2004 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
- Approved calculators may be used
- A table of standard integrals is provided
- All necessary working should be shown in every question

Total marks –120

- Attempt Questions 1 – 10
- All questions are of equal value

Care has been taken to ensure that this paper is free of errors and that it mirrors the format and style of past HSC papers. Moreover, some questions have been adapted from previous HSC examinations as well as from trial examinations from a variety of schools, in an attempt to provide students with exposure to a broad range of possible questions.

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Total marks - 120
Attempt Questions 1 - 10
ALL questions are of equal value

Start each question on a SEPARATE page or in a SEPARATE booklet

Question 1 (12 marks) Use SEPARATE page/ booklet. **Marks**

(a) Find the exact value of $81^{\frac{1}{4}} \times 125^{\frac{1}{3}}$ **1**

(b) Solve for $x: |3x-4| = 2$ **2**

(c) Find: $\int (3x^4 - e^{2x})\,dx$ **2**

(d) Rationalise the denominator $\dfrac{1}{1-\sqrt{3}}$ **2**

(e) There are 15 teams in a football competition. The probability that one team will win the competition is $\dfrac{1}{15}$. Comment briefly on this statement. **1**

(f) Find the primitive function of $3\sec^2 2x$ **2**

(g) Evaluate correct to 3 significant figures $\dfrac{67.78-14.43}{(6.8\times 5.4)^2}$ **2**

Question 2 (12 marks) Use SEPARATE page/ booklet. **Marks**

(a) Find the equation of the tangent on $y = \ln(2x^2 + 1)$ at the point $(2, \ln 9)$. Express your answer in general equation form. **3**

(b) Find the values of k for which the equation $x^2 + (k+3)x - k = 0$ has real roots. **3**

(c) Find the derivative of $\cos^2 5x$ **1**

(d) When John started working, his annual salary was \$36 400. At the start of each following year his salary increased by \$4 500. What was his salary during his ninth year of working? **1**

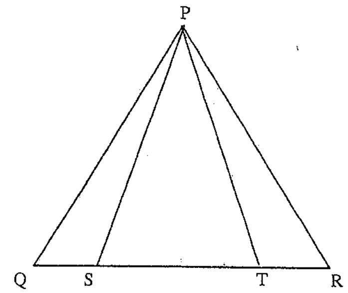
(e) Find the exact value of $\cos 210^0 + \tan 480^0$ **2**

(f) Solve for θ: $\sin\theta = \dfrac{-\sqrt{3}}{2}$, $0 \le \theta \le 360^0$ **2**

Question 3 (12 marks) Use SEPARATE page/ booklet. **Marks**

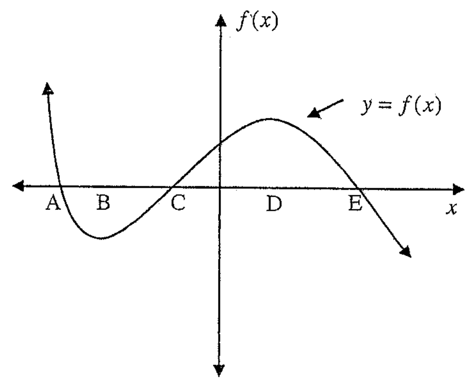
(a) Show by shading on a number plane, the regions where $y \le x^2 + 1$ and $x^2 + y^2 \le 4$ hold simultaneously. (Note: do not find the point of intersection) **4**

(b) In Δ PQR, PQ = PR, $\angle PST = \angle PTS$
Copy this diagram into your booklet and prove $\angle QPS = \angle RPT$, give reasons. **3**



NOT TO SCALE

(c)



The above diagram shows a sketch of the function $y = f(x)$.
Copy this diagram into your writing booklet, and on the same diagram sketch the gradient function $y = f'(x)$. **2**

(d) Solve for x where $0^0 \le x \le 360^0$

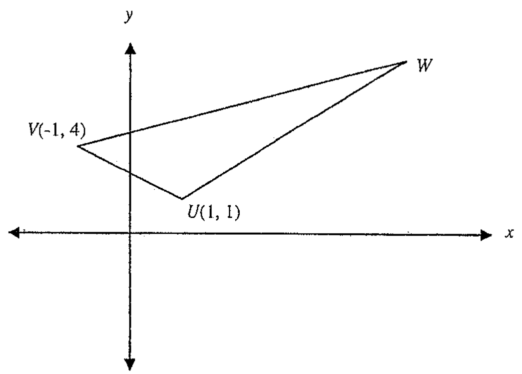
$$\sec(\frac{1}{2}x - 107^0) = 2$$ **3**

Question 4 (12 marks) Use SEPARATE page/ booklet. **Marks**

(a) A ship (S) observes a lighthouse (L) to be on a bearing of $N26^0 E$ and a tower (T) to be on a bearing of $N70^0 E$. The bearing of the lighthouse from the tower is $N15^0 E$ and the distance from the lighthouse to the tower is 6.8 km.

Draw a neat sketch of the above information and find the distance of the ship to the tower to one decimal place. **3**

(b)



NOT TO SCALE

The figure shows a triangle *U, V, W* with *U*(1, 1) and *V*(-1, 4). The gradients of *UV*, *UW* and *VW* are $-3p$, $3p$ and p respectively.

