2005 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension II

General Instructions
- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
- Approved calculators may be used
- A table of standard integrals is provided
- All necessary working should be shown in every question
- Answer each question in a SEPARATE Writing Booklet

Total marks – 120
- Attempt Questions 1 – 8
- All questions are of equal value

Question 1 (15 marks) Use a separate page/booklet **Marks**

(a) Find the indefinite integral:

(i) $\int \frac{dx}{4-9x^2}$ 3

(ii) $\int \frac{dx}{\sqrt{4-9x^2}}$ 2

(iii) $\int \frac{dx}{(4-9x^2)^{3/2}}$ 3

(b) Evaluate:

(i) $\int_0^{1/2} \sin^{-1} x \, dx$ 3

(ii) $\int_0^2 \frac{x^2 \, dx}{x^6+64}$ 2

(iii) $\int_0^a x \sin(a-x) \, dx$ 2

Question 2 (15 marks) Use a separate page/booklet **Marks**

(a) If z and w are complex numbers, $\bar{z}$ and $\bar{w}$ are the respective complex conjugates of z and w.

(i) Prove: $\overline{z+w} = \bar{z} + \bar{w}$ 1

(ii) Show that $|z|^2 = z\bar{z}$ 1

(iii) Prove: $3|z-1|^2 = |z+1|^2$ if and only if $|z-2|^2 = 3$. 2

(iv) Draw a rough sketch of the subset of the complex plane, where

$\sqrt{3}\,|z-1| = |z+1|$ 2

(v) (α) Prove that for all complex numbers z and w

$|z+w|^2 + |z-w|^2 = 2\left[\,|z|^2 + |w|^2\,\right]$ 2

(β) Give a geometrical interpretation of this equation in the complex plane. 1

(b) (i) If $z = \cos\theta + i\sin\theta$, show that $z^n + \frac{1}{z^n} = 2\cos n\theta$ 1

(ii) For $z = r(\cos\theta + i\sin\theta)$, find r and the smallest value of θ which may satisfy the equation $2z^3 = 9+3\sqrt{3}\,i$. 2

(c) Shade the region in the complex plane, for which

$\frac{\pi}{6} \le \arg z \le \frac{\pi}{3}$ 1

(d) Two fixed points z_1 and z_2 and a variable point z represent the complex numbers z_1, z_2 and z respectively. Find the locus if

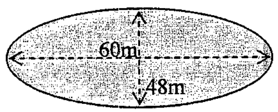
$\arg\left[\frac{z-z_1}{z-z_2}\right] = \beta$ 2

Question 3 (15 marks) Use a separate page/booklet **Marks**

(a) Show that the equation of the circle on the diameter joining the points (x_1, y_1) and (x_2, y_2) is given by

$$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$$ **2**

(b) The village green of Mathematicus is in the shape of an ellipse with external dimensions 60m by 48m.



NOT TO SCALE

(i) Write an equation to model the shape of the green. Assume that the centre of the green is the origin. **1**

(ii) Write the coordinates of the focii and the equations of the directrices. **2**

(c) (i) In the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, $(a>b)$, A and A' are two points where the ellipse cuts the *y- axis*. The tangents at A and A' to the ellipse intersect the tangent at P in Q and Q' respectively. Show that $AQ \times A'Q' = a^2$. **2**

(ii) If the circle on QQ' as diameter meets the *x –axis* at the points R and R', show that $OR \times OR' = a^2 - b^2$, where O is the origin of the ellipse. **3**

(d) If $(2-\sqrt{3})^n = a_n - b_n\sqrt{3}$ for all positive integers *n*, where a_n and b_n are integers, show that

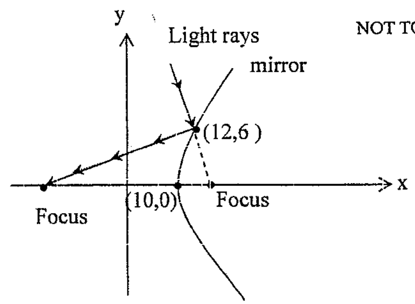
(i) $a_{n+1} = 2a_n + 3b_n$ and $b_{n+1} = a_n + 2b_n$ **1**

(ii) Calculate: $a_n^{\ 2} - 3b_n^{\ 2}$, for $n = 1,2$ and 3 **2**

(iii) Guess a formula for $a_n^{\ 2} - 3b_n^{\ 2}$ and prove your guess is true for all positive integers *n*. **2**

Question 4 (15 marks) Use a separate page/booklet **Marks**

(a) A hyperbolic mirror is used in some telescopes. It has the property that a light ray directed at one focus will be reflected to the other focus. Using the figure given below write an equation to model the hyperbolic mirror's surface. **2**



NOT TO SCALE

(b) P $\left(2p, \frac{2}{p}\right)$ is a point on the hyperbola $xy = 4$

(i) Show that the equation of the normal at P is given by

$$y = p^2x - 2p^3 + \frac{2}{p}$$ **3**

(ii) If this normal meets the $x-$*axis* at Q, find the coordinates of Q **1**

(iii) Find the coordinates the midpoint M of PQ . **2**

(iv) Hence find the locus of M. **1**

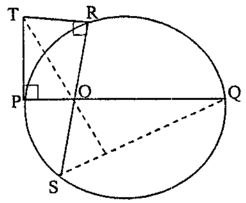
(c) (i) Show that $(\cos\theta + i\sin\theta)^5 = \sum_{r=0}^{5} {}^5c_r\, i^r \cos^{5-r}\theta \sin^r\theta$ **2**

(ii) Hence prove that $\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$ **2**

(iii) Deduce that $\cos\frac{\pi}{10}\cos\frac{3\pi}{10}\cos\frac{7\pi}{10}\cos\frac{9\pi}{10} = \frac{5}{16}$ **2**

Question 5 (15 marks) Use a separate page/booklet **Marks**

(a) NOT TO SCALE



In the diagram, PQ and RS are two chords of the circle intersecting at O. TR and TP are perpendicular to RS and PQ respectively. Prove that the line through T and O is perpendicular to SQ. **3**

(b) If $p > 0$ and $q > 0$, and $p + q = 1$, show that $\frac{1}{p} + \frac{1}{q} \geq 4$ **2**

(c) Find the general solution of the equation $\cos 5\theta - \sin 4\theta = 0$
Hence write down the solutions in $0 \leq x \leq 4\pi$. **3**

(d) If ω is a complex root of the equation $x^3 = 1$, show that the other complex root is ω^2 and $1 + \omega + \omega^2 = 0$ **2**

(e) If $y = \dfrac{x^2}{x^3 + 1}$.

(i) Find the coordinates of the stationary points of the curve and determine their nature. **3**

(ii) Sketch the curve. **1**

(iii) Use the sketch to find the number of real roots of the equation $x^3 - 4x^2 + 1 = 0$ **1**

Question 6 (15 marks) Use a separate page/booklet **Marks**

(a) Sketch each of the following curves on separate axis.

