SYDNEY GIRLS H.S.

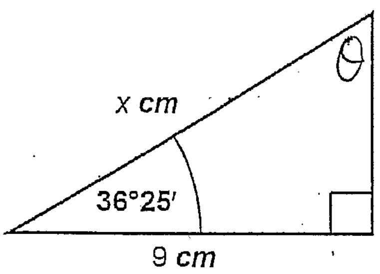
Year 11 Mathematics 2012 — Trigonometry

Name

Result %

DIRECTIONS
- Full working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Use black or blue pen only.

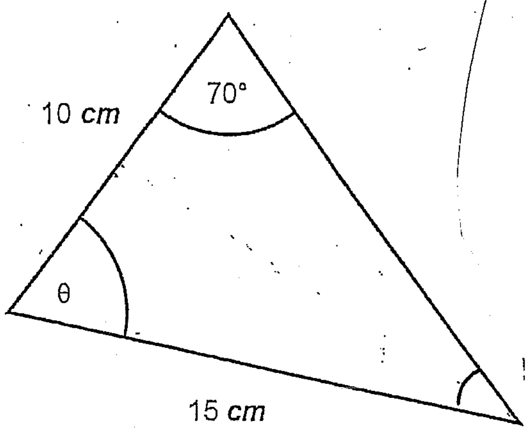
1. Find the value of x correct to 2 decimal places.



2. Find the largest angle in a triangle with side lengths 10 cm, 15 cm and 18 cm. Give your answer correct to the nearest minute.

3. Find the value of y if $\sin(5y) = \cos(y - 10)$.

4. Find the value of θ correct to the nearest minute.



5. Find the area of the triangle in question 4 (above), giving your answer correct to 2 decimal places.

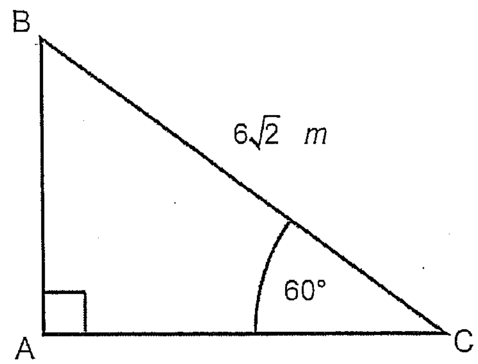
6. Find the exact value of $\tan 330°$.

7. If $\tan\theta = -\frac{1}{3}$ and $\sin\theta > 0$, find the exact value of $\sec\theta$.

8. From the top of a 50 m high tree, an eagle can observe its nest in another tree. The shortest distance between the trees is 12 m and the eagle is 19 m from the nest. Calculate the angle of depression from the eagle to its nest. Give your answer correct to the nearest minute.

9. Solve $\cos x = \frac{1}{3}$ for x where $0° \leq x° \leq 360°$.

10. Find the exact length of AB in the diagram below.



11. Julia and Tony depart from the same position. Julia travels along a straight road due east at 30 km/h. Tony departs 15 minutes after Julia, and travels along another straight road on a bearing of $S\,30^\circ\,E$ at 40 km/h.

(a) How far apart are they 15 minutes after Tony departs? Express your answer in simplest exact form.

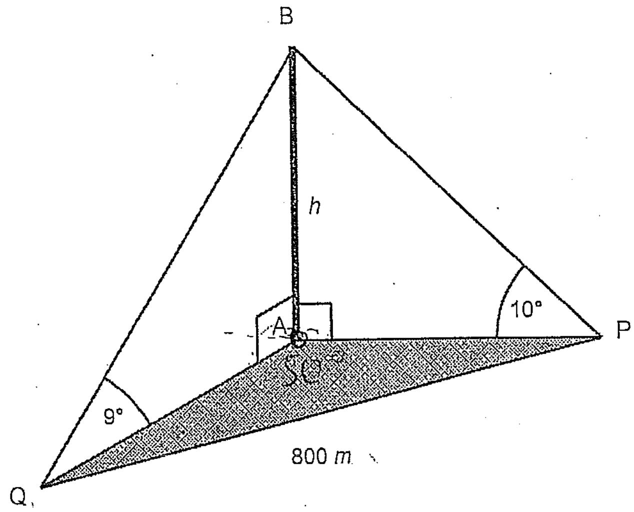
(b) What is the bearing of Tony from Julia ?

12. Sketch the graph of $y = \sec x$ for $0° \le x° \le 360°$.

13. Solve $2 + \sin x = 2\cos^2 x$ for x where $-180° \le x° \le 180°$.

14. Simplify $\dfrac{\sin^2 x + \cos^2 x + \cot^2 x}{2\operatorname{cosec}^2 x}$.

15. The angle of elevation of a tower AB of height h metres from a point P, due east of it, is $10°$. From another point Q, the bearing of the tower is $050°T$ and the angle of elevation is $9°$. The points P and Q are 800 metres apart and on the same level as the base A of the tower.