(i) Show that the value of P is $\frac{1}{2}$. **2**

(ii) Find the coordinates of *W*. **3**

(iii) Show that $WU = 2VU$. **2**

(c) Find the second derivative of $\frac{1}{\sqrt{x}}$. Express the answer as a surd. **2**

Question 5 (12 marks) Use SEPARATE page/ booklet. **Marks**

(a) (i) Show that (1, 1) lies on the curves $y = x^2$ and $y = \sqrt{x}$. **1**

(ii) Find the volume of the solid generated by revolving about the x axis the region enclosed between the curves $y = x^2$ and $y = \sqrt{x}$. **2**

(b) Use the trapezoidal rule with 5 ordinate values, to find the area bounded by the curve $y = \tan x$, the x axis and the lines $x = 0$ and $x = \frac{1}{3}\pi$. Give your answer correct to 1 decimal place. **3**

(c) (i) Sketch the graphs $y = \sin x + 1$ *and* $y = \cos x$ on the same number plane for $0 \le x \le 2\pi$. **2**

(ii) Using the graph, or otherwise, find all solutions to $\sin x - \cos x + 1 = 0$ in the domain $0 \le x \le 2\pi$. **1**

(d) By using a suitable substitution, solve the equation:

$$3^{2x} - 4(3^x) + 3 = 0$$ **2**

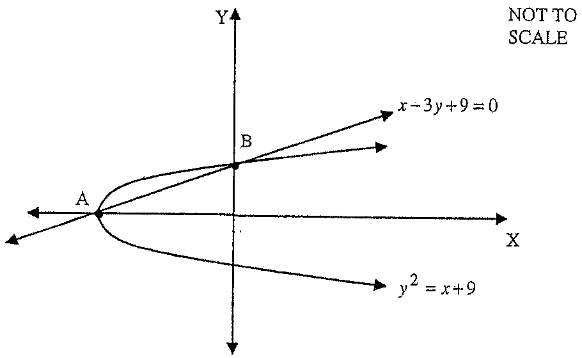
(e) Solve for *x*: $e^{\ln 5x} = 10$ **1**

Question 6 (12 marks) Use SEPARATE page/ booklet. **Marks**

(a) In the following diagram the curve $y^2 = x + 9$ and the line $x - 3y + 9 = 0$. intersect at A and B.

(i) Find the coordinates of A and B. **2**

(ii) Calculate the area bounded by the two curves. **2**



(b) A radioactive by-product decays according to the formula $M = M_0e^{-0.04t}$, where M is the mass after t years and M_0 is the original mass. The company responsible for the safe storage of this radioactive by-product considers that any quantity of the by-product can be transferred from a "HIGH RADIATION COMPLEX" into a "LOWER GRADE RADIATION SHIELD" after 60% of its original mass has decayed.

Find the minimum number of complete years for which the by-product must remain in the "HIGH RADIATION COMPLEX". **3**

(c) The function $f(x)$ is defined by the rule $f(x) = 6x\sqrt{x+2}$

(i) Find the domain of the function $f(x)$. **1**

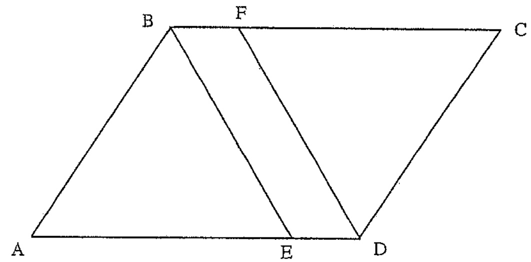
(ii) Find the location of any stationary point(s). **2**

(iii) Determine the nature of the stationary point(s). **1**

(iv) Sketch the function. **1**

Question 7 (12 marks) Use SEPARATE page/ booklet. **Marks**

(a)



In the figure, ABCD is a parallelogram.
BE bisects $\angle ABC$ and FD bisects $\angle CDE$.

(i) Copy the diagram into your answer booklet and prove that ΔABE is congruent to ΔFCD. **3**

(ii) Prove BE = FD. **1**

(b) Find the exact value of $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sin x \, dx$ **3**

(c) John puts \$2 000 into a superannuation account on his 40th birthday. He continues to do this on his birthday up to and including his 60th birthday. The interest he earns is 10% pa compounded yearly.

On his 61st birthday he leaves the accumulated amount into an account which earns 8% pa compounded yearly.

He will collect his accumulated amount on his 65th birthday.

(i) How much does the first \$2 000 accumulate to when John celebrates his 61st birthday. **1**

(ii) How much will John collect on his 65th birthday. **4**

Question 8 (12 marks) Use SEPARATE page/ booklet. **Marks**

(a) Janine has five pairs of stockings in a box, each pair a different colour. She selects one stocking at a time and at random from the box.