(i) $y = \ln|x|$ **2**

(ii) $y = |\ln(x)|$ **2**

(iii) $|y| = |\ln(x)|$ **2**

(iv) $|y| = \ln|x|$ **2**

(b) Five points are placed randomly in the *xy* plane, not on the axes. What is the probability that

(i) exactly one point will lie in the first quadrant? **2**

(ii) at least two of these points will lie in the first quadrant? **2**

(c) A particle moving with simple harmonic motion has a speed of 32m/s and 24 m/s when its distances from the centre of motion are respectively 3 m and 4 m. Find the periodic time of the motion. **3**

Question 7 (15 marks) Use a separate page/booklet **Marks**

(a) Use the shell method to find the volume generated by revolving about the x – axis the region in the coordinate plane bounded by

$y = \frac{1}{4}x^3$ *and* $y = \sqrt{2x}$. **4**

(b) If a >0 and b>0, c >0, show that

(i) $\frac{a+b+c}{3} \geq \sqrt[3]{abc}$ **3**

(ii) $\frac{a}{b} + \frac{b}{c} + \frac{c}{a} \geq 3$ **1**

(c) A circular hole is drilled through the centre of a sphere of radius 1 cm so that the volume of the sphere cut away is half of the volume of the sphere. Find the radius of the drill bit used. **4**

(d) Find the roots of the equation $x^4 + 2x^3 + 6x^2 + 8x + 8 = 0$ given that one of the roots is purely imaginary. **3**

Question 8 (15 marks) Use a separate page/booklet **Marks**

(a) A body of mass *m* in falling from rest, experiences air resistance of magnitude kv^2 per unit mass, where *k* is a positive constant.

(i) Write the equation of motion of the body and find the value of the terminal velocity V of the body in terms of k and g (acceleration due to gravity). [Take g = 9· 8 m/s^2] **3**

(ii) If w is the velocity of the body when it reaches the ground, show that the distance S fallen is given by

$$S = -\frac{1}{2k}\ln\left(1 - \frac{w^2}{V^2}\right).$$ **3**

(iii) With air resistance remaining the same, prove that if the body is projected vertically upwards from the ground with velocity U, then it will attain its greatest height H where $H = \frac{1}{2k}\ln\left(1 + \frac{U^2}{V^2}\right)$, and return to the ground with velocity w given by $w^{-2} = U^{-2} + V^{-2}$. **3**

(b) A section of a road approximates to a circular bend of radius 500 m. The road is inclined at an angle of 4^0 to the horizontal. [Take g = 9 · 8 m/s^2]

(i) Find the best possible speed to drive this bend in order to minimise the frictional sideways force on the tyres. **3**

(ii) If a car is of mass 900 kg and is driven round the road at 100 km/h, find the sideways frictional force on the tyres in Newtons. **3**

Marking Guidelines: Mathematics Extension II Examination

ANSWERS QUESTION 1

Question 1 (a) (i)

Criteria	Marks
• One mark for correct partial fractions, one each for integration	3

Answer:

Let $\frac{1}{4-9x^2} = \frac{A}{2+3x}+\frac{B}{2-3x}$

Then $1 = A(2-3x) + B(2+3x)$

Putting $x=\frac{2}{3}$, $1=4B \therefore B=\frac{1}{4}$

Putting $x=-\frac{2}{3}$, $1=4A \therefore A=\frac{1}{4}$

$= \int\frac{dx}{4-9x^2} = \frac{1}{4}\left(\int\frac{dx}{2+3x}+\int\frac{dx}{2-3x}\right)$

$= \frac{1}{4}\left(\frac{1}{3}\ln(2+3x)-\frac{1}{3}\ln(2-3x)\right)+const$

$= \frac{1}{4}\bullet\frac{1}{3}\ln\left(\frac{2+3x}{2-3x}\right)+const$

$= \frac{1}{12}\ln\left(\frac{2+3x}{2-3x}\right)+const$

Question 1 (a) (ii)

Criteria	Marks
• One mark for integration for writing difference of squares and one integration	2

Answer:

$\int\frac{dx}{\sqrt{4-9x^2}} = \int\frac{dx}{\sqrt{2^2-(3x)^2}}$

$= \frac{1}{3}\sin^{-1}\frac{3x}{2} + const$

Question 1(a) (iii)

Criteria	Marks
• One mark for proper substitutions, one for simplifying and one for integration	3

Answer:

$\int\frac{dx}{(4-9x^2)^{3/2}}$

Let $x=\frac{2}{3}\sin\theta \Rightarrow dx=\frac{2}{3}\cos\theta\, d\theta$

$=\frac{2}{3}\int\frac{\cos\theta\, d\theta}{(4-4\sin^2\theta)^{3/2}} = \frac{2}{3}\int\frac{\cos\theta\, d\theta}{4^{3/2}(\cos^2\theta)^{3/2}}$

$= \frac{1}{12}\int\frac{\cos\theta\, d\theta}{\cos^3\theta}$

$= \frac{1}{12}\int\sec^2\theta\, d\theta = \frac{1}{12}\tan\theta + const$

$= \frac{1}{12}\frac{\sin\theta}{\cos\theta}+const = \frac{1}{12}\frac{\frac{3x}{2}}{\sqrt{1-\frac{9x^2}{4}}}+const$

$= \frac{x}{4}\frac{1}{\sqrt{4-9x^2}} + const$

Question 1 (b) (i)

Criteria	Marks
• One mark for writing in integration by parts form one for integration and one for final answer.	3

Answer:

(i) $\int_0^{1/2}\sin^{-1}x\,dx = \int_0^{1/2}\sin^{-1}x\,\frac{d}{dx}(x)\,dx$

$= \left(x\sin^{-1}x\right)_0^{1/2} - \int_0^{1/2}\frac{x}{\sqrt{1-x^2}}\,dx$

$= \left(\frac{\pi}{6}\times\frac{1}{2}\right)-\left(-\left(\frac{\sqrt{3}}{2}-1\right)\right)$

$= \frac{\pi}{12}+\frac{\sqrt{3}}{2}-1$

$I = \int_0^{1/2}\frac{x\,dx}{\sqrt{1-x^2}}$

Let $1-x^2=t$ then $-2x\,dx=dt$

$I = -\frac{1}{2}\int_1^{3/4}\frac{dt}{t^{1/2}} = -\frac{1}{2}\left(\frac{t^{1/2}}{1/2}\right)_1^{3/4}$

$= -\frac{1}{2}\times\frac{2}{1}\left[\left(\frac{\sqrt{3}}{2}-1\right)\right] = -\left(\frac{\sqrt{3}}{2}-1\right)$

Question 1(b) (ii)

Criteria	Marks
• One mark for substitution and change of limits and one for integration.	2

Answer:

$I = \int_0^2\frac{x^2\,dx}{x^6+64}$ Let $x^3=t$ then $3x^2\,dx=dt$

$I = \frac{1}{3}\int_0^8\frac{dt}{t^2+64} = \frac{1}{3}\times\frac{1}{8}\left(\tan^{-1}\frac{t}{8}\right)_0^8$

$= \frac{1}{24}\left(\tan^{-1}1-\tan^{-1}0\right) = \frac{1}{24}\times\frac{\pi}{4}$

