answer. (c)(ii) One for $\frac{9}{10}=e^{5k}$, one for finding k, one for simplification. (d) (i) One for correct answer. (ii) One for setting up the integral, one for integration, one for simplification.

3(a) (i) $\frac{d}{dx}x^2\ln x = x^2\frac{1}{x}+\ln x\times 2x = x(1+2\ln x)$

(ii) $\frac{d}{dx}\sin^2 2x = 2\sin 2x\times 2\cos 2x = 4\sin 2x\cos 2x$

(b)(i) $\int\cos 2x\,dx = \frac{1}{2}\sin 2x + c$

(ii) $\int_0^1 \frac{3}{x+1}dx = \left[3\ln(x+1)\right]_0^1 = 3\ln 2 - 3\ln 1 = 3\ln 2$

(c) $\frac{dy}{dx} = 6x-1$

$y = 3x^2 - x + c$

Sub $(1, 2.5)$

$2.5 = 2 + c$

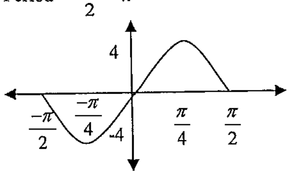
$c = \frac{1}{2}$

$\therefore y = 3x^2 - x + \frac{1}{2}$

(d) $A \approx \frac{1}{2}\left[3+3^4+2(3^2+3^3)\right] \approx \frac{1}{2}[84+72] \approx 78\text{u}^2$

4 (a) Amplitude = 4

Period = $\frac{2\pi}{2} = \pi$



4 (b)

$9^x - 10(3^x) + 9 = 0$

$3^{2x} - 10(3^x) + 9 = 0$

$(3^x)^2 - 10(3^x) + 9 = 0$

$v^2 - 10v + 9 = 0$ where $v = 3^x$

$(v-9)(v-1) = 0$

$v = 3^x = 9 \rightarrow x = 2$

$v = 3^x = 1 \rightarrow x = 0$

(c)(i) $M = Ae^{kt}$

$10 = Ae^0$

$A = 10$

(ii) $M = 10e^{kt}$

$9 = 10e^{5k}$

$\frac{9}{10} = e^{5k}$

$\ln(9/10) = 5k$

$k = \frac{1}{5}\ln(9/10) = -0.021072103$

$M = 10e^{-0.021072103\times 20}$

$M = 6.561...$

$M = 6.6$grams

(d) (i)

$\frac{1}{2x-5} - \frac{1}{2x+5} = \frac{(2x+5)-(2x-5)}{(2x-5)(2x+5)} = \frac{10}{4x^2-25}$

(ii)

$\int\frac{dx}{4x^2-25} = \frac{1}{10}\left[\int\frac{dx}{2x-5} - \int\frac{dx}{2x+5}\right]$

$= \frac{1}{10}\left[\frac{1}{2}\int\frac{2dx}{2x-5} - \frac{1}{2}\int\frac{2dx}{2x+5}\right]$

$= \frac{1}{20}\left[\ln(2x-5) - \ln(2x+5)\right] = \frac{1}{20}\ln\left(\frac{2x-5}{2x+5}\right)$

ANSWERS QUESTION 5 and 6

Criteria

Q5(a) (i) One for finding $\angle QPR$, one for simplification (a)(ii) One for showing two steps of proof, one for conclusion. (b) (i) One for $6x^2$, one for e^{2x^3} (ii) one for $\int_0^1 x^2e^{2x^3}dx = \frac{1}{6}\left(e^{2x^3}\right)_0^1$ and one for simplification (c) One for points of intersection, one for differentiation, one for equation of tangents (d) One for correct answer. **Q6**(a) One for discriminate, one for simplification (b)(i) One for substitution into formula and one for simplification.(ii) One for total amount for Hot computer plan in terms of n, one for $1510+10n^2+110n = 3160-5n^2+505n$, one for $3n^2-79n-330=0$ and one for simplification. (c)(i) One for finding c and one for finding s.(ii) One for second derivative equals zero, one for simplification.

