2005 PRELIMINARY HIGHER SCHOOL CERTIFICATE

EXAMINATION

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
- Approved calculators may be used
- All necessary working should be shown in every question
- Answer each question in a SEPARATE Writing Booklet

Total marks –84

- Attempt Questions 1 –7
- All questions are of equal value

Care has been taken to ensure that this paper is free of errors and that it mirrors the format and style of standard trial papers. Moreover, some questions have been adapted from previous HSC examinations as well as from trial examinations from various schools, in an attempt to provide students with exposure to a broad range of possible questions.

Question 1 (12 marks) **Marks**

(a) Factorise:

(i) $x^2 - x$ **1**

(ii) $x^2 - 1$ **1**

(iii) $8x^2 - 10x + 3$ **1**

(b) Simplify: $\dfrac{x^2 - x}{x^2 - 1}$ **1**

(b) Express $\dfrac{2}{\sqrt{5} - 1}$ in simplest form with rational denominator. **2**

(d) Solve: $|2 - x| < 3$ **2**

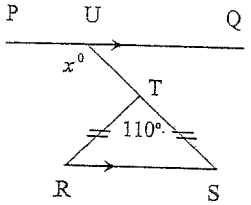
(e) If $f(x) = x^2 - 2x + 2$, find $f(\sqrt{3} + 1)$ **2**

(f) Given $y = \dfrac{2x - 3}{x + 6}$, make x the subject of the equation. **2**

Question 2 (12 marks) Use a SEPARATE page/ booklet. **Marks**

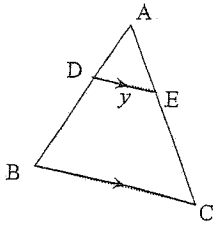
(a) The interior angle of a regular polygon is 170^0. How many sides does the polygon have? **1**

(b) If $\angle PUT = x^0$, $\angle RTS = 110^0$ and RT=TS, find the value of x, giving reasons.



NOT TO SCALE **2**

(c) In the diagram, DE is parallel to BC, AD = 12 cm, DB = 16 cm and BC = 21 cm. Find the value of y. Give reasons. **2**



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(d) Find the value(s) of k for which $x^2 + (k-1)x + 4 = 0$ has equal roots. **2**

(e) Find the exact value of $\cos x^0$ if $\sin x^0 = \frac{1}{3}$ where $0 < x < 90$. **1**

(f) Show that $\cos 300^0 - \sin 585^0 = \frac{1}{2}(1+\sqrt{2})$ **2**

(g) Simplify: $\frac{2^n - 2^{n-1}}{2^{n+1} + 2^n}$ **2**

Question 3 (12 marks) Use a SEPARATE page/ booklet. **Marks**

(a) On a number plane mark the origin O, the point A(2,4) and sketch the line l $x + 2y - 5 = 0$. **2**

(b) Find the coordinates of M the mid-point of OA. **1**

(c) Show that M lies on l. **1**

(d) Show that the line l is the perpendicular bisector of AO. **2**

(e) Find the coordinates of B where l meets the x-axis. **1**

(f) C is a point on l such that AB $\parallel$ OC. Find the coordinates of C. **4**

(g) Which type of quadrilateral is ABOC ? **1**

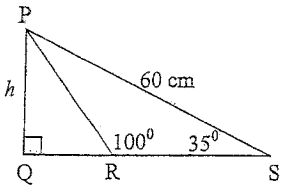
Question 4 (12 marks) Use a SEPARATE page/ booklet. **Marks**

(a) Show that $\frac{1}{1-\sin\theta} + \frac{1}{1+\sin\theta} = 2\sec^2\theta$ **2**

(b) Solve: $4\cos^2\theta = 1$ for $0^0 \le \theta \le 360^0$ **3**

(c) A ship is 35 km due east of a lighthouse. The lighthouse is 20 km due south of a cliff. What is the bearing of the cliff from the ship, to the nearest minute? **2**

(d)



NOT TO SCALE

In the diagram, $\angle PRS = 100^0$, $\angle PSR = 35^0$, PS = 60 cm, $\angle PQR = 90^0$ and PQ = h cm

(i) Show that $PR = \frac{60\sin 35^0}{\sin 100^0}$ **1**

(ii) Hence find h correct to 2 dec. places. **2**

(e) Graph the region given by the inequations: $y \ge 0, x \ge 0$ *and* $y < 4 - x^2$ **2**

