TRINITY GRAMMAR SCHOOL
MATHEMATICS DEPARTMENT

YEAR 12 2002 ASSESSMENT TASK 3.

MATHEMATICS

(EXTENSION 1)

Time Allowed – Two hours

Tuesday May 7th 8.30am – 10.30am.

WEIGHTING 30% towards final result

INSTRUCTIONS:

1. Attempt **ALL** questions.
2. Show all necessary working.
3. **Begin** each question in a new exam book..
4. Mark values are shown at the beginning of each question.
5. Non-programmable silent Board of Studies approved calculators are permitted.

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n<0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x>0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a>0, \quad -a<x<a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x>a>0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x>0$

QUESTION ONE (12 Marks) Start a new exam book

a) (3 Marks)
A is the point (-2, 1) and B is the point (x, y).
The point P (13, -9) divides AB externally in the ratio 5 : 3.

Find the values of x and y.

b) (3 Marks)
Determine (in radians) the acute angle between $x = 4$ and $2x - y - 5 = 0$

c) (3 Marks)
Solve $\frac{x-2}{x+4} > 1$

d) (3 Marks)
Solve the equation $2\ln(3x+1) - \ln(x+1) = \ln(7x+4)$

QUESTION TWO (12 Marks) Start a new exam book

a) (5 Marks)

Find i) $\int \frac{5}{4+x^2}\,dx$

ii) $\int_{-\frac{1}{3}}^{\frac{1}{3}} \frac{dx}{\sqrt{4-9x^2}}$

b) (4 Marks)

Differentiate i) $f(x) = 3\sin^{-1}(\frac{x}{6})$

ii) $f(x) = \cos^{-1}(\sin x)$

c) (2 Marks)

Evaluate giving an exact answer i) $\sin^{-1}(\frac{1}{2}) + \cos^{-1}(1)$

ii) $\tan(\sin^{-1}(\frac{1}{2}))$

d) (1 Mark)

Using the table of standard integrals, find the exact value of $\int_0^{\frac{\pi}{8}} \sec 2x \tan 2x\,dx$

QUESTION THREE (12 Marks) Start a new exam book

a) (2 Marks)
Find the inverse function of $f(x) = \frac{x}{x+5}$

b) (5 Marks)
Consider the function $y = 2\cos^{-1}\frac{x}{3}$

i) State the domain and range for this function
ii) Sketch the graph of this function clearly showing the domain and range.
iii) Find the gradient and hence the angle that the tangent to the curve

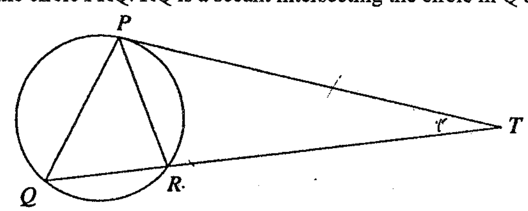
$y = 2\cos^{-1}\frac{x}{3}$ at $x = 0$ makes with the positive direction of the x axis.

c) (3 Marks)
For the function $y = x^2 - 2x + 1$, find a suitable domain such that this function has an inverse. Find the equation of this inverse and state its range.

d) (2 Marks)
Sketch $y = e^x$ and on the same diagram sketch its inverse function showing clearly any cut points on the x and y axes. Mark clearly the line of reflection.

QUESTION FOUR (12 Marks) Start a new exam book

a) (4 Marks)
PT is a tangent to the circle PRQ. RQ is a secant intersecting the circle in Q and R. The line QR intersects PT at T.



i) Prove that triangle PRT is similar to triangle QPT.
ii) Hence prove that $PT^2 = QT \times RT$

b) (2 Marks)
Show that $\frac{dv}{dt} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$

c) (6 marks)
A particle is moving on a straight line. At time t seconds it had displacement x metres from a fixed point, 0 on the line, velocity $v\ m/s$ and acceleration $a\ m/s^2$. The particle starts from 0 and at time t seconds, the velocity is given by $v = (1-x)^2$.

i) Find an expression for a in terms of x. Hint: Use part (b)
ii) Find an expression for x in terms of t.
iii) Find the time taken for the particle to slow down to a speed of 1% of its initial speed.

QUESTION FIVE (12 Marks) Start a new exam book

a) (4 Marks)
Use mathematical induction to prove that, for every positive integer n, $13\times 6^n+2$ is divisible by 5.

b) (6 Marks)
Two points P(2ap, ap^2) and Q (2aq, aq^2) lie on the parabola $x^2 = 4ay$.

i) Show that the equation of the tangent to the parabola at P is
$$y = px - ap^2.$$
ii) The tangent at P and the line through Q parallel to the y axis intersect at T. Find the coordinates of T.
iii) Write down the coordinates of M, the midpoint of PT.
iv) Determine the locus of M when pq = -1

c) (2 Marks)
If $\dfrac{dx}{dt} = \dfrac{1}{x+2}$ and x = 0 when t = 0, find t when x = 4.