(a) Find the size of $\angle PAQ$.

(b) Consider ΔPBA and show that $PA = h\cot 10$.

(c) Find a similar expression for QA.

(d) Calculate h correct to 2 decimal places.

16. Prove that $\dfrac{\cos\theta\left(\sin\theta+\cos\theta\right)}{\left(1+\sin\theta\right)\left(1-\sin\theta\right)}=1+\tan\theta$.

17. Solve $\cot 2\theta=-\dfrac{1}{\sqrt{3}}$ for θ where $0°\leq\theta°\leq 360°$.

Year 11 Mathematics 2012 — Trigonometry

Name: *SOLUTIONS*   Result: %

DIRECTIONS
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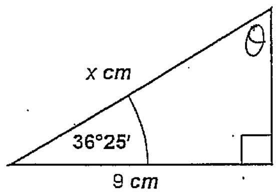
1. Find the value of x correct to 2 decimal places.



$90° + 36°25' + \theta = 180°$ (∠ sum of △)
$\theta = 180° - (90° + 36°25')$
$\theta = 53°35'$

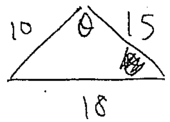
$\frac{\sin 53°35'}{9} = \frac{\sin 90°}{x}$

~~$(\sin 53°35')$~~ $x = \frac{9}{\sin 53°35'}$

$x \doteqdot 11.18$ cm (2 d.p) ✓ 3

2. Find the largest angle in a triangle with side lengths 10 cm, 15 cm and 18 cm. Give your answer correct to the nearest minute.

let ~~θ be~~ the largest angle be known as θ ∴



$\cos\theta = \frac{15^2 + 10^2 - 18^2}{2 \times 15 \times 10}$

$\cos\theta = \frac{1}{300}$

$\theta = 89°48'32.45''$

$\theta \doteqdot 89°49'$ (nearest minute) ✓ 3

3. Find the value of y if $\sin(5y) = \cos(y-10)$.

$\sin(90-\theta) = \cos\theta$
$\cos(90-\theta) = \sin\theta$

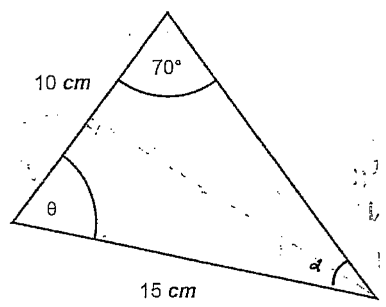
~~sin(90 − y − 10°~~
$\cos(90° - 5y) = \cos(y-10)$
$90° - 5y = y - 10$
$100 = 6y$

$y = \frac{50}{3}$

$\therefore y \doteqdot 16.67$ ✓ 2

(8)

4. Find the value of θ correct to the nearest minute.



Using Sine rule

$\frac{\sin\alpha}{10} = \frac{\sin 70°}{15}$

$\therefore \sin\alpha = 0.6265$ (to 4dp)

$\therefore \alpha = 38°47'$

$\therefore \theta = 180° - 70° - 38°47'$

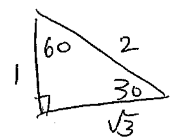
$= 71°13'$ (to the nearest min)

5. Find the area of the triangle in question 4 (above), giving your answer correct to 2 decimal places.

$\frac{1}{2}ab\sin C = \frac{1}{2} \times 15 \times 10 \times \sin 71°$
$A = 70.71$
$A \doteqdot 70.7$ cm² (1 d.p) ✓ 2

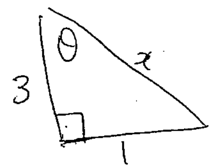
6. Find the exact value of $\tan 330°$.

$\tan(360° - 330°)$
$= \tan 30°$
$\therefore \tan 330° = -\frac{1}{\sqrt{3}}$ ✓ 3



7. If $\tan\theta = -\frac{1}{3}$ and $\sin\theta > 0$, find the exact value of $\sec\theta$.

S | A
T | C



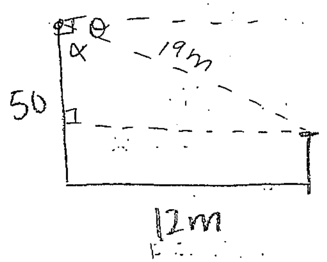
$x^2 = 3^2 + 1^2$
$x = \sqrt{9+1}$
$x = \sqrt{10}$

~~sec~~ $\cos\theta = -\frac{3}{\sqrt{10}}$

$\therefore \sec\theta = -\frac{\sqrt{10}}{3}$ ✓ 3

(10)

8. From the top of a 50 m high tree, an eagle can observe its nest in another tree. The shortest distance between the trees is 12 m and the eagle is 19 m from the nest. Calculate the angle of depression from the eagle to its nest. Give your answer correct to the nearest minute.