Question 5 (12 marks) Use a SEPARATE page/ booklet. **Marks**

(a) Solve: $3x^2 \geq 4-x$ **2**

(b) The height of an equilateral triangle is 12 cm. Find the exact area of the triangle. **2**

(c) Simplify: $\dfrac{x^2-144}{x^3+8} \div \dfrac{x+12}{x+2}$ **2**

(d) If α and β are the roots of $2x^2+4x-5=0$, find the value of

(i) $\alpha+\beta$ **1**

(ii) $\alpha\beta$ **1**

(iii) $\alpha^2+\beta^2$ **1**

(iv) $\alpha^2\beta+\beta^2\alpha$ **1**

(e) Solve: $\sqrt{2}\sin\beta-1=0$ for $0° \leq \beta \leq 360°$. **2**

Question 6 (12 marks) Use a SEPARATE page/ booklet. **Marks**

(a) A function is defined as : $g(x) = \begin{cases} |2x| & \text{for } x < 0 \\ x^2 & \text{for } 0 \leq x \leq 2 \\ 1 & \text{for } x > 2 \end{cases}$

(i) Find the value of $g(-a^2)$ where $a \neq 0$ **1**

(ii) Draw a sketch of the function. **2**

(b) $f(x)$ is a function defined by $f(x) = 1 - \dfrac{2}{x^2+1}$

(i) Evaluate $f(0)$ **1**

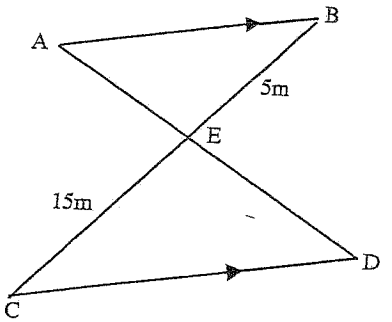
(ii) What values does $f(x)$ approach as $x \to \infty$? **1**

(iii) Show that $f(x)$ is an even function. **1**

(iv) Find the domain and range of $f(x)$. **1**

(v) Draw a sketch of the function. **1**

(c)



Not to Scale

In the diagram above, AB is parallel to CD. BE = 15m, CE = 5m,

(i) Prove that ΔABE is similar to ΔDCE. **3**

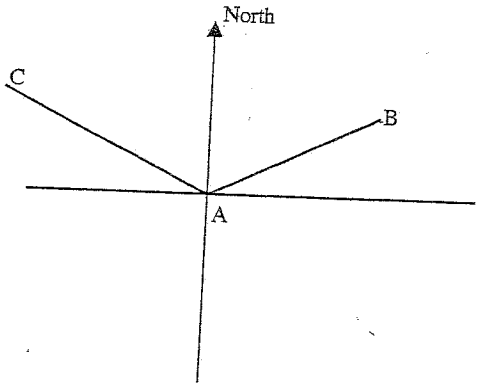
(ii) Find the ratio AE : AD. **1**

Question 7

Start a new page

Marks

(a) The diagram below shows three Fire Observation Towers A, B and C. From Tower A, Tower B bears 057°T and is at a distance of 7 kilometres. Tower C bears 296°T from Tower A and is 9 kilometres from it.



Not to scale

(i) Copy the diagram onto your worksheet and mark on it the information given above — 1

(ii) Calculate the distance from Tower B to Tower C (give your answer correct to one decimal place). — 2

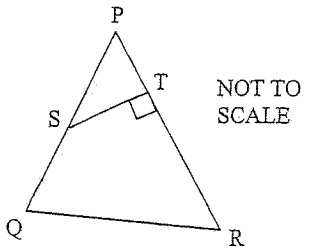
(iii) Find the bearing of Tower B from Tower C. (give your answer correct to the nearest degree) — 2

(b) A triangle has sides of length x^2-1, x^2+1 and $2x$ where x is an integer greater than 1.

(i) Show that the triangle is right-angled with x^2+1 as the hypotenuse. — 2

(ii) Explain briefly why $x \neq 1$. — 1

(c) In the diagram, PS = SQ = 5, TR = 2 × PT = 8 and $\angle RTS = 90^0$. Find the length of QR — 2



NOT TO SCALE

Marking Guidelines: Preliminary Mathematics
2005 HSC Trial Examination

ANSWERS QUESTION 1

Question1 (a) to (f)