$= \frac{\pi}{96}$

Question 1(b) (iii)

Criteria	Marks
• One mark for writing in integration by parts form and one for integration.	2

Answer:

$\int_0^a x\sin(a-x)\,dx = \int_0^a x\frac{d}{dx}[\cos(a-x)]dx$

$= [x\cos(a-x)]_0^a - \int_0^a\cos(a-x)\,dx$

$= (a\cos 0-0) - [-\sin(a-x)]_0^a$

$= a-\sin a$

ANSWERS QUESTION 2

Question 2 (a)(i)(ii)

Criteria	Marks
• (i) One mark for correct answer	1
• (ii) One mark for correct answer.	1

Answer:

(a) (i) let $z = a + ib$ and $w = c + id$

$\overline{z+w} = \overline{(a+ib)+(c+id)}$

$= \overline{(a+c)+i(b+d)}$

$= (a+c) - i(b+d)$

$= (a-ib)+(c-id)$

$= \bar{z}+\bar{w}$

(ii) $|z|^2 = \left(\sqrt{a^2+b^2}\right)^2$

$= a^2+b^2$

$= (a+ib)(a-ib)$

$= z \cdot \bar{z}$

Question 2 (a) (iii)

Criteria	Marks
• One mark for $\Rightarrow 3(z-1)\overline{(z-1)} = (z+1)\overline{(z+1)}$ and one for simplification	2

Answer:

(iii) $3|z-1|^2 = |z+1|^2$

$\Rightarrow 3(z-1)\overline{(z-1)} = (z+1)\overline{(z+1)}$ .... {using (ii)}

$\Rightarrow 3(z-1)(\bar{z}-1) = (z+1)(\bar{z}+1)$ .... {using (i)}

$\Rightarrow 3\left[z\bar{z}-z-\bar{z}+1\right] = \left[z\bar{z}+z+\bar{z}+1\right]$

$\Rightarrow 2z\bar{z}-4z-4\bar{z}+2 = 0$

$\Rightarrow z\bar{z}-2z-2\bar{z}+1 = 0$

$\Rightarrow z\bar{z}-2z-2\bar{z}+4 = 3$

$\Rightarrow (z-2)(\bar{z}-2) = 3$

$\Rightarrow |z-2|^2 = 3$ ....{using (ii)}

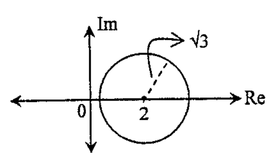
Question 2 (a) (iv)

Criteria	Marks
• One mark for the locus and one for the diagram.	2

(iv) From above we have

$\sqrt{3}|z-1| = |z+1|$ leads to $|z-2| = \sqrt{3}$

which is a circle with centre at (2,0) and radius √3.



Question 2 (a) (v) (α) and (β)

Criteria	Marks
• (α) One mark for step1 and one for step 2.	2
• (β) One mark for the geometrical interpretation.	1

Answer:

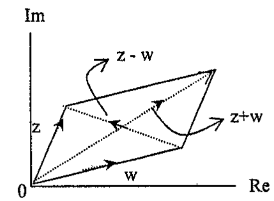
(α) $|z-w|^2 + |z+w|^2 = (z-w)\overline{(z-w)} + (z+w)\overline{(z+w)}$ from (i) step 1

$= (z-w)(\bar{z}-\bar{w}) + (z+w)(\bar{z}+\bar{w})$

$= z\bar{z} - z\bar{w} - w\bar{z} + w\bar{w} + z\bar{z} + z\bar{w} + w\bar{z} + w\bar{w}$

$= 2\,z\bar{z} + 2\,w\bar{w} = 2|z|^2 + 2|w|^2 = 2\left[|z|^2 + |w|^2\right]$ from (ii) Step 2

(β)



Thus as $|z-w|$ and $|z+w|$ are the lengths of the diagonals of a parallelogram with sides of length $|z|$ and $|w|$ we observe: 'The sum of squares of the lengths of diagonals of a parallelogram is equal to the sum of squares of the lengths of the sides of the parallelogram'

Question 2 (b) (i) (ii)

Criteria	Marks
• (i) One mark for showing $2\cos n\theta$	1
• (ii) One for r and one for θ	2

Answer:

(i)

$z = \cos\theta + i\sin\theta$

$\therefore z^n = \cos n\theta + i\sin n\theta$ [By De Moivre's theorem]

and $z^{-n} = \cos n\theta - i\sin n\theta$

$z^n + \dfrac{1}{z^n} = 2\cos n\theta$

(ii) $z = r(\cos\theta + i\sin\theta)$

Given $2\,z^3 = 9 + 3\sqrt{3}\,i$

$2r^3(\cos 3\theta + i\sin 3\theta) = 6\sqrt{3}\left(\dfrac{\sqrt{3}}{2} + \dfrac{1}{2}i\right)$

$= 6\sqrt{3}\,[\cos\dfrac{\pi}{6} + i\sin\dfrac{\pi}{6}]$

$\therefore 2r^3 = 6\sqrt{3}$ *and* $3\theta = \dfrac{\pi}{6}$

$\therefore r = \sqrt{3}$ *and* $\theta = \dfrac{\pi}{18}$

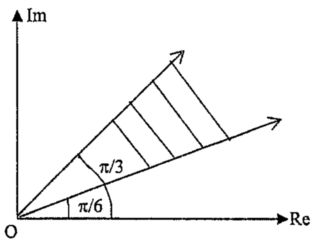
Question 2 (c) (d)

Criteria	Marks
• (c) One mark for the diagram • (d) One mark for the diagram and one for stating the locus.	1 2

Answer:

(c)

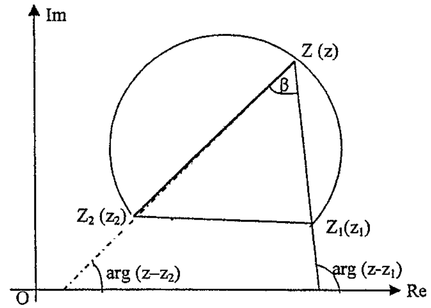
$$\frac{\pi}{6} \le \arg Z \le \frac{\pi}{3}$$



(d)

$$\arg\left[\frac{z-z_1}{z-z_2}\right] = \beta$$

The locus of Z is the major arc of a circle terminated at Z_2 and Z_1.