(5)(a)(i)

PQ = PR (given)

$\angle PQR = \angle PRQ = 70^0$ (base angles iss. triangle)

$\angle QPR = 40^0$ (third angle)

$\angle QPR = \angle PSR + x$ (ext. angle of triangle)

$x = 40^0 - 30^0 = 10^0$

(ii)

$\Delta TPQ \equiv \Delta SPR$ since

QP = RP (given)

PT = PS (given)

$\angle TPQ = \angle SPR$ (vertically opp)

$\therefore \Delta TPQ \equiv \Delta SPR$ (SAS)

$\therefore \angle TQP = \angle SRP = x$ (corr. $\angle$'s of cong. Δ's)

(b) (i)

$y = e^{2x^3}\ \therefore \frac{dy}{dx} = 6x^2e^{2x^3}$

(ii)

$y = e^{2x^3}$

$\frac{dy}{dx} = 6x^2e^{2x^3}$

$\frac{1}{6}\int_0^1\frac{dy}{dx}dx = \int_0^1 x^2e^{2x^3}dx$

$\int_0^1 x^2e^{2x^3}dx = \frac{1}{6}\left(e^{2x^3}\right)_0^1$

$= \frac{1}{6}\left(e^2-1\right)$

(c)

$y = x^2 - 2x - 3$ and $y = 5$

$x^2 - 2x - 3 = 5$

$x^2 - 2x - 8 = 0$

$(x-4)(x+2) = 0$

i.e. (4,5) and (−2,5)

Gradient of tangents at x=4 and -2

6 (a)

$\Delta = b^2 - 4ac$

$= (-m)^2 - 4(4)(9) = 0$

$m^2 - 144 = 0$

$m = \pm 12$

(b) (i)

Pel computers

total amount paid

$= 1510 + \frac{n}{2}[2\times 120 + (n-1)20]$

$= 1510 + \frac{n}{2}(240 + 20n - 20)$

$= 1510 + 10n^2 + 110n$

(ii)

Total amount paid for Hot computers

$= 3160 + \frac{n}{2}[2\times 500 - 10(n-1)]$

$= 3160 + \frac{n}{2}(1000 - 10n + 10)$

$= 3160 - 5n^2 + 505n$

Pel computers = Hot computers

$1510 + 10n^2 + 110n = 3160 - 5n^2 + 505n$

$15n^2 - 395n - 1650 = 0$

$3n^2 - 79n - 330 = 0$

$n = \frac{79 \pm\sqrt{(-79)^2 - 4(3)(-330)}}{(2)(3)} = -3\frac{2}{3}$ or 30

It will take 30 months before the total amount paid into the Pel and Hot computer plans will be the same.

(5) (c) continued

$\frac{dy}{dx} = 2x - 2$

$= 6 \text{ at } x = 4$

$= -6 \text{ at } x = -2$

Equation of tangent at x = 4

y-5 = 6(x-4)

y = 6x-19

Equation of tangent at x = -2

y-5 = -6(x+2)

y = -6x − 7

(d) Test for GP:

$\frac{\ln x^2}{\ln x} = \frac{2\ln x}{\ln x} = 2$

$\frac{\ln x^3}{\ln x^2} = \frac{3\ln x}{2\ln x} = \frac{3}{2} \therefore$ not GP

Test for *AP*:

$\ln x^2 - \ln x = 2\ln x - \ln x = \ln x$

$\ln x^3 - \ln x^2 = 3\ln x - 2\ln x = \ln x$

$\therefore$ AP

(c) (i)

$v = 60 - 3t^2$

$s = 60t - t^3 + c$

$15 = 60(10) - 10^3 + c$

$c = 415$

$\therefore s = 60t - t^3 + 415$

when $t = 0$

$s = 415\,cm$

(ii)