$\sin\alpha = \frac{12}{19}$

$\alpha = \sin^{-1}(\frac{12}{19})$

$\alpha = 39°10'0.16''$

$\theta + \alpha = 90°$

$\theta + 39°10'0.16'' = 90°$

$\theta = 90° - 39°10'0.16''$

$\theta = 50°49'59.84''$

∴ The angle of depression is ≈ 50°50' (nearest minute)

✓ 3

9. Solve $\cos x = \frac{1}{3}$ for x where $0° \le x° \le 360°$.

S | A
T | C

acute ∠ = $\cos^{-1}(\frac{1}{3})$

acute ∠ = 70°31'43.61''

∴ x = 70°31'43.61'', (360° − 70°31'43.61'')

x ≑ 71°32', 289°28' (nearest minute)

✓ 3

10. Find the exact length of AB in the diagram below.

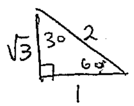
$\sin 60° = \frac{AB}{6\sqrt{2}}$

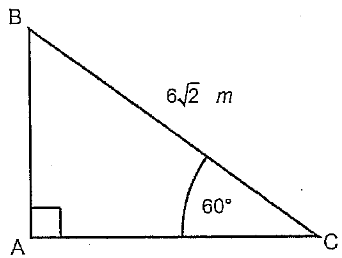
$\frac{\sqrt{3}}{2} = \frac{AB}{6\sqrt{2}}$

$6\sqrt{6} = 2AB$

∴ $AB = 3\sqrt{6}$ m

✓ 3

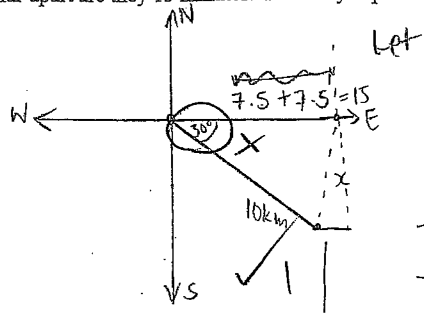




(9)

11. Julia and Tony depart from the same position. Julia travels along a straight road due east at 30 km/h. Tony departs 15 minutes after Julia, and travels along another straight road on a bearing of $S\,30°\,E$ at 40 km/h.

(a) How far apart are they 15 minutes after Tony departs? Express your answer in simplest exact form.



15 − 10 = 5km

Let their distance be known as 'x'

$x^2 = (15)^2 + (10)^2 - 2(15)(10)\cos 30°$

~~$x^2 = 225 + 100 - 259.8076211$~~

~~$x^2 = 65.19237886$~~

~~$x = 8.074179764$~~

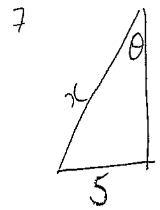
$x^2 = 225 + 100 - (300 \times \frac{\sqrt{3}}{2})$

$x^2 = 325 - 150\sqrt{3}$

∴ $x = \sqrt{325 - 150\sqrt{3}}$ km

✓ 2

(b) What is the bearing of Tony from Julia ?



$\sin\theta = \frac{5}{\sqrt{325 - 150\sqrt{3}}}$

$\theta = \sin^{-1}\left(\frac{5}{\sqrt{325 - 150\sqrt{3}}}\right)$

$\theta = 38°15'43.08''$ ✗✗

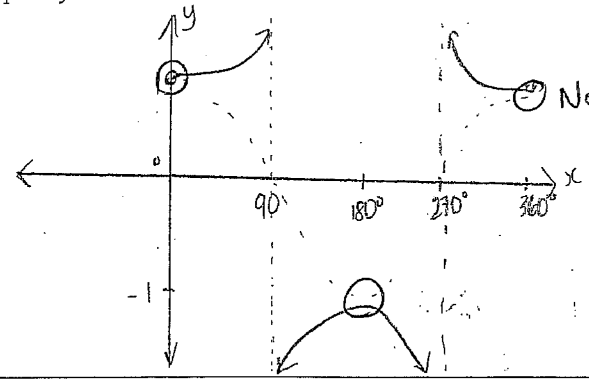
$180° + \theta = 218°15'43.08''$

≑ 218° (nearest degree) ✓ 1

∴ Tony's bearing from Julia is T 218°

(4)