ANSWERS QUESTION 3

Question 3 (a)

Criteria	Marks
• One mark for writing ∠APB = 90° and one for the final answer.	2

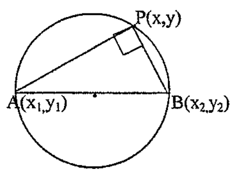
Answer:

If P is point on the circumference and AB is a diameter then angle APB = 90^0 or AP ⊥ PB

i.e. grad of AP × grad. of PB = - 1

$$\therefore \frac{y-y_1}{x-x_1} \times \frac{y-y_2}{x-x_2} = -1$$

or $\quad (y-y_1)(y-y_2) + (x-x_1)(x-x_2) = 0$



Question 3 (b) (i) (ii)

Criteria	Marks
• (i) One mark for the equation. • (ii) One mark for finding the coordinates of focus and one for the finding directricies.	1 2

Answer:

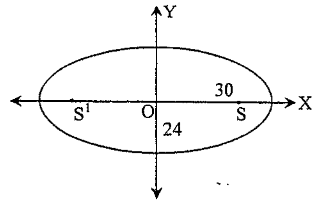
(i)
Here $a = 30$ and $b = 24$
The equation of the ellipse is given by

$$\frac{x^2}{900} + \frac{y^2}{576} = 1$$

(ii) $b^2 = a^2(1-e^2)$ or $e = \sqrt{1-\frac{b^2}{a^2}}$

$$e = \sqrt{1-\frac{576}{900}} = \frac{3}{5}$$

S $(ae,0)$ is (18, 0) and S^1 is (-18,0)



(iii) Eqns of directricies are

$$x = \pm\frac{a}{e} = \pm\frac{30}{3/5} = \pm 50$$

Question 3 (c) (i) (ii)

Criteria	Marks
• (i) One mark for finding AQ and AQ^1 and one for the product. • (ii) One mark for finding the equation of the circle, one for the equation where the circle cuts the x-axis and one for the final answer.	2 3

The equation of the tangent at P is
$bx\cos\theta + ay\sin\theta - ab = 0$
Substitute $y = b$ to get x cord. of Q
$bx\cos\theta + ab\sin\theta - ab = 0$

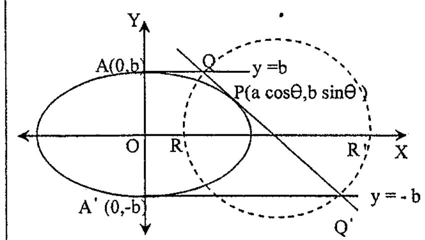
or $x = \dfrac{ab\,(1-\sin\theta)}{b\cos\theta} = \dfrac{a(1-\sin\theta)}{\cos\theta}$

i.e. $AQ = \dfrac{a(1-\sin\theta)}{\cos\theta}$

Substitute y = - b to get x cord. of Q'
$bx\cos\theta - ab\sin\theta - ab = 0$

or $x = \dfrac{ab\,(1+\sin\theta)}{b\cos\theta} = \dfrac{a(1+\sin\theta)}{\cos\theta}$

$A'Q' = \dfrac{a(1+\sin\theta)}{\cos\theta}$



$$\therefore AQ \times A'Q' = \frac{a^2(1-\sin^2\theta)}{\cos^2\theta} = a^2$$

(ii) The equation of the circle on QQ ' is given by

$$[x-\frac{a(1-\sin\theta)}{\cos\theta}][x-\frac{a(1+\sin\theta)}{\cos\theta}]+(y-b)(y+b)=0$$

This meets the X – axis at $y = 0$

$$\therefore [x-\frac{a(1-\sin\theta)}{\cos\theta}][x-\frac{a(1+\sin\theta)}{\cos\theta}]+(0-b)(0+b)=0$$

$$[x-\frac{a(1-\sin\theta)}{\cos\theta}][x-\frac{a(1+\sin\theta)}{\cos\theta}] = b^2$$

$$[x\cos\theta - a(1-\sin\theta)][x\cos\theta - a(1+\sin\theta)=b^2\cos^2\theta$$

$$x^2\cos^2\theta - ax\cos\theta(1+\sin\theta) - ax\cos\theta(1-\sin\theta) + a^2(1-\sin^2\theta) = b^2\cos^2\theta$$

$$x^2\cos^2\theta - ax\cos\theta - ax\cos\theta\sin\theta - ax\cos\theta + ax\cos\theta\sin\theta + a^2\cos^2\theta = b^2\cos^2\theta$$

$$x^2\cos^2\theta - 2ax\cos\theta + \cos^2\theta(a^2-b^2) = 0$$

or $x^2 - \frac{2ax}{\cos\theta} + (a^2-b^2) = 0$ If x_1 and x_2 are the roots of this equation, then $x_1 \times x_2 = a^2 - b^2$

OR and OR' can be represented by x_1 and x_2 $\therefore$ OR $\times$ OR' $= a^2 - b^2$

Question3 (d) (i) (ii) (iii)

Criteria	Marks
• (i) One mark for both a_{n+1} and b_{n+1}	1
• (ii) One mark for calculating when $n = 1,2$ &3 and one for guessing the formula	2
• (iii) One mark upto the assertion for $n = k$ and one for showing it is true for $n = k+1$	2

Answer:

(i) $(2-\sqrt{3})^n = a_n - b_n\sqrt{3}$

$\therefore$ $(2-\sqrt{3})^{n+1} = a_{n+1} - b_{n+1}\sqrt{3}$

Thus $a_{n+1} - b_{n+1}\sqrt{3} = (2-\sqrt{3})^n(2-\sqrt{3})$

$= (a_n - b_n\sqrt{3})\cdot(2-\sqrt{3})$

$= (2a_n + 3b_n) - \sqrt{3}(a_n + 2b_n)$

Equating rational and irrational parts, we get

$a_{n+1} = 2a_n + 3b_n$ and $b_{n+1} = a_n + 2b_n$

(ii) $(2-\sqrt{3})^1 = 2-\sqrt{3}$ $\therefore a_1 = 2,\ b_1 = 1$

$(2-\sqrt{3})^2 = 7-4\sqrt{3}$ $\therefore a_2 = 7,\ b_2 = 4$

$(2-\sqrt{3})^3 = 26-15\sqrt{3}$ $\therefore a_3 = 26,\ b_3 = 15$

Now $a_1^2 - 3b_1^2 = 4 - 3\times 1 = 1$

$a_2^2 - 3b_2^2 = 49 - 3\times 16 = 1$

$a_3^2 - 3b_3^2 = 676 - 3\times 225 = 1$

(iii) Guess : $a_n^2 - 3b_n^2 = 1$ for all positive integers n

Proof:

It is true for $n = 1$ as shown above.

Let it be true for $n = k$ i.e $a_k^2 - 3b_k^2 = 1$

We have to show that it it is true for $n = k+1$

i.e. $a_{k+1}^2 - 3b_{k+1}^2 = 1$

Now $a_{k+1}^2 - 3b_{k+1}^2$

$= (2a_k + 3b_k)^2 - 3(a_k + 2b_k)^2$

$= 4a_k^2 + 12a_kb_k + 9b_k^2 - 3(a_k^2 + 4a_kb_k + 4b_k^2)$

$= a_k^2 - 3b_k^2$

$= 1$

Thus if it is true for $n = k$, then it is true for $n = k+1$.

But it true for $n = 1$, therefore it is true for $n = 1+1 = 2, 2+1 = 3$, and so on,.$\therefore$ for all positive values of n.