Maximum velocity occurs when

$\frac{d^2v}{dt^2} = 0$

$\therefore -6t = 0$

$t = 0$

$\therefore v = 60 cm/s$

ANSWERS QUESTION 7 and 8

Criteria

Q7 (a)One for 0.8=sin(t/2), one for simplification (b)(i)One for sketching both curves, one for points of intersection (ii) One for $A = \int_1^6 (\frac{1}{5}(6-y)dy + \int_0^1 \sqrt{y}dy$, one for the integration and one for simplification. (c) One for st. Pts, one for second derivative and one for finding the nature of both. (d) One for the differentiation, one for simplification.**Q8**(a)(i) One for making x the subject, one for simplification. (ii) One for integration and one for simplification. (b) (i) One for the correct answer. (ii) One for sum and product, one for simplification.(c) One for integration, one for substitution, one for finding p (d) One for forming expression, one for simplification.

7(a)

$T = 21.7 + \sin(\frac{t}{2})$

$22.5 = 21.7 + \sin(\frac{t}{2})$

$0.8 = \sin(\frac{t}{2})$

$\frac{t}{2} = 0.927$

$t = 1.854$ minutes $= 1$ minute 51 seconds

(8)(a)(i)

$y = 4\ln x$

if $x = 1, y = 4\ln 1 = 0$

if $x = e, y = 4\ln e = 4$

now $y = 4\ln x$

$\frac{y}{4} = \ln x$

$e^{\frac{y}{4}} = x$

$x^2 = \left(e^{\frac{y}{4}}\right)^2$

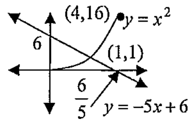
(b) (i)

$y = x^2 = -5x + 6$

$x^2 + 5x - 6 = 0$

$(x+6)(x-1) = 0$

i.e. (1,1)



(ii)

$A = \int_1^6 (\frac{1}{5}(6-y)dy + \int_0^1 \sqrt{y}dy$

$= \left[\frac{6y}{5} - \frac{y^2}{10}\right]_1^6 + \left[\frac{2}{3}y^{3/2}\right]_0^1$

$= (\frac{36}{5} - \frac{36}{10}) - (\frac{6}{5} - \frac{1}{10}) + \frac{2}{3}$

$= 3\frac{1}{6}u^2$

(c) $y = \frac{x^3}{3} - 3x^2 + 8x + 4$

$\frac{dy}{dx} = x^2 - 6x + 8$

St pts

$0 = (x-4)(x-2)$

$\therefore \left(4, 9\frac{1}{3}\right)$ and $\left(2, 10\frac{2}{3}\right)$

$\frac{d^2y}{dx^2} = 2x - 6$

at $x = 4$, $\frac{d^2y}{dx^2} > 0 \therefore$ rel.min.at $\left(4, 9\frac{1}{3}\right)$

at $x = 2$, $\frac{d^2y}{dx^2} < 0 \therefore$ rel.max.at $\left(2, 10\frac{2}{3}\right)$

(d)

$V = \frac{\pi t^3}{3} - \frac{\pi t^2}{6} + \frac{1}{2}$

$\frac{dv}{dt} = \pi t^2 - \frac{\pi t}{3}$

when $t = 2$

$\frac{dv}{dt} = 4\pi - \frac{2\pi}{3}$

$= \frac{10\pi}{3} cm^3/s$

$x^2 = e^{\frac{y}{2}}$

$\therefore v = \pi \int_0^4 e^{\frac{y}{2}} dy = \pi \int_0^4 e^{0.5y} dy$

(ii)

$v = \pi \left[\frac{e^{\frac{y}{2}}}{\frac{1}{2}}\right]_0^4 = \pi\left[2e^2 - 2e^0\right]$

$= \pi\left[2e^2 - 2\right] = 2\pi\left[e^2 - 1\right]\ u^3$

(b)(i)

$(\alpha+\beta)^3 = (\alpha+\beta)(\alpha^2+2\alpha\beta+\beta^2)$

$= \alpha^3 + 2\alpha^2\beta + \alpha\beta^2 + \alpha^2\beta + 2\alpha\beta^2 + \beta^3$

$= \alpha^3 + 3\alpha^2\beta + 3\alpha\beta^2 + \beta^3$

(ii)