(i) Find the probability when she chooses her second stocking it will be a matching pair. **1**

(ii) Find the probability that she does not have a matching pair after selecting the third stocking. **2**

(b) Two particles A and B move along a straight line so that their displacements, in centimetres, from the origin at time t seconds are given by

$X_A = 4t^2 - t^3$ and $X_B = 4t - 6$

(i) Which is moving faster when $t = 1$. **2**

(ii) When do the particles travel at the same speed. **1**

(iii) What is the acceleration of the particle A when $t = 3$. **2**

(iv) What is the maximum positive displacement of particle A. **2**

(c) Find the coordinates of the vertex of the parabola $y^2 - 6y - 9x - 9 = 0$.. **2**

Question 9 (12 marks) Use SEPARATE page/ booklet **Marks**

(a) The surface area of a sphere is increasing a constant rate of $6cm^2 per\sec$. Find the rate of increase of

(i) the radius at the instant when the radius is $5cm$. **3**

(ii) the volume at the instant when the radius is $5cm$. **2**

(b) (i) Show that $\frac{d}{dx}\left[\ln\sqrt{\frac{2+x}{2-x}}\right] = \frac{2}{4-x^2}$. **2**

(ii) Hence or otherwise, evaluate $\int_0^1 \frac{4}{4-x^2}dx$. **2**

(c) Local health officials have advised the citizens of Mathsville that a flu epidemic is spreading throughout the city. Furthermore, x days after January 1, the officials expect that new cases of the flu will occur at the rate of $700x - 6x^2$ cases per day.

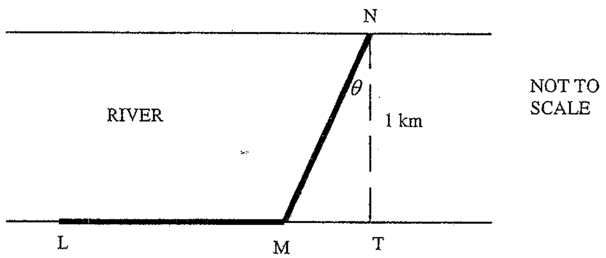
How many new cases will there be during the period January 3 to January 8. **3**

Question 10 (12 marks) Use SEPARATE page/ booklet. **Marks**

(a) A circle is divided into 6 unequal sectors, the angles of which are in arithmetic progression. Given that the largest angle is three times the smallest angle, find the angle of each sector. **3**

(b) Given $y = xe^{-2x}$, prove that $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 0$ **3**

(c) It is desired to construct a cable link between two points L and N, which are situated on opposite banks of a river of width 1km. L lies 3km upstream from N. It costs 3 times as much to lay a length of cable underwater as it does to lay the same length overland. The following diagram is a sketch of the cables, where θ is the angle where NM makes with the direct route across the river.



(i) Prove $MN = \sec\theta$ and $MT = \tan\theta$. **1**

(ii) If segment LM costs c dollars per km, prove the total cost (T) of laying the cable is given by
$$T = 3c - c\tan\theta + 3c\sec\theta.$$ **2**

(iii) At what angle should the cable cross the river in order to minimize the total cost of laying it. **3**

STANDARD INTEGRALS

$\int x^n dx$	$= \frac{1}{n+1}x^{n+1}, n \neq -1;\ x \neq 0,$ if $n < 0$
$\int \frac{1}{x} dx$	$= \ln x,\ x > 0$
$\int e^{ax} dx$	$= \frac{1}{a}e^{ax},\ a \neq 0$
$\int \cos ax dx$	$= \frac{1}{a}\sin ax,\ a \neq 0$
$\int \sin ax dx$	$= -\frac{1}{a}\cos ax,\ a \neq 0$
$\int \sec^2 ax dx,$	$= \frac{1}{a}\tan ax,\ a \neq 0$
$\int \sec ax \tan ax dx$	$= \frac{1}{a}\sec ax,\ a \neq 0$
$\int \frac{1}{a^2+x^2} dx$	$= \frac{1}{a}\tan^{-1}\frac{x}{a},\ a \neq 0$
$\int \frac{1}{\sqrt{a^2-x^2}} dx$	$= \sin^{-1}\frac{x}{a},\ a > 0,\ -a < x < a$
$\int \frac{1}{\sqrt{x^2-a^2}} dx$	$= \ln(x + \sqrt{x^2-a^2}),\ x > a > 0$
$\int \frac{1}{\sqrt{x^2+a^2}} dx$	$= \ln(x + \sqrt{x^2+a^2})$

NOTE : $\ln x = \log_e x, \quad x > 0$

2004 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics

- **Solutions including marking scale**
- **Mapping grid**

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ANSWERS QUESTION 1

Question 1 (a)

Criteria	Marks
• One mark for final answer	1

Answer:

$$3\times 5=15$$

Question 1 (b)

Criteria	Marks
• One mark for each for solving both equations correctly	2

Answer:

$$3x-4=2 \quad and \quad 3x-4=-2$$
$$3x=6 \quad\quad 3x=2$$
$$x=2 \quad\quad x=\frac{2}{3}$$

Question 1 (c)

Criteria	Marks
• One mark for either $\frac{3x^5}{5}$ or $\frac{e^{2x}}{2}$ and one for final answer	2

Answer:

$$\int(3x^4-e^{2x})dx=\frac{3x^5}{5}-\frac{e^{2x}}{2}+c$$

Question 1 (d)

Criteria	Marks
One mark for multiplying by $\frac{1+\sqrt{3}}{1+\sqrt{3}}$ and one for simplification	2

Answer:

$$\frac{1}{1-\sqrt{3}}=\frac{1}{1-\sqrt{3}}\times\frac{1+\sqrt{3}}{1+\sqrt{3}}$$
$$=\frac{1+\sqrt{3}}{-2}$$
$$=\frac{-1-\sqrt{3}}{2}$$

Question 1 (e)

Criteria	Marks
• One mark for correct statement	1

Answer:

Statement is incorrect as all teams are not of the same ability

Question 1 (f)

Criteria	Marks
• One mark for $\tan 2x$ and one for final answer	2

Answer:

$$\int 3\sec^2 2x.dx = \frac{3}{2}\tan 2x + c$$

Question 1 (g)