ANSWERS QUESTION 4

Question 4 (a) (b) (i) (ii) (iii) (iv)

Criteria	Marks
• (a) One mark for writing the value of a and one for for the equation	2
• (b) (i) One mark for grad. at any point, one for grad. of normal at P and one for the equation of normal	3
• (ii) One mark for finding coordinates of Q	1
• (iii) One for x value and one for y value	2
• (iv) One for the locus	1

Answer:

(a) Let the equation of the hyperbola be

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Here $a = 10$ and (12, 6) is a point on it

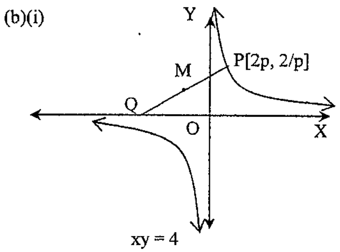
$\therefore$ $\frac{12^2}{10^2} - \frac{6^2}{b^2} = 1$ or $\frac{36}{b^2} = \frac{144}{100} - 1$

or $b^2 = \frac{100\times 36}{44} = \frac{900}{11}$

The equation is

$$\frac{x^2}{100} - \frac{y^2}{\frac{900}{11}} = 1 \quad or \quad \frac{x^2}{100} - \frac{11y^2}{900} = 1$$

(b)(i)



$y = \frac{4}{x}$ $\therefore \frac{dy}{dx} = -\frac{4}{x^2}$ or grad.at $x = 2p$ is $\frac{-4}{4p^2} = \frac{-1}{p^2}$

Grad. of normal at $x = 2p$ is p^2

$\therefore$ Eqn. of normal at P is given by

$y - \frac{2}{p} = p^2(x - 2p)$ or $y = p^2x - 2p^3 + \frac{2}{p}$

(b) (ii) This normal meets X – axis where $y = 0$

$0 = p^2x - 2p^3 + \frac{2}{p}$ $\therefore x = 2p - \frac{2}{p^3}$

$\therefore$ Q is $\left(2p - \frac{2}{p^3}, 0\right)$ and P is $\left(2p, \frac{2}{p}\right)$

(b) (iii) If M is the mid point of QP, then

M is $\left(\frac{2p - \frac{2}{p^3} + 2p}{2}, \frac{0 + \frac{2}{p}}{2}\right) = \left(2p - \frac{1}{p^3}, \frac{1}{p}\right)$

(b) (iv) i.e. $x = 2p - \frac{1}{p^3}$ and $y = \frac{1}{p}$

The locus of M is $x = \frac{2}{y} - y^3$

Question 4 (c) (i)(ii)(iii)

Criteria	Marks
• (i) One mark for the binomial expansion and one for writing in sigma notation	2
• (ii) One for De Moivre's theorem and one for equating the real parts and simplifying	2
• (iii) One mark for finding the values of Θ and one for showing it is the product of the roots	2

(c) (i)

$(\cos\theta + i\sin\theta)^5 = {}^5c_0\cos^5\theta + {}^5c_1\cos^4\theta(i\sin\theta) + {}^5c_2\cos^3\theta(i\sin\theta)^2 + {}^5c_3\cos^2\theta(i\sin\theta)^3$

$+\ {}^5c_4\cos\theta(i\sin\theta)^4 + {}^5c_5(i\sin\theta)^5$

$= \sum_{r=0}^{5} {}^5c_r(i)^r\cos^{5-r}\theta\sin^r\theta$

(ii) By De Moivres Theorem $(\cos\theta + i\sin\theta)^5 = \cos 5\theta + i\sin 5\theta$

Equating real parts, $\cos 5\theta = {}^5c_0\cos^5\theta + {}^5c_2\cos^3\theta(i\sin\theta)^2 + {}^5c_4\cos\theta(i\sin\theta)^4$

$= \cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta$

$= \cos^5\theta - 10\cos^3\theta(1-\cos^2\theta) + 5\cos\theta(1-\cos^2\theta)^2 = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$

(iii) $16\cos^5\theta - 20\cos^3\theta + 5\cos\theta = 0$

if $\cos 5\theta = 0$

$\therefore\ 5\theta = 2n\pi \pm \frac{\pi}{2}$

or $\theta = \frac{2n\pi}{5} \pm \frac{\pi}{10}$

i.e. $\theta = \frac{\pi}{10}, \frac{3\pi}{10}, \frac{\pi}{2}, \frac{7\pi}{10}, \frac{9\pi}{10}$

$\therefore$ roots of $16\cos^5\theta - 20\cos^3\theta + 5\cos\theta = 0$

are $\cos\frac{\pi}{10}, \cos\frac{3\pi}{10}, \cos\frac{\pi}{2}, \cos\frac{7\pi}{10}$ and $\cos\frac{9\pi}{10}$

Now $16\cos^5\theta - 20\cos^3\theta + 5\cos\theta = 0$

or $\cos\theta(16\cos^4\theta - 20\cos^2\theta + 5) = 0$

$\therefore$ roots of $16\cos^4\theta - 20\cos^2\theta + 5 = 0$ are

$\cos\frac{\pi}{10}, \cos\frac{3\pi}{10}, \cos\frac{7\pi}{10}$ and $\cos\frac{9\pi}{10}$

product of roots $= \frac{5}{16}$

$\therefore\ \cos\frac{\pi}{10}\cdot\cos\frac{3\pi}{10}\cdot\cos\frac{7\pi}{10}\cdot\cos\frac{9\pi}{10} = \frac{5}{16}$

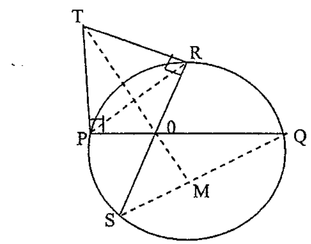
ANSWERS QUESTION 5

Question 5 (a)

Criteria	Marks
• One mark for showing that PTRO is cyclic quadrilateral, one for showing that the angles PTO, PRO and PQS are equal and one for showing that ∠OMQ is 90⁰.	3

Answer:

Proof: Produce TO to meet SQ in M.
PTRO is a cyclic quadrilateral since
$\angle P + \angle R = 180^0$
$\therefore \angle PTO = \angle PRO$ (angles in the same segment)
But $\angle PRO = \angle PQS$ (angles in the same segment in the cyclic quad. PRQS)
$\angle POT = \angle MOQ$ (Vertically opposite angles)
$\angle POT + \angle PTO = 90^0$ (PTO is a rt. angled Δ)
$\therefore\ \angle MOQ + \angle PQS = 90^0$
In Δ OQM, $\angle OMQ = 180^0 - (\angle MOQ + \angle PQS)$
$= 90^0$



Question 5 (b) (c)

Criteria	Marks
• (b) One mark for showing $\frac{(p+q)^2}{4} \geq pq$ and one for the final answer.	2
• (c) One mark for $5\theta = 2n\pi \pm \left(\frac{\pi}{2} - 4\theta\right)$, one for $5\theta = 2n\pi + \left(\frac{\pi}{2} - 4\theta\right)$ or $5\theta = 2n\pi - \frac{\pi}{2} + 4\theta$ and one for simplification	3

Answer:

(b) $(\sqrt{p} - \sqrt{q})^2 \geq 0 \Rightarrow p + q - 2\sqrt{pq} \geq 0$

$\Rightarrow p + q \geq 2\sqrt{pq}$ (equality iff $p = q$)