12. Sketch the graph of $y = \sec x$ for $0° \le x° \le 360°$.



No gap. No open circle

× 1

13. Solve $2 + \sin x = 2\cos^2 x$ for x where $-180° \le x° \le 180°$.

$2 + \sin x = 2(1 - \sin^2 x)$

$2 + \sin x = 2 - 2\sin^2 x$

$\sin x + 2\sin^2 x = 0$

$\sin x(1 + 2\sin x) = 0$

$1 + 2\sin x = 0$

$2\sin x = -1$

$\sin x = -\frac{1}{2}$

acute $\angle = 30°$

$\therefore x = (-180° + 30°), -30°$

$\sin x = 0$

$x = -150°, -30°$

✓ 3

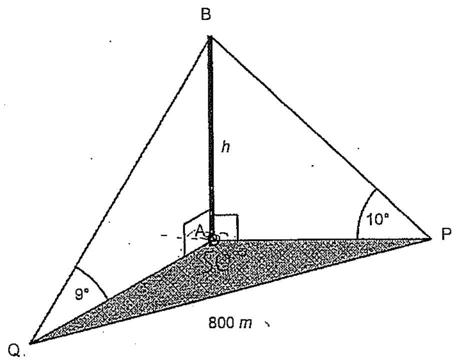
14. Simplify $\frac{\sin^2 x + \cos^2 x + \cot^2 x}{2\operatorname{cosec}^2 x}$.

$= \frac{1 + \cot^2 x}{2(1 + \cot^2 x)}$

$= \frac{1}{2}$ ✓ 3

(7)

15. The angle of elevation of a tower AB of height h metres from a point P, due east of it, is $10°$. From another point Q, the bearing of the tower is 050°T and the angle of elevation is $9°$. The points P and Q are 800 metres apart and on the same level as the base A of the tower.



(a) Find the size of $\angle PAQ$. $= \angle QAR + 90°$

$= 50° + 90° = 140°$

~~$\angle PAQ = 50°$ (given)~~ ×

(b) Consider ΔPBA and show that $PA = h\cot 10$.

$\tan 10° = \frac{h}{PA}$ ✓

$PA \tan 10° = h$

$PA = \frac{h}{\tan 10°}$ ✓ 2

$\therefore PA = h \cot 10°$

(c) Find a similar expression for QA.

$\tan 9° = \frac{h}{QA}$

$QA \tan 9° = h$

$QA = \frac{h}{\tan 9°}$

$QA = h\cot 9°$ ✓ 1

(d) Calculate h correct to 2 decimal places.

Using Cosine rule in ΔPAQ

$800^2 = (h\cot 10°)^2 + (h\cot 9°)^2 - 2(h\cot 10°)(h\cot 9°)\cos 140°$

$= h^2\cot^2 10° + h^2\cot^2 9° - h^2 . 2\cot 10°\cot 9°\cos 140°$

$\therefore h^2 = 8880.294367$

$\therefore h = 94.24$ (to 2 d.p.)

16. Prove that $\frac{\cos\theta(\sin\theta+\cos\theta)}{(1+\sin\theta)(1-\sin\theta)}=1+\tan\theta$.

$LHS = \frac{\cos\theta(\sin\theta+\cos\theta)}{1-\sin^2\theta}$ ✓

$= \frac{\cos\theta(\sin\theta+\cos\theta)}{\cos^2\theta}$

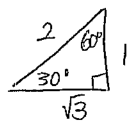
$= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\cos\theta}$

$= \tan\theta + 1$

$= RHS$.

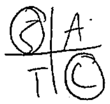
17. Solve $\cot 2\theta = -\frac{1}{\sqrt{3}}$ for θ where $0° \le \theta° \le 360°$.

$\tan 2\theta = -\sqrt{3}$ for 2θ; $0° \le 2\theta \le 720°$



$2\theta = 60°$ ✓ (acute ∠)

~~$\theta = 30°$~~



~~$\theta = 30°, (180°-30°)$,~~

~~$2\theta = 60°, (180°-60°), (180°+60°), (360°-60°), (60°+360$~~

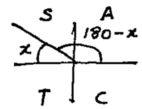
$2\theta = (180°-60°), (360°-60°), (180°-60°+360°), (360°-60°+360°)$

$2\theta = 120°, 300°, 480°, 660°$

$\theta = 60°, 150°, 240°, 330°$ ✓ 3 (5)

$\therefore \theta = 150°, 330°$ ✗ Why did you leave off 60°, 240°?

18. Simplify $\frac{\tan(180-x)}{\tan(90-x)} - 1$



$= \frac{-\tan x}{\cot x} - 1$

$= -(\tan^2 x + 1)$

$= -\sec^2 x$