due: Thursday

EXT 1. TRIG. REVISION Assignment

1. Express $\tan 15^\circ$ in exact form with a rational denominator

2. Find $\lim\limits_{x \to 0} \dfrac{\tan 2x}{\sin x}$

3. Prove $\tan x = \dfrac{\sin 2x + \sin x}{1 + \cos 2x + \cos x}$

4. On the same axes, draw neat sketches of $y = \cos x$ and $y = \cos^2 x$ for $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$

   What is the periodicity of $y = \cos^2 x$?

5. Show that $\cos(A+B) + \cos(A-B) = 2\cos A . \cos B$

   Hence or otherwise evaluate $\int_0^{\pi/6} 2\cos 3x . \cos 2x \, dx$

6. Write $\cos 2\theta$ in terms of $\cos\theta$. hence express $\cos^2\left(\frac{\theta}{2}\right)$ in terms of $\cos\theta$

7. $\int \cos^2 2x . dx$ evaluate.

8. Express $6\sin x + 8\cos x$ in the form $A\sin(x+\alpha)$
   Hence sketch $y = 6\sin x + 8\cos x$. and state how many solutions to the equation $6\sin x + 8\cos x = 5$ for $0 \leq x \leq 2\pi$

9. Find the acute angle between the following straight lines $x + 2y - 3 = 0$ & $3x + y + 4 = 0$ to the nearest degree

10. Solve $3\sin\theta - \cos 2\theta = -2$ if $0 \leq \theta \leq 2\pi$

EXT1 : TRIG. REVISION ASSIGNMENT

(1) $\tan 15^\circ = \tan(45^\circ - 30^\circ)$

$$= \frac{\tan 45^\circ - \tan 30^\circ}{1 + \tan 45^\circ \tan 30^\circ}$$

$$= \frac{1 - \frac{1}{\sqrt{3}}}{1 + 1.\frac{1}{\sqrt{3}}}$$

$$= \left(\frac{\sqrt{3}-1}{\sqrt{3}}\right) \div \left(\frac{\sqrt{3}+1}{\sqrt{3}}\right)$$

$$= \left(\frac{\sqrt{3}-1}{\sqrt{3}}\right) \times \left(\frac{\sqrt{3}}{\sqrt{3}+1}\right)$$

$$= \frac{\sqrt{3}-1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$$

$$= \frac{3+1-2\sqrt{3}}{(\sqrt{3})^2 - 1^2}$$

$$= \frac{2(2-\sqrt{3})}{2}$$

$$= 2 - \sqrt{3}$$

2. $\lim_{x \to 0} \dfrac{\tan 2x}{\sin x}$

$\lim_{x \to 0} 2x = \dfrac{\tan 2x}{2\cancel{x}} \times \dfrac{\cancel{x}}{\sin x}$