$\Rightarrow \frac{(p+q)^2}{4} \geq pq$

Now $\frac{1}{p} + \frac{1}{q} = \frac{p+q}{pq} \geq \frac{p+q}{\frac{(p+q)^2}{4}} = \frac{4}{p+q} = 4$

since $p + q = 1$

(c) $\cos 5\theta = \sin 4\theta$

$\therefore\ \cos 5\theta = \cos\left(\frac{\pi}{2} - 4\theta\right)$

$\therefore\ 5\theta = 2n\pi \pm \left(\frac{\pi}{2} - 4\theta\right)$

$\therefore\ 5\theta = 2n\pi + \left(\frac{\pi}{2} - 4\theta\right)$ or $5\theta = 2n\pi - \frac{\pi}{2} + 4\theta$

or $9\theta = \frac{(4n+1)\pi}{2}$ or $\theta = \frac{(4n-1)\pi}{2}$

$\therefore\ \theta = \frac{(4n+1)\pi}{18}, \frac{(4n-1)\pi}{2}$

Question 5 (d)

Criteria	Marks
• One mark for finding the roots and one for the sum	2

(d) $x^3 - 1 = (x-1)(x^2 + x + 1) = 0$

$\therefore$ The roots are $1, \ \dfrac{-1+\sqrt{3}\,i}{2} \ \& \ \dfrac{-1-\sqrt{3}i}{2}$

If $\omega = \dfrac{-1+\sqrt{3}i}{2}$ then

$\omega^2 = \left(\dfrac{-1+\sqrt{3}i}{2}\right)^2 = \left(\dfrac{1-2\sqrt{3}i-3}{4}\right) = \dfrac{-1-\sqrt{3}i}{2}$

is the other complex root.

$\therefore$ sum of the roots $= 1+\omega+\omega^2 =$

$1 + \dfrac{-1+\sqrt{3}\,i}{2} + \dfrac{-1-\sqrt{3}}{2} = 1 - 1 = 0$

Question 5 (e) (i) (ii) (iii)

Criteria	Marks
• (i) One mark for finding asymptote and one each for stationary points and their nature	3
• (ii) One mark for sketch	1
(iii) One mark for answer	1

(i) $y = \dfrac{x^2}{x^3+1} = \dfrac{x^2}{(x+1)(x^2-x+1)}$

(0,0) lies on the curve.

As $x \to \pm\infty$, $y \to 0$ $\therefore$ x –axis is a horizontal asymptote.

When $x \to -1$, $y \to \infty$, $\therefore x = -1$ is a vertical asymptote.

Now $y' = \dfrac{(x^3+1)\times 2x - x^2(3x^2)}{(x^3+1)^2} = \dfrac{x(2-x^3)}{(x^3+1)^2}$

$y' = 0$ *when* $x = 0$ *and* $\sqrt[3]{2}$

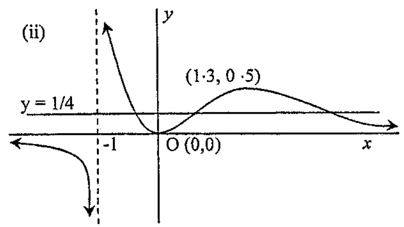
y' changes sign from - to + in passing through $x = 0$ and hence at $x = 0$, the curve has a minima.

y' changes sign from + to - in passing through $x = \sqrt[3]{2}$ and hence at $x = \sqrt[3]{2}$, the curve has a maxima.

When $x = \sqrt[3]{2}$, $y = \dfrac{(\sqrt[3]{2})^2}{\left[(\sqrt[3]{2})^3 + 1\right]} = \dfrac{\sqrt[3]{4}}{3}$

i.e. max point $\cong$ (1·3, 0·5)

(ii)



(iii) $x^3 - 4x^2 + 1 = 0$ can be written as $x^3 + 1 = 4x^2$

or $\dfrac{x^2}{x^3+1} = \dfrac{1}{4}$

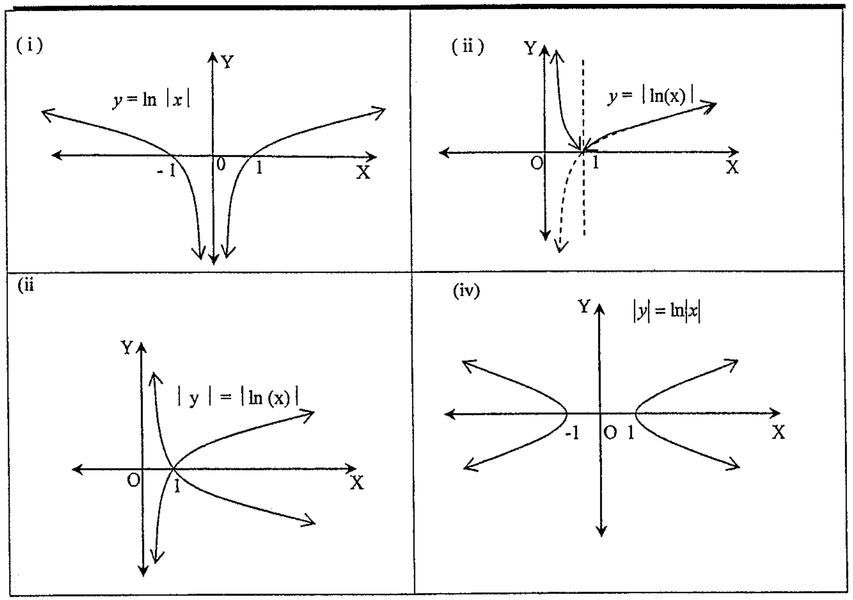
The roots of this eqn are the x - coordinates of the points where $y = 1/4$ cuts the curve. There are three real and distinct roots.

ANSWERS QUESTION 6

Question 6 (a) (i) (ii) (iii)(iv)

Criteria	Marks
• (i) One mark for showing where the curve cuts x –axis and for the general shape of the curve.	2
• (ii) One mark for showing where the curve cuts x –axis and for the general shape of the curve.	2
• (iii) One mark for showing where the curve cuts x –axis and for the general shape of the curve.	2
• (iv) One mark for showing where the curve cuts x –axis and for the general shape of the curve.	2

Answer:



Question 6 (b) (i) (ii)

Criteria	Marks
• (i) One mark for the binomial generator, one for the numerical answer • (ii) One mark for = 1- [Pr. no points in Q_1 or 1 point in Q_1] and one for simplification.	2 2

Answer:

(i) P (a) = Pr.of point in Q_1 = $\frac{1}{4}$

P ($\bar{a}$) = Pr.of point not in $Q_1 = \frac{3}{4}$

Pr. generator = $\left(\frac{1}{4}+\frac{3}{4}\right)^5$

Pr.one point in Q_1= ${}^5c_1 \cdot \left(\frac{1}{4}\right)^1\left(\frac{3}{4}\right)^4 = \frac{405}{1024}$

(ii) Pr. at least 2 points in Q_1

= 1- [Pr. no points in Q_1 or 1 point in Q_1]

= 1 - [Pr. no points in Q_1 + Pr of 1 point in Q_1]

$= 1 - \{\left(\frac{3}{4}\right)^5 + \frac{405}{1024}\}$

$= 1 - \frac{243+405}{1024} = \frac{47}{128}$

Question 6 (c)

Criteria	Marks
• One for writing and substituting in the formula, one for finding the amplitude and one for the period.	3

Answer:

The speed is given by

$v^2 = n^2(a^2 - x^2)$

$32^2 = n^2(a^2 - 9)$ -------(1)

and $24^2 = n^2(a^2 - 16)$ ------(2)

Dividing equations (1) and (2) we get

$\frac{32^2}{24^2} = \frac{a^2-9}{a^2-16}$

$\frac{16}{9} = \frac{a^2-9}{a^2-16}$ or $16a^2 - 256 = 9a^2 - 81$

$\therefore a^2 = 25$ i.e. amplitude is 5 m.