$2x^2 - 3x - 1 = 0$ where $\alpha+\beta = \frac{3}{2}$ and $\alpha\beta = -\frac{1}{2}$

$\therefore \alpha^3 + \beta^3 = (\alpha+\beta)^3 - 3\alpha^2\beta - 3\alpha\beta^2$

$= (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)$

$= \left(\frac{3}{2}\right)^3 - 3\left(\frac{-1}{2}\right)\left(\frac{3}{2}\right) = \left(\frac{3}{2}\right)^3 + \frac{9}{4} = 5\frac{5}{8}$

(c)

$\int_0^p \sqrt{x}\,dx = \int_p^{16} \sqrt{x}dx$

$\left[\frac{2}{3}x^{\frac{3}{2}}\right]_0^p = \left[\frac{2}{3}x^{\frac{3}{2}}\right]_p^{16}$

$\frac{2}{3}p^{\frac{3}{2}} = \frac{2}{3}(16)^{\frac{3}{2}} - \frac{2}{3}p^{\frac{3}{2}}$

$\therefore \frac{4}{3}p^{\frac{3}{2}} = \frac{2}{3}(16)^{\frac{3}{2}}$

$p = \left[\frac{1}{2}(16)^{\frac{3}{2}}\right]^{\frac{2}{3}} = 32^{\frac{2}{3}} = 10.079... = 10.08(2\text{dp})$

8(d)

$S_n = 3^n + 2n^2$

$T_{13} = S_{13} - S_{12}$

$= (3^{13} + 2\times 13^2) - (3^{12} + 2\times 12^2)$

$= 1062932$

ANSWERS QUESTION 9

Criteria

(9) (a) (i) One for derivative, one for conclusion. (ii) One for finding x values, one for finding y values. (iii) One for the correct answer. (b) One for differentiation, one for completing the square and one for simplification (c)(i) One for finding equation after two months, one for simplification. (ii) One for noting it is a GP, one for simplification.

9 (a)

(i) $y = x + 5 - \frac{4}{x-1} \quad x \neq 1$

$\frac{dy}{dx} = 1 + \frac{4}{(x-1)^2}$

As $\frac{dy}{dx}$ is always positive, the function is always increasing.

(ii) $1 + \frac{4}{(x-1)^2} = 2$

$\frac{4}{(x-1)^2} = 1$

$(x-1)^2 = 4$

$x - 1 = \pm 2$

i.e. (3,6) and (−1,6)

(iii)

At (3,6) gradient = 2

$y - 6 = 2(x-3)$

$y = 2x$

meets x axis at P(0,0)

At (−1,6)

$y - 6 = 2(x+1)$

$y = 2x + 8$

meets x axis at Q (−4,0)

∴ Length of PQ = 4 units

(b)

$v = \frac{4t^3}{3} - 4t^2 + 6t + 1$

$a = 4t^2 - 8t + 6$

$= 4(t^2 - 2t) + 6$

$= 4(t^2 - 2t + 1) + 6 - 4$

$= 4(t-1)^2 + 2$

now $(t-1)^2 \geq 0$ for all t

∴ the least value of a is 2m/s^2

(c) (i)

Let A_n be the amount owing after n months.

$A_1 = 50\,000R - M$

$A_2 = (50\,000R - M)R - M$

$= 50\,000R^2 - MR - M$

$A_3 = (50\,000R^2 - MR - M)R - M$

$= 50000R^3 - MR^2 - MR - M$

$= 50000R^3 - M(1 + R + R^2)$

$\therefore A_n = 50000R^n - M(1 + R + R^2 + ... + R^{n-1})$

(ii)

$A_{144} = 0$

$M = \frac{50000R^{144}}{(1 + R + R^2 + ...R^{n-1})}$

$= \frac{50000R^{144}(R-1)}{R^n - 1}$ (note GP)

$= \frac{50000(1.005)^{144} \times 0.005}{(1.005)^{144} - 1}$

$= \frac{512.6877039...}{1.0507508...}$

= $487.925114..