$= 2 \times 1 \times 1$

$= 2$

3. RHS $\cancel{\text{LHS}} = \dfrac{\sin 2x + \sin x}{1 + \cos 2x + \cos x}$

$= \dfrac{2\sin x \cos x + \sin x}{1 + (2\cos^2 x - 1) + \cos x}$

$= \dfrac{2\sin x \cos x + \sin x}{\cancel{1} + 2\cos^2 x \cancel{-1} + \cos x}$

$= \dfrac{2\sin x \cos x + \sin x}{2\cos^2 x + \cos x}$

$\dfrac{2\sin x \cos x + \sin x}{\cos x(2\cos x + 1)}$

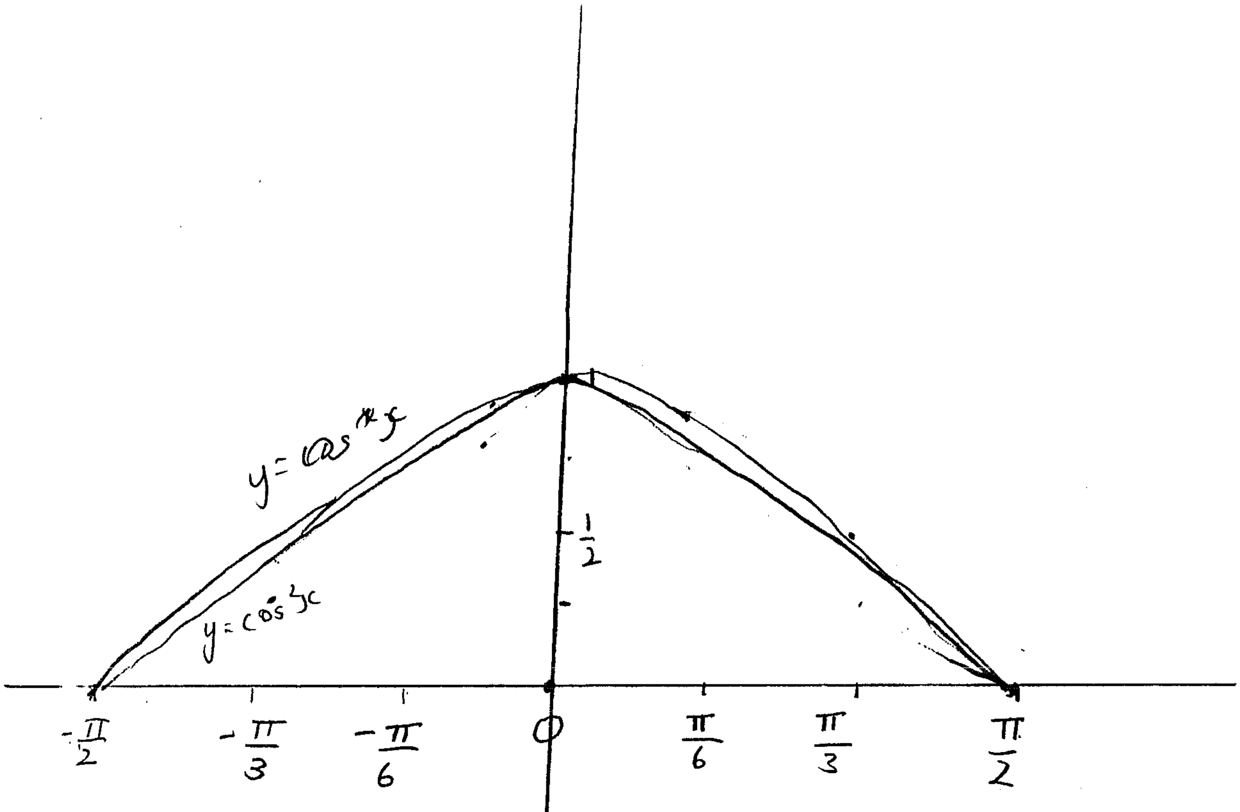
$\dfrac{\sin x(2\cancel{\cos x + 1})}{\cos x(\cancel{2\cos x + 1})}$

$= \dfrac{\sin x}{\cos x}$

$= \tan x$

$=$ LHS.

4.



what is the periodicity of $y = \cos^2 x$

$= \pi$

LHS

5. $\cos(A+B) + \cos(A-B)$

$\cos A\cos B - \sin A\sin B + (\cos A\cos B + \sin A\sin B)$

$\cos A\cos B - \cancel{\sin A\sin B} + \cos A\cos B + \cancel{\sin A\sin B}$

$= 2\cos A\cos B$

$= RHS$

$\cancel{\int_0^{\frac{\pi}{6}}}$

$2\cos 3x \cdot \cos 2x$

$= \cos(3x+2x) + \cos(3x-2x)$

$= \cos 5x + \cos x$

$\therefore \int_0^{\frac{\pi}{6}} \cos 5x + \cos x \, dx = \left[\frac{1}{5}\sin 5x + \sin x\right]_0^{\frac{\pi}{6}}$

$= \left[\frac{\sin 5x}{5} + \sin x\right]_0^{\frac{\pi}{6}}$

$= [0.1 + 0.5] - [0] = \frac{3}{5}$

6. $\cos 2\theta = \cancel{\cos(\theta+\theta)}$
$2\cos^2\theta - 1$

$\therefore$

$2\cos^2 x = \cos 2x + 1$

$\cos^2 x = \frac{1}{2}(\cos 2x + 1)$

$\cos^2\left(\frac{\theta}{2}\right) = \frac{1}{2}\left(\cos 2\left(\frac{\theta}{2}\right) + 1\right)$

$\cos^2\left(\frac{\theta}{2}\right) = \frac{1}{2}\left(\cos \frac{2\theta}{2} + 1\right)$

$\cos^2\left(\frac{\theta}{2}\right) = \frac{\cos\theta}{2} + \frac{1}{2}$

$= \frac{\cos\theta + 1}{2}$

7. $\cos 2x = 2\cos^2 x - 1$

$\cancel{\cos 2x}$ $\cos 2x + 1 = 2\cos^2 x$

$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$

$\therefore \int \cos^2 2x \, dx = \frac{1}{2}(1 + \cos 4x)$

$\frac{1}{2}\int 1 + \cos 4x \, dx = \frac{1}{2}\left[x + \frac{1}{4} \cancel{\cos} \sin 4x\right] + c$