Criteria	Marks
• (a)(i) One mark for the correct answer.	1
• (a)(ii) One mark for the correct answer	1
• (a)(iii) One mark for the correct answer	1
• (b) One mark for the answer.	1
• (c) One mark for writing the expression for r and one for the evaluation.	2
• (d) One mark for correct process and one for simplification.	2
• (e) One mark for writing $f(x)$ as $(x-1)^2+1$ and one for simplification.	2
• (f) One mark for removing the fraction ie $yx+6y=2x-3$ and one for simplification	2

(a) (i) $x^2-x=x(x-1)$

(ii) $x^2-1=(x+1)(x-1)$

(iii) $8x^2-10x+3=(4x-3)(2x-1)$

(b) $\dfrac{x^2-x}{x^2-1}=\dfrac{x(x-1)}{(x-1)(x+1)}=\dfrac{x}{x+1}$

(c) If r is the radius of the sphere then

$\frac{4}{3}\pi r^3 = 40$

$r=\sqrt[3]{\dfrac{10\times 3}{4\pi}}=2.121\ldots\approx 2.1cm$ (1 dec.pl.)

(d) $|2-x|<3 \Leftrightarrow -3<2-x<3$

Subtracting 2 we have,
$-5<-x<1$

Multiplying by -1 we have,
$5>x>-1$

i.e. $-1<x<5$ is the solution

(e) $f(x)=(x-1)^2+1$

$=\left(\sqrt{3}+1-1\right)^2+1=4$

(f) $y=\dfrac{2x-3}{x+6}$

$y(x+6)=2x-3$ *i.e.* $yx+6y=2x-3$

$x(y-2)=-3-6y$

or $x=\dfrac{-3(1+2y)}{y-2}$

ANSWERS QUESTION 2

Question2 (a) to (g)

Criteria	Marks
• (a) One mark for the correct answer.	1
• (b) One mark for showing $\angle TSR=\angle TUQ$ and one for the final answer.	2
• (c) One mark for proving the triangles are similar and one for finding y.	2
• (d) One mark for writing $(k-1)^2-16=0$ and one for simplification.	2
• (e) One mark for the correct answer	1
• (f) One mark for both sin and cos values and one for simplification.	2
• (g) One mark for factorization and one for simplification.	2

(a) Exterior angle of the polygon
$=180^0-170^0=10^0$
No. of sides $=360\div 10=36$

(b) $\angle TSR=\angle TRS$
$=(180^0-110^0)\div 2=35^0$ (isosceles triangle TR=TS given)
$=\angle TUQ$ (alt.angles, PQ ∥ RS)
But $\angle TUP+\angle TUQ=180^0$ (Adj.angles on a st.line)
$\therefore \angle TUP=180^0-35^0=145^0$

(c) In triangles ADE and ABC,
$\angle ADE=\angle ABC$ (Corresponding angles, DE ∥ BC)
$\angle A$ is common
Δ ADE and Δ ABC are equiangular and hence similar.

$\dfrac{AD}{AB}=\dfrac{DE}{BC}$ *i.e.* $\dfrac{12}{28}=\dfrac{y}{21}$

or $y=\dfrac{12}{28}\times 21=9$

(d) For equal roots, the discriminant = 0

$(k-1)^2-16=0$

$k-1=\pm\sqrt{16}$

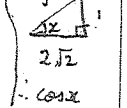
or $k=1\pm 4=5$ or -3

(e) $\cos x=\sqrt{1-\sin^2 x}=\sqrt{1-\frac{1}{9}}$

$=\sqrt{\frac{8}{9}}=\frac{2\sqrt{2}}{3}$

OR

$\sin x=\frac{1}{3}$



$\therefore \cos x=\frac{2\sqrt{2}}{3}$

(f) $\cos 300^0=\cos 60^0=\dfrac{1}{2}$

$\sin 585^0=\sin 225^0$
$=-\sin 45^0=-\dfrac{1}{\sqrt{2}}$

$\cos 300^0-\sin 585^0=\dfrac{1}{2}+\dfrac{1}{\sqrt{2}}=\dfrac{1+\sqrt{2}}{2}$

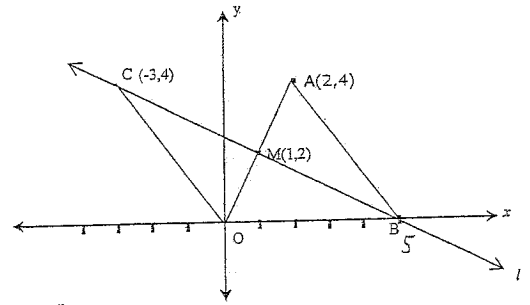
(g) $\dfrac{2^n - 2^{n-1}}{2^{n+1} + 2^n} = \dfrac{2^n\left(1 - 2^{-1}\right)}{2^n\left(2^1 + 1\right)} = \dfrac{\left(1 - \frac{1}{2}\right)}{3} = \dfrac{1}{6}$