Substituting we get,

$24^2 = n^2(25-16)$

$64 = n^2$

$8 = n$

$\therefore$ periodic time = $2\pi/n = \pi/4$ s

ANSWERS QUESTION 7

Question 7 (a)

Criteria	Marks
• One mark for the pts of intersection, one for showing volume of an elementary shell, one for the definite integral and one for the final answer.	4

Answer:

To find the pts of intersection we solve the equations $y = \frac{x^3}{4}$ and $y = \sqrt{2x}$

$y^2 = \frac{x^6}{16}$ and $y^2 = 2x$

$\therefore \frac{x^6}{16} = 2x$ or $x(x^5 - 32) = 0$

$\therefore x = 0$ or $x = 2$

The points of intersection are (0 ,0) and (2 ,2)
When the region between the curves is rotated about the X- axis, a typical strip ABCD between these curves and parallel to the X– axis generates a cylindrical shell of volume

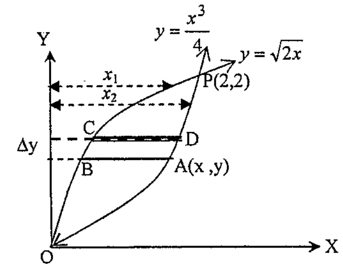
$\Delta V = 2\pi\,(x_2 - x_1)\,y \cdot \Delta y$ where

x_1 is given by $2x_1 = y^2$ or $x_1 = \frac{y^2}{2}$

and x_2 is given by $x_2^3 = 4y$ or $x_2 = (4y)^{1/3}$

The required volume is

$V = \int_0^2 2\pi y[(4y)^{1/3} - \frac{y^2}{2}]\,dy$



$V = 2\pi\int_0^2\left(4^{1/3}y^{4/3} - \frac{y^3}{2}\right)dy$

$V = 2\pi\left[4^{1/3} \times \frac{3}{7}y^{7/3} - \frac{y^4}{8}\right]_0^2$

$= 2\pi\left(\frac{24}{7} - 2\right) = \frac{20\pi}{7}$ *cubic units*

Question 7 (b) (i)

Criteria	Marks
• One mark for showing $a^2+b^2 \ge 2ab$, one for $a^2+b^2+c^2-ab-bc-ca \ge 0$ and one for the $\frac{a+b+c}{3} \ge \sqrt[3]{abc}$	3

Answers:

(i) If $a>0$, $b>0$ and $c>0$, then

$(a-b)^2 \ge 0$ equality iff $a=b$

i.e. $a^2+b^2-2ab \ge 0$ or $a^2+b^2 \ge 2ab$

Similarly it can be shown that

$b^2+c^2 \ge 2bc$ and $c^2+a^2 \ge 2ca$

$\therefore a^2+b^2+b^2+c^2+c^2+a^2 \ge 2ab+2bc+2ca$

or $2(a^2+b^2+c^2) \ge 2(ab+bc+ca)$

or $a^2+b^2+c^2-ab-bc-ca \ge 0$ equality iff $a=b=c$

It can be shown by expansion that

$(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

$=a^3+b^3+c^3-3abc$

since $a+b+c>0$ and $a^2+b^2+c^2-ab-bc-ca \ge 0$

$a^3+b^3+c^3-3abc \ge 0$ equality iff $a=b=c$

or $a^3+b^3+c^3 \ge 3abc$

If we substitute $a \to a^3$, $b \to b^3$ and $c \to c^3$ we get

$a+b+c \ge 3a^{1/3}b^{1/3}c^{1/3}$

$\therefore \frac{a+b+c}{3} \ge \sqrt[3]{abc}$

Question 7 (b) (ii)

Criteria	Marks
• One mark for the answer.	1

Answer:

Now substituting $a \to \frac{a}{b}$, $b \to \frac{b}{c}$, $c \to \frac{c}{a}$ in $\frac{a+b+c}{3} \ge \sqrt[3]{abc}$, we get

$$\frac{\frac{a}{b}+\frac{b}{c}+\frac{c}{a}}{3} \ge \sqrt[3]{\frac{abc}{bca}}=1$$

Hence $\frac{a}{b}+\frac{b}{c}+\frac{c}{a} \ge 3$

Question 7 (c)

Criteria	Marks
• One mark for writing the integral for the remaining volume, one for the integration, one for evaluation and one for finding *r*.	4

Answer:

The diagram shows the upper half of the cross-section across the centre plane of the sphere of radius I cm. If a drill of radius *r* units cuts through the centre of the sphere, the remaining volume can be found by revolving the shaded area around the x-axis. Therefore, the remaining volume

$$=\pi\int_{-\sqrt{1-r^2}}^{\sqrt{1-r^2}}\left(\sqrt{1-x^2}\right)^2dx-\pi\int_{-\sqrt{1-r^2}}^{\sqrt{1-r^2}}r^2dx$$

$$=\pi\int_{-\sqrt{1-r^2}}^{\sqrt{1-r^2}}(1-x^2)dx-\pi r^2\int_{-\sqrt{1-r^2}}^{\sqrt{1-r}}1\,dx$$

$$=\pi\left[x-\frac{x^3}{3}\right]_{-\sqrt{1-r^2}}^{\sqrt{1-r^2}}-\pi r^2[x]_{-\sqrt{1-r^2}}^{\sqrt{1-r^2}}$$