Criteria	Marks
• One mark for 0.0395666...and one for final answer	2

Answer :

$$\frac{67.78-14.43}{(6.8\times5.4)^2} = 0.0395666\ldots$$

$$= 0.0396 \ (3sig\ figs)$$

ANSWERS QUESTION 2

Question 2 (a)

Criteria	Marks
• One mark for $\frac{dy}{dx} = \frac{4x}{2x^2+1}$, one for the equation in non general form and one for the equation in general form	3

Answer:

$y = \ln(2x^2+1)$

$\frac{dy}{dx} = \frac{4x}{2x^2+1}$

at x=2, $\frac{dy}{dx} = \frac{8}{9}$

Equation of tangent

$y - \ln 9 = \frac{8}{9}(x-2)$

$8x - 9y + 9\ln 9 - 16 = 0$

Question 2 (b)

Criteria	Marks
• One mark for $(k+3)^2 + 4k \ge 0$, one for $(k+9)(k+1) \ge 0$, one for $k \le -9$ *and* $k \ge -1$	3

Answer:

$\Delta = (k+3)^2 - 4(1)(-k)$

For real roots $\Delta \ge 0$

ie $(k+3)^2 + 4k \ge 0$

$k^2 + 10k + 9 \ge 0$

$(k+9)(k+1) \ge 0$

$k \le -9$ *and* $k \ge -1$

Question 2 (c)

Criteria	Marks
• One mark for correct answer	1

Answer:

$$\frac{d}{dx}(\cos^2 5x) = 2(\cos 5x)\times -5\sin 5x$$

$$= -10\cos 5x \sin 5x$$

Question 2 (d)

Criteria	Marks
• One mark for the correct answer	1

Answer:

$T_9 = 36400 + 8\times4500$ (This is an AP where a=36 400, d = 4 500 and n = 8)

$= \$72400$

Question 2 (e)

Criteria	Marks
• One mark for $-\cos 30^0 - \tan 60^0$, one for simplification	2

Answer:

$$\cos 210^0 + \tan 480^0 = \cos(180+30)^0 + \tan(360^0 + 120^0)$$

$$= -\cos 30^0 - \tan 60^0$$

$$= \frac{-\sqrt{3}}{2} - \sqrt{3}$$

$$= \frac{-3\sqrt{3}}{2}$$

Question 2 (f)

Criteria	Marks
• One mark for using 60^0 and one for final answers	2

Answer:

$$\sin\theta = \frac{-\sqrt{3}}{2}$$

$\therefore \theta$ *is in 3rd and 4th quadrants*

$\theta = 180^0 + 60^0$ *and* $360^0 - 60^0$

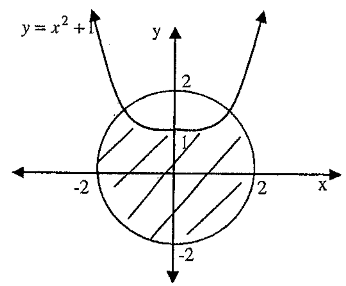
$\theta = 240^0$ *and* 300^0

ANSWERS QUESTION 3

Question 3 (a)

Criteria	Marks
• One mark each for both diagrams, one mark for region inside circle or region below parabola and one for the common region	4

Answer:



Question 3 (b)

Criteria	Marks
• One mark for $\angle PST = \angle QPS + \angle PQS$ *(external angle of triangle)* and $\angle PTS = \angle TPR + \angle PRT$ *(external angle of triangle)*, one for stating property of isosceles triangle, one for stating all reasons	3

Answer:

$\angle PST = \angle QPS + \angle PQS$ *(external angle of triangle)*

$\therefore \angle QPS = \angle PST - \angle PQS$

$\angle PTS = \angle TPR + \angle PRT$ *(external angle of triangle)*

$\therefore \angle RPT = \angle PTS - \angle PRT$

now $PQ = PR$ *(given)*

$\therefore \Delta PQR$ *is isosceles*

$\angle PQS = \angle PRT$ *(base angles are equal)*

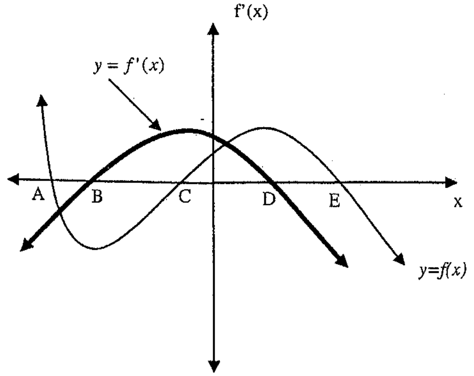
$\angle PST = \angle PTS$ *(given)*

$\therefore \angle QPS = \angle RPT$

Question 3 (c)

Criteria	Marks
• One mark for correct shape, one for location of x intercepts	2

Answer:



Question 3 (d)

Criteria	Marks
• One mark for the basic angle 60^0, one for $\frac{1}{2}x - 107^0 = -60^0,\ 60^0$ and one for simplification	3

Answer:

$\sec(\frac{1}{2}x - 107^0) = 2$

$\frac{1}{\cos(\frac{1}{2}x - 107^0)} = 2$

$\cos(\frac{1}{2}x - 107^0) = \frac{1}{2}$

the basic angle is 60^0

$\frac{1}{2}x - 107^0 = -60^0, 60^0$

$\frac{1}{2}x = 47^0, 167^0$

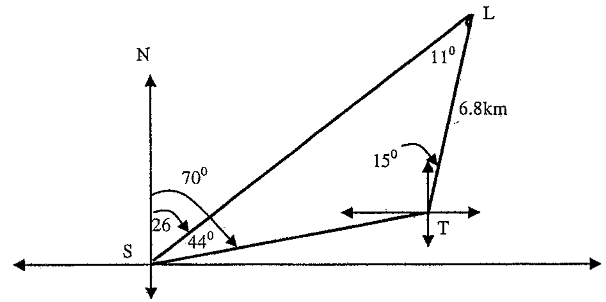
$\therefore x = 334^0, 94^0$ sin ce $0^0 \le x \le 360^0$

ANSWERS QUESTION 4

Question 4 (a)

Criteria	Marks
• One for correct diagram, one for using sine rule, one for simplification	3

Answer:



Let d be the distance of the ship to the tower

$$\frac{d}{\sin 11} = \frac{6.8}{\sin 44}$$

$$d = 1.867...km$$

$$= 1.9km(1dp)$$

Question 4 (b) (i)

Criteria	Marks
• One mark for correct use of formula, one for answer	2

Answer:

Gradient UV: $-3p = \frac{4-1}{-1-1}$

$$p = \frac{1}{2}$$

Question 4 (b) (ii)

Criteria	Marks
• One mark each for equations of VW and UW and one for x and y values	3

Answer:

Eqn VW: $y - 4 = \frac{1}{2}(x+1)$

$$y = \frac{1}{2}x + 4\frac{1}{2}$$

Eqn UW: $y - 1 = \frac{3}{2}(x-1)$

$$y = \frac{3}{2}x - \frac{1}{2}$$

$\therefore \frac{1}{2}x + \frac{9}{2} = \frac{3}{2}x - \frac{1}{2}$

$x + 9 = 3x - 1$

$x = 5$ *and* $y = 7$

$\therefore$ *Coordinates of w is* $(5, 7)$

Question 4 (b) (iii)

Criteria	Marks
• One mark for distance of WU and one for distance VU	2

Answer:

$d_{WU} = \sqrt{(7-1)^2+(5-1)^2}$
$= \sqrt{36+16}$
$= \sqrt{52}$
$= 2\sqrt{13}$

$d_{VU} = \sqrt{(4-1)^2+(-1-1)^2}$
$= \sqrt{9+4}$
$= \sqrt{13}$
$\therefore WU = 2VU$

Question 4 (c)

Criteria	Marks
• One mark for first derivative, one for second derivative in surd form	2

Answer:

$y = x^{-\frac{1}{2}}$

$y' = -\frac{1}{2}x^{\frac{-3}{2}}$

$y'' = \frac{3}{4}x^{\frac{-5}{2}}$

$y'' = \frac{3}{4\sqrt{x^5}}$

ANSWERS QUESTION 5

Question 5 (a) (i)

Criteria	Marks
• One mark for correct answer	1

Answer:

$y = x^2$
$sub(1,1) \therefore 1 = 1^2 = 1$

$y = \sqrt{x}$
$sub(1,1) \therefore 1 = \sqrt{1} = 1$

Question 5 (a) (ii)

Criteria	Marks
• One mark for $\left[\frac{x^2}{2} - \frac{x^5}{5}\right]_0^1$, one for simplification	2

Answer:

$v = \pi\int_0^1 x\,dx - \pi\int_0^1 x^4\,dx$

$= \pi\int_0^1 x - x^4\,dx$

$= \pi\left[\frac{x^2}{2} - \frac{x^5}{5}\right]_0^1$

$v = \pi\left[\left(\frac{1}{2}-\frac{1}{5}\right)-0\right]$

$v = \frac{3\pi}{10}$ u^3

Question 5 (b)

Criteria	Marks
• One mark for $h = \frac{\pi}{12}$, one for $A \approx \frac{\frac{\pi}{12}}{2}\left[(\tan 0 + \tan\frac{\pi}{3}) + 2(\tan\frac{\pi}{12} + \tan\frac{\pi}{6} + \tan\frac{\pi}{4})\right]$, one for simplification	3

Answer:

$A \approx \frac{\frac{\pi}{12}}{2}\left[(\tan 0 + \tan\frac{\pi}{3}) + 2(\tan\frac{\pi}{12} + \tan\frac{\pi}{6} + \tan\frac{\pi}{4})\right]$

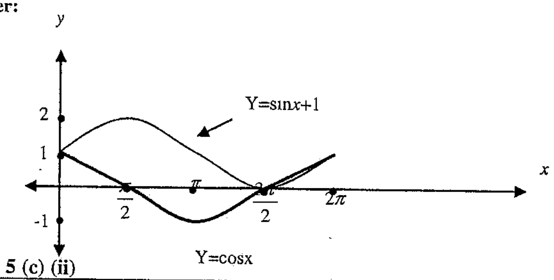
$A \approx \frac{\frac{\pi}{12}}{2}\left[(0+\sqrt{3}) + 2(\tan\frac{\pi}{12} + \tan\frac{\pi}{6} + \tan\frac{\pi}{4})\right]$

$\approx \frac{\pi}{24}\left[\sqrt{3} + 2(1.845299...)\right]$
$\approx 0.709...$
$\approx 0.7u^2 (1dp)$