= $487.93

ANSWERS QUESTION 10

Criteria

(10) (a)(i) One mark for probability of selecting one sock i.e. x/6, one mark for forming equation for selecting two socks, one mark for simplification (b) One mark for forming equation $\log_2 \frac{x}{x-2} = \log_2 9$,one mark for simplification. (c)(i) One mark for first and second derivative, one mark for substitution and one mark for simplification.(ii) One mark for first and second derivative, one mark for simplification of second derivative, one for substitution into formula and one for simplification.

(10) (a)

Let x be the number of white socks in the drawer. The probability that Sonia picks a white sock is $\frac{x}{6}$ and the probability that she picks two white socks is

$\frac{x}{6} \times \frac{x-1}{5} = \frac{x(x-1)}{30}$

$\therefore \frac{2}{3} = \frac{x(x-1)}{30}$

$x(x-1) = 20$

$x^2 - x - 20 = 0$

$(x-5)(x+4) = 0$

$x = 5$

So Sonia has 5 white socks and 1 black sock

Thus probability (black pair) = 0

(b)

$\log_2 x - \log_2 (x-2) = \log_2 27^{2/3} = \log_2 9$

$\log_2 \frac{x}{x-2} = \log_2 9$

$\frac{x}{x-2} = 9$

$x = 9x - 18$

$8x = 18$

$x = \frac{9}{4} = 2\frac{1}{4}$

(c) (i) $y = x^2$

$\frac{dy}{dx} = 2x$ and $\frac{d^2y}{dx^2} = 2$

$\therefore$ radius of curvature $= \frac{\left[1 + (2x)^2\right]^{\frac{3}{2}}}{|2|} = \frac{\left(1 + 4x^2\right)^{\frac{3}{2}}}{2}$

at $x = 1$

$= \frac{(1+4)^{\frac{3}{2}}}{2} = \frac{1}{2}\sqrt{5^3} = 5.590.... = 5.6$

(c)

(ii) $y = \left(r^2 - x^2\right)^{1/2}$

$\frac{dy}{dx} = \frac{1}{2}\left(r^2 - x^2\right)^{-\frac{1}{2}} \times (-2x) = -x\left(r^2 - x^2\right)^{-1/2}$

$\frac{d^2y}{dx^2} = (-x) \times \frac{-1}{2}\left(r^2 - x^2\right)^{\frac{-3}{2}} \times (-2x) + (-1) \times \left(r^2 - x^2\right)^{\frac{-1}{2}}$

$= -x^2\left(r^2 - x^2\right)^{\frac{-3}{2}} - \left(r^2 - x^2\right)^{\frac{-1}{2}}$

$= \left(r^2 - x^2\right)^{\frac{-3}{2}}\left[-x^2 - \left(r^2 - x^2\right)\right]$

$= -r^2\left(r^2 - x^2\right)^{\frac{-3}{2}}$

∴ radius of curvature

$= \frac{\left\{1 + \left[-x\left(r^2 - x^2\right)^{\frac{-1}{2}}\right]^2\right\}^{\frac{3}{2}}}{r^2\left(r^2 - x^2\right)^{\frac{-3}{2}}} = \frac{\left[1 + x^2\left(r^2 - x^2\right)^{-1}\right]^{\frac{3}{2}}}{r^2\left(r^2 - x^2\right)^{\frac{-3}{2}}}$

$= \frac{\left[1 + \frac{x^2}{r^2 - x^2}\right]^{\frac{3}{2}}}{\frac{r^2}{\left(r^2 - x^2\right)^{\frac{3}{2}}}} = \frac{\frac{\left(r^2 - x^2 + x^2\right)^{\frac{3}{2}}}{\left(r^2 - x^2\right)^{\frac{3}{2}}}}{\frac{r^2}{\left(r^2 - x^2\right)^{\frac{3}{2}}}} = \frac{\left(r^2\right)^{\frac{3}{2}}}{r^2}$

$= \frac{r^3}{r^2}$

$= r$