QUESTION SIX (12 Marks) Start a new exam book

a) (3 Marks)
The displacement x metres of a particle from the origin that is moving in simple harmonic Motion, is given by $x = 5\cos \pi t$, where the time t is in seconds.

i) What is the period of oscillation
ii) What is the speed v of the particle as it moves through the origin.

b) (3 Marks)
A sphere is expanding such that its surface area is increasing at the rate of 0.01cm/sec ^{2}
Calculate the rate of change of
i) its radius
ii) its volume
at the instant when the radius is 5 cm.

c) (6 Marks)
The velocity v m/s^{-1} of a body moving in simple harmonic motion along the x axis is given by
$$v^2 = 15 + 2x - x^2$$
a) Between which points is the body oscillating
b) Calculate the amplitude of the motion
c) Find the acceleration of the particle

QUESTION SEVEN (12 Marks) Start a new exam book

a) (5 Marks)
At time t the temperature T° of a body in a room of constant temperature 20° is decreasing according the equation $\dfrac{dT}{dt} = -k(T-20)$ for some constant k, where $k > 0$.

i) Verify that $T = 20 + Ae^{-kt}$, A constant, is the solution of the equation.
The initial temperature of the body is 90° and it falls to 70° after 10 minutes.

Find the temperature of the body after a further 5 minutes.

b) (7 marks)

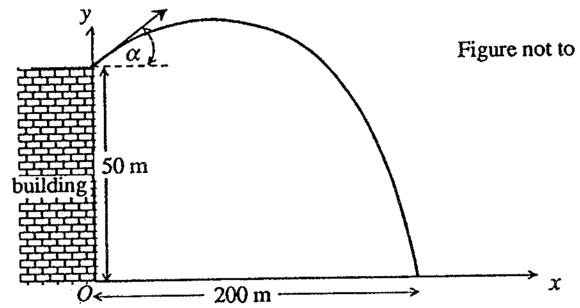


Figure not to scale.

The diagram shows the path of a projectile launched at an angle of elevation of α, from the top of a building 50 m high with an initial velocity of 40 m/s. The acceleration due to gravity is assumed to be 10m/sec ^{2}. Take the origin to be the base of the tower.

i) Given that $\dfrac{d^2x}{dt^2} = 0$ and $\dfrac{d^2y}{dt^2} = -10$ show that
$x = 40t\cos\alpha$ and $y = -5t^2 + 40t\sin\alpha + 50$

where x and y are the horizontal and vertical displacements of the projectile in metres fr the origin at time t seconds after launching.

ii) The projectile lands on the ground 200 metres from the base of the building.
Find the two possible angles for α. Give your answers to the nearest degree.

Question 1

a) Externally in ratio $5:3 \rightarrow 5:-3$

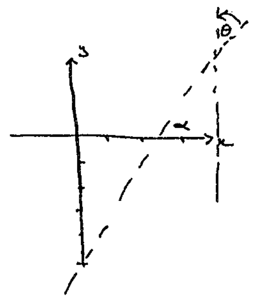
$\therefore \quad 13 = \dfrac{5(x) + (-3)(-2)}{5+(-3)} \qquad -9 = \dfrac{5(y) + (-3)(1)}{5+(-3)}$

$13 = \dfrac{5x+6}{2}, \quad -9 = \dfrac{5y-3}{2}$ Using $\left(\dfrac{kx_2 + lx_1}{k+l}, \dfrac{ky_2 + ly_1}{k+l}\right)$

$5x = 20 \qquad 5y = -15$

$\boxed{x=4} \qquad \boxed{y=-3}$ The point B is $(4,-3)$.

b)



Gradient of $y = 2x - 5$ $(2x - y - 5 = 0)$

$m_1 = 2$

$\tan\alpha = 2$

$\alpha = \tan^{-1}(2)$

$\alpha = 63^\circ 26' 5.82''$

$\theta = 180^\circ - (90^\circ + 63^\circ 26' 5.82'')$

$\theta = 26^\circ 33' 54.18''$

$\theta = 0.463647\ldots$

$\theta = 0.46$ radians (2 d.p.)

c) $(x+4)^2 \dfrac{x-2}{x+4} > 1\,(x+4)^2$ noting $x \neq -4$.

$(x+4)(x-2) > (x+4)^2$

$x^2 + 2x - 8 > x^2 + 8x + 16$

$-6x > 24$

$\boxed{x < -4}$

d) Solving $2\ln(3x+1) - \ln(x+1) = \ln(7x+4)$

$\ln(3x+1)^2 - \ln(x+1) = \ln(7x+4)$

$\ln\dfrac{(3x+1)^2}{(x+1)} = \ln(7x+4)$

$\therefore \quad 9x^2 + 6x + 1 = 7x^2 + 11x + 4$

$2x^2 - 5x - 3 = 0$

$(2x+1)(x-3) = 0$

$x = 3$ or $x = -\frac{1}{2}$

$x \neq -\frac{1}{2}$ (not a valid solution)

$\therefore \boxed{x = 3}$

(3)