$= \frac{1}{2}\left(x + \frac{\sin 4x}{4}\right) + c$

$= \frac{x}{2} + \frac{\sin 4x}{8} + c$

8. $6\sin x + 8\cos x$

$A = \sqrt{6^2 + 8^2}$

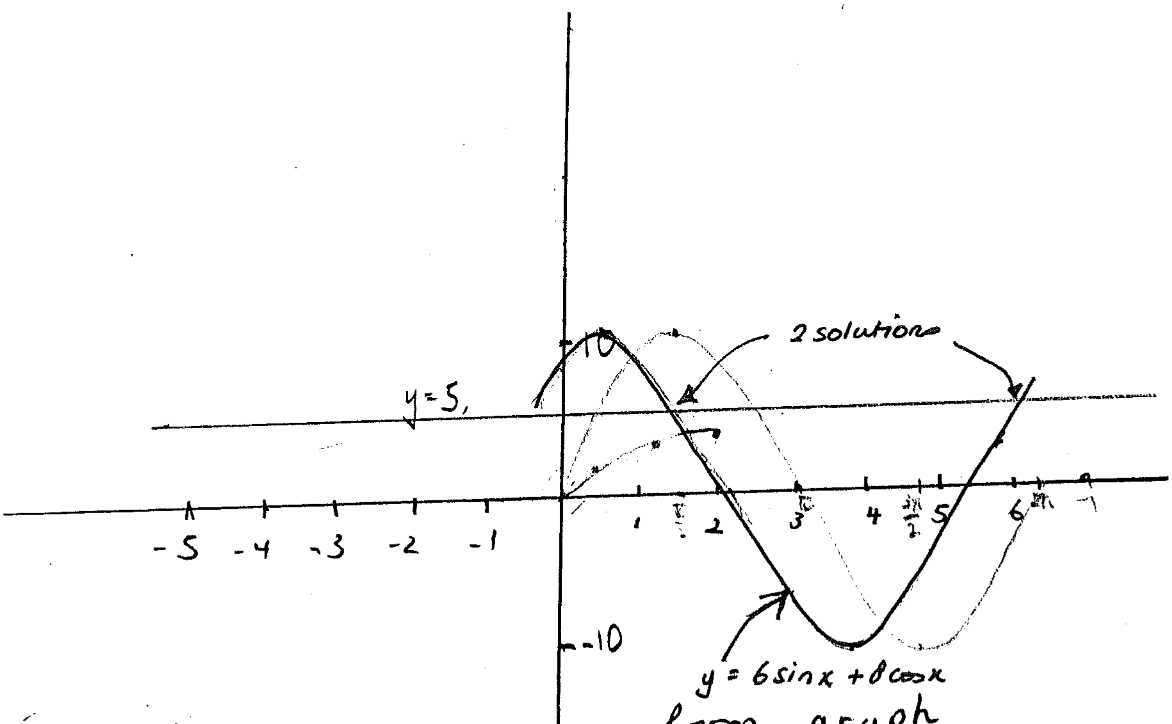
~~$A = 10$~~

$A = 10\sin(x + \alpha)$

$\tan\alpha = \frac{8}{6}$

$\alpha \doteq 0.9273$

$\therefore 6\sin x + 8\cos x = 10\sin(x + 0.9273)$



from graph
$6\sin x + 8\cos x = 5$
has two solutions

9. $2y = 3 - x$

$y = \frac{3}{2} - \frac{x}{2}$

$\therefore m_1 = -\frac{1}{2}$

$y = -3x - 4$

$\therefore m_2 = -3$

$\therefore \tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$= \left|\frac{-\frac{1}{2} + 3}{1 + -3 \times -\frac{1}{2}}\right|$ $= \left|\cancel{\frac{3.5}{2.5}}\right|$ $= \cancel{54°28'}$

$= \left|\frac{2\frac{1}{2}}{2\frac{1}{2}}\right| = |1|$ $\boxed{\theta = 45°}$

[redacted]

10. $3\sin\theta - \cos 2\theta = -2$

$3\sin\theta - (1 - 2\sin^2\theta) = -2$

$3\sin\theta - 1 + 2\sin^2\theta = -2$

$\cancel{3\sin\theta + 2\sin^2\theta = -1}$

$\cancel{\sin\theta(3 + 2\sin\theta) = -1}$

$\cancel{\therefore \sin\theta = -1 \text{ and } 3 + 2\sin\theta = -1}$

$\cancel{2\sin\theta = -4}$

$\cancel{\sin\theta = -\frac{1}{2}}$

S | A
T | C

$3\sin\theta - 1 + 2\sin^2\theta + 2 = 0$

$\cancel{3st}\ 2\sin^2\theta + 3\sin\theta + 1 = 0$

$(2\sin\theta + 1)(\sin\theta + 1) = 0$

$\therefore 2\sin\theta + 1 = 0 \qquad \sin\theta + 1 = 0$

$2\sin\theta = -1 \qquad \sin\theta = -1$

$\sin\theta = \frac{-1}{2}$

8/6

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$\therefore \theta = \frac{7\pi}{6}, \frac{11\pi}{6} \qquad \theta = \frac{3\pi}{2}$