Question 5 (c) (i)

• One mark for y=sinx+1, one for y=cosx,	2

Answer:



Question 5 (c) (ii)

Criteria	Marks
• One mark for the correct answer	1

Answer:

$0, \frac{3\pi}{2}, 2\pi$

Question 5(d)

Criteria	Marks
• One mark for $3^x = 1$ *and* $3^x = 3$ and one for simplification	2

Answer:

$let\ v = 3^x$

$\therefore v^2 - 4v + 3 = 0$

$v = 3\ and\ v = 1$

$ie\ 3^x = 3\ and\ 3^x = 1$

$x = 1\ or\ 0.$

Question 5 (e)

Criteria	Marks
• One mark for correct answer	1

Answer:

$e^{\ln 5x} = 10$

$\ln e^{\ln 5x} = \ln 10$

$\therefore 5x = 10$

$\therefore x = 2$

ANSWERS QUESTION 6

Question 6 (a) (i)

Criteria	Marks
• One mark each for the x and y coordinate	2

Answer:

$(i)\ x - 3y + 9 = 0$

$y = \frac{x+9}{3} \rightarrow (1)$

$y^2 = x + 9 \rightarrow (2)$

$sub(1) \text{int}\, o(2)$

$\left(\frac{x+9}{3}\right)^2 = x + 9$

$x^2 + 18x + 81 = 9x + 81$

$x(x+9) = 0$

$x = 0 or -9$

$ie\ A(-9, 0)\ and\ B(0, 3)$

Question 6 (a) (ii)

Criteria	Marks
• One mark $= \left[\frac{2}{3}(x+9)^{\frac{3}{2}}\right]_{-9}^{0} - \left[\frac{x^2}{6} + 3x\right]_{-9}^{0}$ and one for simplification	2

Answer:

$A = \int_{-9}^{0} (x+9)^{\frac{1}{2}} dx - \int_{-9}^{0} \frac{x+9}{3} dx$

$= \left[\frac{2}{3}(x+9)^{\frac{3}{2}}\right]_{-9}^{0} - \left[\frac{x^2}{6} + 3x\right]_{-9}^{0}$

$A = 18 - 13.5$

$A = 4\frac{1}{2} u^2$

Question 6 (b)

Criteria	Marks
• One mark $0.4M_0 = M_0 e^{-0.04t}$, one for simplification, one for rounding up years	3

Answer:

$0.4M_0 = M_0 e^{-0.04t}$

$0.4 = e^{-0.04t}$

$t = \frac{\ln(0.4)}{-0.04}$

$t = 22.907..$

$t = 23\ years$

Question 6 (c) (i)

Criteria	Marks
• One mark for the correct answer	1

Answer:

$x \geq -2$

Question 6 (c) (ii)

Criteria	Marks
• One mark for differentiation, one for finding x and y	2

Answer:

$f(x) = 6x\sqrt{x+2}$

$f'(x) = 6x.\frac{1}{2}(x+2)^{\frac{-1}{2}} + (x+2)^{\frac{1}{2}}.6$

$ie \frac{3x}{\sqrt{x+2}} + 6\sqrt{x+2} = 0\ at\ S.P$

$3x + 6(x+2) = 0$

$\left(-1\frac{1}{3}, -8\sqrt{\frac{2}{3}}\right)$

Question 6 (c) (iii)

Criteria	Marks
• One mark for method	1

Answer:

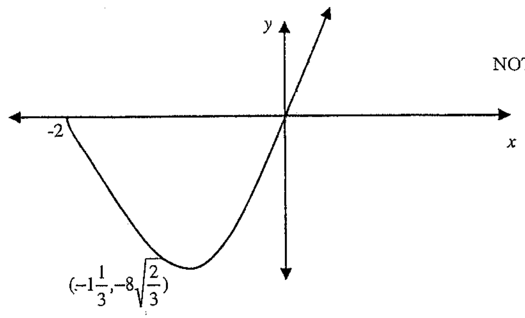
$f'(-1\frac{1}{3} - \delta x)$	$f'(-1\frac{1}{3})$	$f'(-1\frac{1}{3} + \delta x)$
-	0	+

Minimum turning point at $(-1\frac{1}{3}, -8\sqrt{\frac{2}{3}})$

Question 6 (c) (iv)

Criteria	Marks
• One mark for sketch	1

Answer:



NOT TO SCALE

ANSWERS QUESTION 7

Question 7 (a) (i)

Criteria	Marks
• One mark for stating two correct reasons, one for stating a third reason and one a correct conclusion	3

Answer:

$\angle BAD = \angle BCD$ *(opposite angles in a parm are equal)*

$\angle ABC = \angle ADC$ *(opposite angles in a parm are equal)*

Since EB bisects $\angle ABC$ and FD bisects $\angle ADC$

$\therefore \angle ABE = \angle FDC$

$BA = CD$ *(opposite sides of parm)*

$\therefore \Delta ABE \equiv \Delta FCD (AAS)$

Question 7 (a) (ii)

Criteria	Marks
• One for the correct answer	1

Answer:

BE=FD *(corresponding sides in congruent triangles)*

Question (7) (b)

Criteria	Marks
• One mark for integral of $\sin x$, one for substituting, one for simplification	3

Answer:

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sin x\,dx = (-\cos x)_{\pi/6}^{\pi/3}$$

$$= \left(-\frac{1}{2} - -\frac{\sqrt{3}}{2}\right)$$

$$= \frac{-1+\sqrt{3}}{2}$$

Question 7 (c) (i)

Criteria	Marks
• One mark for the correct answer	1

Answer:

$$\$2000(1.1)^{21} = \$14800.499989$$

$$= \$14\,800.50$$

Question 7 (c) (ii)

Criteria	Marks
• One mark for $[2000(1.1)^{21} + ... + 2000(1.1)^1]$, one for 1.08^4, one for $\frac{2000(1.1)(1.1^{21}-1)}{0.1} 1.08^4$ • One mark for simplification	4

Answer:

$$[2000(1.1)^{21} + ... + 2000(1.1)^1]1.08^4 = \left[\frac{2000(1.1)(1.1^{21}-1)}{0.1}\right]1.08^4$$

$$= \$191\,564.3266$$

$$= \$191\,564.33$$

ANSWERS QUESTION 8

Question 8 (a) (i)

Criteria	Marks
• One mark for final answer	1

Answer:

$P(\text{second stocking will be a matching pair}) = 1 \times \frac{1}{9}$

$= \frac{1}{9}$

Let the stockings be G G R R O O B B P P

If G is selected first then probability of G second is $\frac{1}{9}$

Question 8 (a) (ii)

Criteria	Marks
• One mark for noting numerators going down by 2 and one for simplification	2

Answer:

$P(\text{not having a matching pair after selecting the third shoe}) = \frac{10}{10} \times \frac{8}{9} \times \frac{6}{8}$

$= \frac{2}{3}$

Let G be first and R be second

Not G second and not G or R third $= \frac{2}{3}$

Question 8 (b) (i)

Criteria	Marks
• One mark for the velocity equations for both particles, one for substitution and comparing answers	2

Answer:

$X_A = 4t^2 - t^3$

$\dot{X}_A = 8t - 3t^2$

$when\ t = 1$

$\dot{X}_A = 5\,cm/\sec$

$X_B = 4t - 6$

$\dot{X}_B = 4$

$when\ t = 1$

$\dot{X}_B = 4\,cm/\sec$

$\therefore \dot{X}_A\ is\ faster\ at\ t = 1$

Question 8 (b) (ii)

Criteria	Marks
• One mark for the correct answer	1

Answer:

$8t - 3t^2 = 4$

$3t^2 - 8t + 4 = 0$

$t = \frac{2}{3}\sec\ \ and\ \ t = 2\sec$

Question 8 (b) (iii)

Criteria	Marks
• One mark for second derivative, one for substitution	2

Answer:

$\ddot{X}_A = 8 - 6t$

$at\ t = 3$

$\ddot{X}_A = -10cm/\sec^2$

Question 8 (b) (iv)

Criteria	Marks
• One mark for velocity equals zero, one for solving and stating correct positive displacement	2

Answer:

$\dot{X}_A = 8t - 3t^2$

$8t - 3t^2 = 0$

$t(8 - 3t) = 0$

$t = 0\ and\ t = \frac{8}{3}$

$At\ t = 0, X_A = -6\,cm$

$At\ t = 2\frac{2}{3}, X_A = 9\frac{13}{27}\,cm$

$\therefore Max\ positive\ displacement\ is\ 9\frac{13}{27}\,cm$

Question 8 (c)

Criteria	Marks
One mark for completing the square, one for determining vertex	2

Answer:

$y^2 - 6y - 9x - 9 = 0$

$y^2 - 6y + 9 = 9x + 9 + 9$

$(y-3)^2 = 9(x+2)$

$(y-3)^2 = 4(\frac{9}{4})(x+2)$

$Vertex = (-2, 3)$

ANSWERS QUESTION 9

Question 9 (a) (i)

Criteria	Marks
• One mark for $\frac{dr}{dA} = \frac{1}{8\pi r}$, one for using $\frac{dr}{dt} = \frac{dr}{dA} \times \frac{dA}{dt}$, one for simplification	3

Answer:

$\frac{dr}{dt} = \frac{dr}{dA} \times \frac{dA}{dt}$

$Since\ A = 4\pi r^2$

$\frac{dA}{dr} = 8\pi r$

$ie\ \frac{dr}{dA} = \frac{1}{8\pi r}$

$\therefore \frac{dr}{dt} = \frac{1}{8\pi r} \times \frac{dA}{dt}$

$= \frac{1}{40\pi} \times 6$

$= \frac{3}{20\pi}\,cm/\sec$

Question 9 (a) (ii)

Criteria	Marks
• One mark for $\frac{dv}{dt} = 4\pi r^2 \frac{dr}{dt}$ and one for simplification	2

Answer:

$v = \frac{4}{3}\pi r^3$

$\frac{dv}{dr} = 4\pi r^2$

$\frac{dv}{dt} = \frac{dv}{dr} \times \frac{dr}{dt}$

$\frac{dv}{dt} = 4\pi r^2 \frac{dr}{dt}$

$= 4\pi(25)\frac{3}{20\pi}$

$= 15\,cm^3/\sec$

Question 9 (b) (i)

Criteria	Marks
• One mark for $\ln\sqrt{\frac{2+x}{2-x}} = \frac{1}{2}[\ln(2+x)-\ln(2-x)]$, and one for simplification	2