ANSWERS QUESTION 3

QUESTION 3 (a) to (g)

Criteria	Marks
• (a) One mark for plotting both the points (0,0) and A(2,4) and one for sketching the line	2
• (b) one mark for the correct answer	1
• (c) one mark for the correct answer	1
• (d) One mark for showing perpendicularity and one for bisection	2
• (e) One mark for the answer.	1
• (f) One mark for finding the gradient of AB, one for writing the equation of OC and one each for x and y coordinate	4
• (g) One mark for stating it is a parallelogram	1

(a)



(b) M is (1,2)

(c) l is $x + 2y - 5 = 0$. Substituting (1,2) in l we have,
$1 + 4 - 5 = 0$ i.e. (1,2) satisfies the equation and hence it is on l.

(d) Gradient of AO = 2

Rewriting l, $(x + 2y - 5 = 0)$, we have $y = -\dfrac{1}{2}x + \dfrac{5}{2}$

Gradient of $l = -\dfrac{1}{2}$ Gradient of AO × Gradient of $l = 2 \times -\dfrac{1}{2} = -1$

l is perpendicular to AO and since passes through the mid point of AO, it is the perpendicular bisector of AO

(e) Putting y=0 in $x+2y-5=0$ we have $x=5$, B is (5,0)

(f) Gradient of AB $= -\dfrac{4}{3}$ and A is (2,4)

OC has $-\dfrac{4}{3}$ as its gradient and it passes through (0,0)

Equation of OC is $y = -\dfrac{4}{3}x$

Solving the equations of AB and OC we get the coordinates of C

Substituting for y in $x + 2y - 5 = 0$, we have

$x + 2\left(-\dfrac{4}{3}x\right) - 5 = 0$ or $x - \dfrac{8x}{3} = 5$ or $\dfrac{3x - 8x}{3} = 5$

$x = -3$ and hence $y = -\dfrac{4}{3} \times -3 = 4$

or C is $(-3, 4)$

(g) In Quad. ABOC, AB ∥ CO and OB ∥ CA ∴ ABOC is a parallelogram. rhombus

ANSWERS QUESTION 4

Question 4 (a) to (e)

Criteria	Marks
• (a) One mark for showing LHS is $\dfrac{2}{1 - \sin^2\theta}$ and one for $2\sec^2\theta$	2
(b) One mark for finding the values of $\cos\theta$ and one in for 50% of answers and one for the other 50% of answers	3
• (c) One mark for finding θ and one for the bearing.	2
• (d) (i) One mark for the correct answer	1
• (ii) One mark for $h = \dfrac{60\sin 35^0 \times \sin 80^0}{\sin 100^0}$, one for simplification	2
• (e) One mark for drawing parabola and one for correct shading.	2

(a) L.H.S $= \dfrac{1 + \sin\theta + 1 - \sin\theta}{(1 + \sin\theta)(1 - \sin\theta)} = \dfrac{2}{(1 - \sin^2\theta)}$

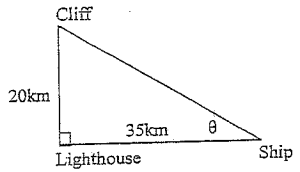
$= \dfrac{2}{\cos^2\theta} = 2\sec^2\theta =$ R.H.S

(b) $4\cos^2\theta = 1$

$\cos^2\theta = \dfrac{1}{4}$ or $\cos\theta = \pm\sqrt{\dfrac{1}{4}} = \pm\dfrac{1}{2}$

$\theta = 60^0, 120^0, 240^0$ and 300^0 4/5

(c)



$\tan\theta = \frac{20}{35}$ $\theta \approx 29^0 45'$

Bearing of cliff from the ship

$= (270^0 + 29^0 45')$

$= 299^0 45'$

(d) (i) Using sine rule in ΔPRS,

$\frac{PR}{\sin 35^0} = \frac{60}{\sin 100^0}$ $PR = \frac{60 \sin 35^0}{\sin 100^0}$

(d) (ii) $\angle PRQ = 180^0 - 100^0 = 80^0$

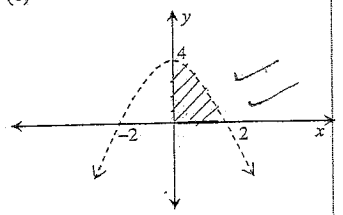
In right angled triangle PQR,

$\frac{h}{PR} = \sin 80^0$

or $h = PR\sin 80^0 = \frac{60 \sin 35^0 \times \sin 80^0}{\sin 100^0}$

$= 34.414... \doteq 34 \cdot 41$ (2dec.pl.)