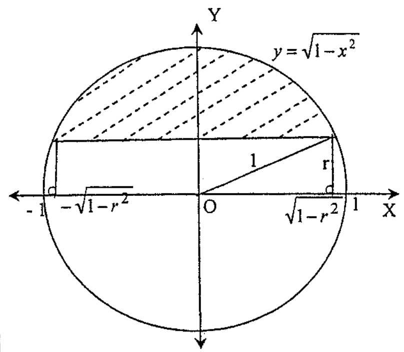
$$=\pi\left[\sqrt{1-r^2}-\frac{\left(\sqrt{1-r^2}\right)^3}{3}-\left(-\sqrt{1-r^2}+\frac{\left(\sqrt{1-r^2}\right)^3}{3}\right)\right]$$

$$-2\pi r^2\sqrt{1-r^2}$$

$$=2\pi\sqrt{1-r^2}-\frac{2\pi}{3}\left(\sqrt{1-r^2}\right)^3-2\pi r^2\sqrt{1-r^2}$$

$$=2\pi\sqrt{1-r^2}(1-r^2)-\frac{2\pi}{3}\left(\sqrt{1-r^2}\right)^3$$

$$=2\pi\left(\sqrt{1-r^2}\right)^3-\frac{2\pi}{3}\left(\sqrt{1-r^2}\right)^3$$



$$=\frac{4\pi}{3}\left(\sqrt{1-r^2}\right)^3$$

Volume of the sphere (radius=1cm)

$$=\frac{4\pi}{3}\ cm^3$$

Since the volume remaining is half of the volume of sphere of radius 1 cm,

$$\frac{4\pi}{3}\left(\sqrt{1-r^2}\right)^3=\frac{1}{2}\times\frac{4\pi}{3}$$

$$\therefore 1-r^2=\left(\frac{1}{2}\right)^{2/3}\approx 0\cdot630$$

or $r \approx \sqrt{(1-0\cdot63)}=0\cdot608\ (3sf)$

Question 7 (d)

Criteria	Marks
One mark for writing that $\pm ai$ are roots, one for showing the polynomial as the product of two quadratic factors and one for writing the roots.	3

Answer:

Let ai be a purely imaginary root, then $-ai$ will also be a root.
Thus P(x) has factors of $(x + ai)$ and $(x - ai)$
or $(x-ai)(x+ai) = x^2 + a^2$ is a factor

$$\therefore\ x^4 + 2x^3 + 6x^2 + 8x + 8 = (x^2 + a^2)\left(x^2 + bx + \frac{8}{a^2}\right)$$

Equating the coefficients we get,

$2 = b$

$6 = \frac{8}{a^2} + a^2$ and $8 = a^2 b$

Thus $b = 2$ and $a^2 = 4$ or $a = \pm 2$

$\therefore\ P(x) = (x^2 + 4)(x^2 + 2x + 2)$

This has the roots of

$\pm 2i,\ \frac{-2 \pm \sqrt{4-8}}{2} = \frac{-2 \pm 2i}{2}$

or $\pm 2i,\ -1 \pm i$

ANSWERS QUESTION 8

Question 8 (a) (i) (ii) (iii)

Criteria	Marks
• (i) One mark for writing the equation of motion, one for showing terminal velocity V, occurs when $f = 0$, and one for finding V.	3
• (ii) One mark for finding the integral for the distance x, one for S and one for evaluating the integral.	3
(iii) One mark for the equation of motion, one for finding H and one for deriving the expression for the velocity w.	3

Answer:

(i) The forces acting on the body are its weight mg acting downwards and the air resistance mkv^2 acting upwards.
Equation of motion of body is given by

$mf = mg - mkv^2$ or $f = g - kv^2$

Terminal velocity, V occurs when $f = 0$
(there is no acceleration and the velocity is constant)

$0 = g - kV^2\ \therefore\ V = \sqrt{\frac{g}{k}}$

(ii) Now $f = v\frac{dv}{dx} = g - kv^2$

i.e. $\frac{dx}{dv} = \frac{v}{g - kv^2}$

$\therefore\ x = \int \frac{v\ dv}{g - kv^2}$

Now the distance S fallen from rest $(v = 0)$ until the body hits the ground $(v = w)$ is given by

(iii) On its upward motion, the forces acting on the body are its weight acting downwards and the air resistance acting against its motion.

$\therefore$ The equation of motion of the body is

$mf = -mg - mkv^2$ or $f = -g - kv^2$

i.e. $v\frac{dv}{dx} = -(g + kv^2)$

or $x = \int \frac{-v\,dv}{g + kv^2} = -\frac{1}{2k}\left[\ln(g + kv^2)\right] + c$

when $x = 0$, $v = U$

$\therefore\ 0 = -\frac{1}{2k}\left[\ln(g + kU^2)\right] + c$ or $c = \frac{1}{2k}\left[\ln(g + kU^2)\right]$

Hence $x = \frac{1}{2k}\left[\ln\left(\frac{g + kU^2}{g + kv^2}\right)\right]$

Now at the highest point $x = H$ and $v = 0$

Part (ii) Con't

$$S = \int_0^w \frac{v\ dv}{g - kv^2} = -\frac{1}{2k}\left[\ln(g - kv^2)\right]_0^w$$

$$= -\frac{1}{2k}\ln\left(\frac{g - kw^2}{g}\right) = -\frac{1}{2k}\ln\left[1 - \frac{k}{g}w^2\right]$$

$$= -\frac{1}{2k}\ln\left[1 - \frac{w^2}{V^2}\right] \quad \because\ V^2 = \frac{g}{k}$$

Part (iii) Con't

$$\therefore\ H = \frac{1}{2k}\left[\ln\left(\frac{g + kU^2}{g}\right)\right]$$

Since the distance for going up and coming down is same $S = H$ and hence

$$-\frac{1}{2k}\left[\ln\left(1 - \frac{W^2}{V^2}\right)\right] = \frac{1}{2k}\left[\ln\left(1 + \frac{U^2}{V^2}\right)\right]$$

i.e. $\left[1 - \frac{W^2}{V^2}\right]^{-1} = \left(1 + \frac{U^2}{V^2}\right)$

or $V^4 = V^4 - V^2W^2 + U^2V^2 - U^2W$

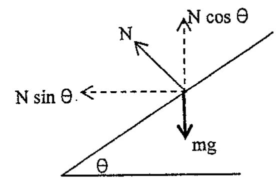
Dividing throughout by $U^2V^2W^2$ and rearranging

Terms we get, $W^{-2} = U^{-2} + V^{-2}$

Question 8 (b) (i) (ii)

Criteria	Marks
• One mark for resolving the forces, one for the expression involving tanΘ and one mark for finding speed.	3
• One mark for showing all forces and resolving them, one for finding F and one substituting all the values to get F.	3

(i) With no sideways force



Resolving forces,

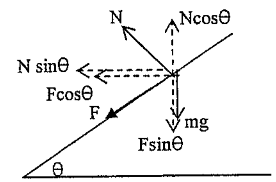
Horizontally : $N \sin\theta = \frac{mv^2}{r}$

Vertically : $N\cos\theta = mg$

$\therefore\ \tan\theta = \frac{v^2}{rg}$ or $v^2 = rg\tan\theta$

$\therefore\ v = \sqrt{500 \times 9{\cdot}8 \times \tan 4^0} \approx 18{\cdot}5\,m/s$

(ii)



Resolving forces:

Horizontally: $N\sin\theta + F\cos\theta = \frac{mv^2}{r}$

Vertically: $N\cos\theta = mg + F\sin\theta$

Solving for F, we get

$F = \frac{mv^2}{r}\cos\theta - mg\sin\theta$

Given

$m = 900,\ v = 100km/h = \frac{250}{9}m/s\ \&\ g = 9{\cdot}8$

$$F = \frac{900 \times \left(\frac{250}{9}\right)^2 \cos 4^0}{500} - 900 \times 9{\cdot}8 \times \sin 4^0$$

$\approx 770{\cdot}25\ N$