Answer:

$$\frac{d}{dx}\ln\sqrt{\frac{2+x}{2-x}} = \frac{d}{dx}\left(\frac{1}{2}\ln\left(\frac{2+x}{2-x}\right)\right)$$
$$= \frac{d}{dx}\frac{1}{2}[\ln(2+x)-\ln(2-x)]$$
$$= \frac{1}{2}\left(\frac{1}{2+x}-\frac{-1}{2-x}\right)$$
$$= \frac{1}{2}\left(\frac{1}{2+x}+\frac{1}{2-x}\right)$$
$$= \frac{1}{2}\left(\frac{4}{4-x^2}\right) \quad ie \quad \frac{2}{4-x^2}$$

Question 9 (b) (ii)

Criteria	Marks
• One mark for $\int_0^1 \frac{4}{4-x^2}dx = 2\left[\ln\sqrt{\frac{2+x}{2-x}}\right]_0^1$, and one for simplification	2

Answer:

$$\int_0^1 \frac{4}{4-x^2}dx = 2\left[\ln\sqrt{\frac{2+x}{2-x}}\right]_0^1$$
$$\int_0^1 \frac{4}{4-x^2}dx = 2\ln\sqrt{3}$$

Question 9 (c)

Criteria	Marks
• One mark for knowing F(x) represents the derivative, one for integration, one for simplification	3

Answer:

Let F(x) be the number of new cases of flu. The daily rate at which new cases are occurring x days after January 1 represents the derivative of F(x). Thus

$$\int_3^8 (700x-6x^2)dx = \left(\frac{700x^2}{2}-\frac{6x^3}{3}\right)_3^8$$
$$= (350x^2-2x^3)_3^8$$
$$= 18280$$

ANSWERS QUESTION 10

Question 10 (a)

Criteria	Marks
• One mark for forming $5d=2a$, one for using sum to n terms, one for solving simultaneous equations and stating the 6 angles	3

Answer:

Let the six sectors be a, $a+d$, $a+2d$,....$a+5d$

$$\therefore a+5d = 3a$$
$$\therefore 5d = 2a \rightarrow (1)$$

Sum to n terms is $S_n = \frac{n}{2}(a+l)$

$$ie \frac{6}{2}(a+a+5d) = 360^0$$
$$ie\, 2a+5d = 120^0 \rightarrow (2)$$

Sub (1) *into* (2)

$$5d+5d = 120^0$$
$$d = 12^0 \text{ and } a = 30^0$$

Hence the 6 angles are $30^0, 42^0, 54^0, 66^0, 78^0, 90^0$

Question 10 (b)

Criteria	Marks
• One mark for correct use of product rule, one for correct method used in second derivative, one for simplification	3

Answer:

$$y = xe^{-2x}$$
$$\frac{dy}{dx} = x\left(-2e^{-2x}\right)+e^{-2x}(1)$$
$$= -2xe^{-2x}+e^{-2x}$$
$$\frac{d^2y}{dx^2} = -2x\left(-2e^{-2x}\right)+e^{-2x}(-2)-2e^{-2x}$$
$$= 4xe^{-2x}-4e^{-2x}$$

$$\therefore \frac{d^2y}{dx^2}+4\frac{dy}{dx}+4y$$
$$= 4xe^{-2x}-4e^{-2x}+4(-2xe^{-2x}+e^{-2x})+4xe^{-2x}$$
$$= 4xe^{-2x}-4e^{-2x}-8xe^{-2x}+4e^{-2x}+4xe^{-2x}$$
$$= 0$$

Question 10 (c) (i)

Criteria	Marks
• One mark for the correct answer	1

Answer:

$$\cos\theta = \frac{1}{MN} \qquad \text{and} \qquad \tan\theta = \frac{MT}{1} = MT$$
$$\therefore MN = \sec\theta$$

Question 10 (c) (ii)

Criteria	Marks
• One for distance LM, one for distance MN and total cost	2

Answer:

Distance $LM = 3-Tan\,\theta$

$\therefore$ *Cost for distance* $LM = c(3-Tan\,\theta)$

Distance $MN = Sec\,\theta$

$\therefore$ *Cost for distance* $MN = 3c\,Sec\,\theta$

$\therefore$ *Total cost* $T = 3c - c\,Tan\,\theta + 3c\,Sec\,\theta$

Question 10 (c) (iii)

Criteria	Marks
• One mark for finding first derivative, one for finding the angle, one for proving it is a minimum	3

Answer:

$T = 3c - c\tan\theta + 3c\sec\theta$

$T = 3c - c\tan\theta + 3c(\cos\theta)^{-1}$

$\frac{dT}{d\theta} = -c\sec^2\theta - 3c(\cos\theta)^{-2} \times -\sin\theta$

$\frac{dT}{d\theta} = -c\sec^2\theta + 3c\sec\theta\tan\theta$

$= 0$ *at a S.P*

$\therefore -c\sec^2\theta + 3c\sec t\tan\theta = 0$

$3\sec\theta\tan\theta = \sec^2\theta$

multiplying both sides by $\cos^2\theta$

$3\sin\theta = 1$

$\sin\theta = \frac{1}{3}$ *ie* $\theta = 19^0 28'$

Test for minimum

$f'(19^0 28' - \delta x) < 0$

$f'(19^0 28') = 0$

$f'(19^0 28' + \delta x) > 0$

$\therefore$ *The min cost is achieved by laying the line across the river at an angle of* $19^0 28'$ *to the direct route across.*