(e) NOT TO SCALE



ANSWERS QUESTION 5

Question5 (a) to (g)

Criteria	Marks
• (a) One mark for factorization and one for the solution.	2
• (b) One mark for finding the length of side and one the area.	2
• (c) One mark for finding a and one for the equation.	2
• (d) (i) One mark for the correct answer	1
• (d) (ii) One mark for the correct answer	1
• (d) (iii) One mark for the correct answer	1
• (d) (iv) One mark for the correct answer	1
• (e) One mark for writing as a quadratic equation and one for simplification	2

(a) Rewriting we have, $3x^2 + x - 4 \geq 0$

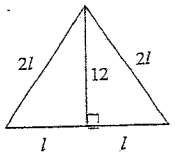
$3x^2 + 4x - 3x - 4 \geq 0$

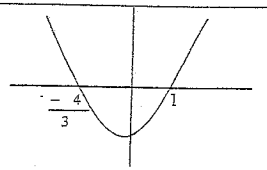
$x(3x+4) - 1(3x+4) \geq 0$

or $(3x+4)(x-1) \geq 0$

$x \leq -\frac{4}{3}$ or $x \geq 1$

(b)





$4l^2 - l^2 = 12^2$ or $l = \sqrt{\frac{144}{3}} = \frac{12}{\sqrt{3}}$

Area of triangle

$= 12 \times \frac{12}{\sqrt{3}} = \frac{12 \times 12}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 48\sqrt{3}$ $Unit^2$

(c) The length from the focus to the directrix = 4+2 = 6 units

∴ focal length = a = 3 units

Vertex is at (−1,−1)

Equation of the parabola is

$(x+1)^2 = -4 \times 3(y+1)$

i.e. $(x+1)^2 = -12(y+1)$

(d) (i) Sum of roots $= \frac{-b}{a} = \frac{-4}{2} = -2$

(ii) Product of roots $= \frac{c}{a} = \frac{-5}{2}$

(iii) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= (-2)^2 - 2 \times \frac{-5}{2} = 4 + 5 = 9$

(iv) $\alpha^2\beta + \beta^2\alpha = \alpha\beta(\alpha+\beta)$

$= -2 \times \frac{-5}{2} = 5$

(e) $4^x - 2^x \cdot 5 + 4 = 0$

$\Rightarrow 2^{2x} - 5 \cdot 2^x + 4 = 0$

Let $2^x = m$, then

$2^{2x} - 5 \cdot 2^x + 4 = m^2 - 5m + 4 = 0$

$\Rightarrow (m-1)(m-4) = 0$

$m = 1$ or $m = 4$

i.e. $2^x = 1$ or $2^x = 4$

$x = 0$ or 2

ANSWERS QUESTION 6

Question 6 (a) to (c)

Criteria	Marks
• (a) (i) One mark for the correct answer	1
(ii) One mark for the half parabola and one for the other curves.	2
• (b) (i) One mark for each correct answer	1
• (b) (ii) One mark for each correct answer	1
• (b) (iii) One mark for each correct answer	1
• (b) (iv) One mark for each correct answer	1
• (b) (v) One mark for each correct answer	1
• (c) (i) One mark for the centre	1
(ii) One for the finding radius.	1
(iii) One mark for finding the perpendicular distance and one for explanation.	2

Question 5

(c) $\dfrac{x^2-144}{x^3+8} \times \dfrac{x+2}{x+12}$

$= \dfrac{(x+12)(x-12)}{(x+2)(x^2-2x+4)} \times \dfrac{(x+2)}{(x+12)}$

$= \dfrac{x-12}{x^2-2x+4}$

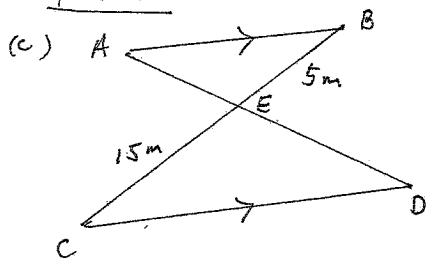
(e) $\sqrt{2}\sin\beta - 1 = 0;\ 0° \le \beta \le 360°$

$\sin\beta = \dfrac{1}{\sqrt{2}} \Rightarrow \beta = 45°$ in 1st and 2nd quadrants.

$\therefore \beta = 45°, 135°$

Question 6

(c)



(i) In $\triangle ABE$, $\triangle DCE$,

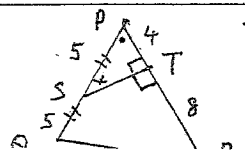
$\angle A = \angle D$ (alt $\angle$s, // lines)

$\angle B = \angle C$ (" " " ")

$\angle AEB = \angle CED$ (vert. opp. $\angle$s)

$\therefore \triangle ABE \equiv \triangle DCE$.

(ii) $\dfrac{AE}{AD} = \dfrac{BE}{BC} = \dfrac{5}{20} = \dfrac{1}{4}$

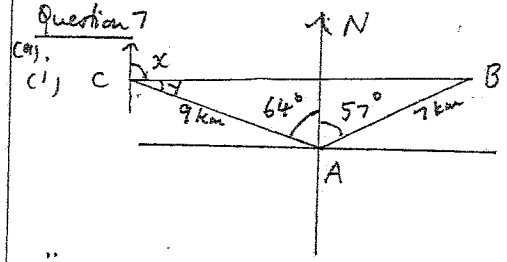


$TR = 2PT$

$\therefore PT = 4$ ⇒

Question 7

(a) (i)



(ii) $BC^2 = 9^2 + 7^2 - 2\times9\times7\times\cos 121°$

$BC = \sqrt{194.89\ldots}$

$= 13.96$

$= 14.0$ (1d.p.)

(iii) $\cos y = \dfrac{9^2+14^2-7^2}{2\times9\times14}$

$y = 25°$

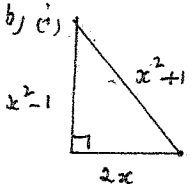
$x + y = 180° - 64°$ (co-int $\angle$s)

$\therefore x = 180° - 64° - 25°$

$= 91°$

$\therefore$ Bearing of B from C $= 091°T$.

(b) (i)



$(x^2-1)^2 + (2x)^2$

$= x^4 - 2x^2 + 1 + 4x^2$

$= x^4 + 2x^2 + 1$

$= (x^2+1)^2$

= square of the hyp.

$\therefore \triangle$ is right-angled

(ii) If $x = 1$, then $2x = 0$

and length cannot be zero.

$\therefore x \neq 1$.

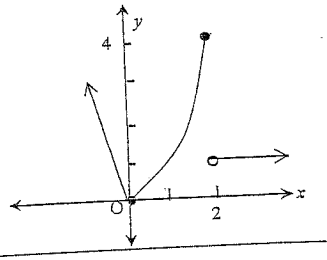
$\cos P = \dfrac{4}{5}$, $PQ = 10$, $PR = 12$

$QR^2 = 10^2 + 12^2 - 2\times10\times12\times\cos P$

$QR = \sqrt{52}$

(a) (i) $g(-a^2) = |2(-a^2)| = 2a^2$

(a) (ii)



(b) (i) $f(0) = -\dfrac{2}{0+1} = -2$

(ii) As $x \to \infty$, $f(x) \to 0$

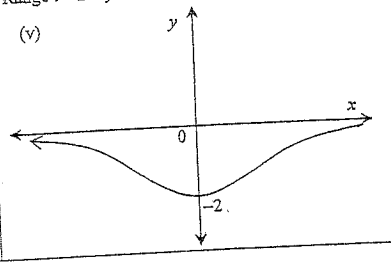
(iii) $f(-x) = -\dfrac{2}{(-x)^2+1} = -\dfrac{2}{x^2+1} = f(x)$

Since $f(-x) = f(x)$, $f(x)$ is even

(iv) Domain: All x

Range: $-2 \le y < 0$

(v)



(c) $x^2 + y^2 - 2x + 4y - 11 = 0$ can be written as $(x^2 - 2x + 1) + (y^2 + 4y + 4) - 5 - 11 = 0$

i.e. $(x-1)^2 + (y+2)^2 = 16$

(i) Centre is $(1, -2)$

(ii) Radius = 4 units

(iii) Perpendicular distance of $2x - y + 8 = 0$ from $(1, -2)$

$\dfrac{|2\times1 - (-2) + 8|}{\sqrt{2^2+1^2}} = \dfrac{12}{\sqrt{5}} > 4$

Since the distance of the line from the centre of circle is more than the length of the radius, it does not intersect the circle.