Question 2

a) (i) $\int \frac{5}{4+x^2}\,dx = 5\int \frac{1}{4+x^2}$

$= \frac{5}{2}\tan^{-1}\frac{x}{2} + c$

(ii) $\int_{-1/3}^{1/3} \frac{dx}{\sqrt{4-9x^2}} = \frac{1}{3}\int_{-1/3}^{1/3}\frac{dx}{\sqrt{\frac{4}{9}-x^2}}$

$= \frac{1}{3}\left[\sin^{-1}\frac{3x}{2}\right]_{-1/3}^{1/3}$

$= \left(\frac{1}{3}\sin^{-1}\frac{1}{2}\right) - \left(\frac{1}{3}\sin^{-1} -\frac{1}{2}\right)$

$= \frac{1}{3}\left(\frac{\pi}{6}\right) - \frac{1}{3}\left(-\frac{\pi}{6}\right)$

$= \frac{\pi}{9}.$

b) i) $f(x) = 3\sin^{-1}\left(\frac{x}{6}\right)$

$f'(x) = 3 \cdot \frac{1}{\sqrt{1-\left(\frac{x}{6}\right)^2}} \cdot \frac{d}{dx}\left(\frac{x}{6}\right)$

$= 3 \cdot \frac{1}{\sqrt{\frac{36-x^2}{36}}} \cdot \frac{1}{6}$

$= \frac{3}{\sqrt{36-x^2}}$

ii) $f(x) = \cos^{-1}(\sin x)$

$f'(x) = \frac{-1}{\sqrt{1-(\sin x)^2}} \cdot \frac{d}{dx}(\sin x)$

$= \frac{-1}{\sqrt{1-\sin^2 x}} \cdot \cos x$

$= \frac{-\cos x}{\sqrt{\cos^2 x}} = \frac{-\cos x}{\cos x}$

$= -1.$

Question 2 (cont.)

c) i) $\sin^{-1}\left(\frac{1}{2}\right) + \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6} + \frac{\pi}{3}$

$= \frac{\pi}{2}$

ii) $\tan\left\{\sin^{-1}\left(\frac{1}{2}\right)\right\} = \tan\frac{\pi}{6}$

$= \frac{1}{\sqrt{3}}.$

d) $\int_0^{\pi/8} \sec 2x \tan 2x\,dx = \left[\frac{1}{2}\sec 2x\right]_0^{\pi/8}$

$= \frac{1}{2}\left(\sec\frac{\pi}{4} - \sec 0\right)$

$= \frac{1}{2}\left(\frac{1}{\cos\frac{\pi}{4}} - \frac{1}{\cos 0}\right)$

$= \frac{1}{2}\left(\sqrt{2}-1\right)$ or $\frac{\sqrt{2}}{2} - \frac{1}{2}$

Question 3

(5)

a) Let $y = \frac{x}{x+5}$ $\longrightarrow$ Inverse function $x = \frac{y}{y+5}$

$xy + 5x = y$

$xy - y = -5x$

$y(x-1) = -5x$

$y = \frac{-5x}{x-1}$ $\therefore y = \frac{5x}{1-x}$ is the inverse.

b) $y = 2\cos^{-1}\frac{x}{3}$

i) $y = \cos^{-1}x$ has domain $-1 \le x \le 1$

range $0 \le y \le \pi$

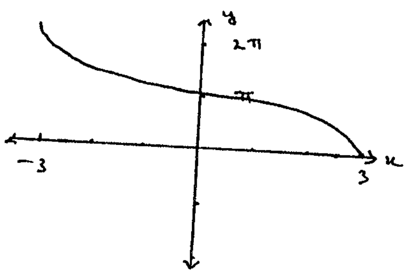
$\therefore y = 2\cos^{-1}\frac{x}{3}$ " domain $-1 \le \frac{x}{3} \le 1$

range $0 \le 2\cos^{-1}\frac{x}{3} \le \pi$

i.e Domain $-3 \le x \le 3$

Range $0 \le y \le 2\pi$.

ii)



iii) Gradient of tangent at $x=0$, $y = 2\cos^{-1}\frac{x}{3}$

$\frac{dy}{dx} = \frac{-2}{\sqrt{1-\left(\frac{x}{3}\right)^2}} \cdot \frac{d}{dx}\left(\frac{x}{3}\right)$

$= \frac{-2}{\sqrt{9-x^2}} \cdot 3 \cdot \frac{1}{3}$

m_T at pt. $x = 0 \longrightarrow m_T = \frac{-2}{\sqrt{9-0^2}}$

$= \frac{-2}{3}$

b) iii) Continued.

$m = \tan\theta$

$\tan\theta = -\frac{2}{3}$

$\theta = -33^\circ 41' 24.24''$

$\therefore$ angle made with positive direction

$= 180^\circ - 33^\circ 41' 24.24''$

$= 146^\circ 18' 35''$.

c) $y = x^2 - 2x + 1$ has domain $x \in \mathbb{R}$

range $y \geq 0$

$\therefore$ Inverse must have a domain $x \geq 0$.

Inverse of $y = x^2 - 2x + 1$ is $x = y^2 - 2y + 1$

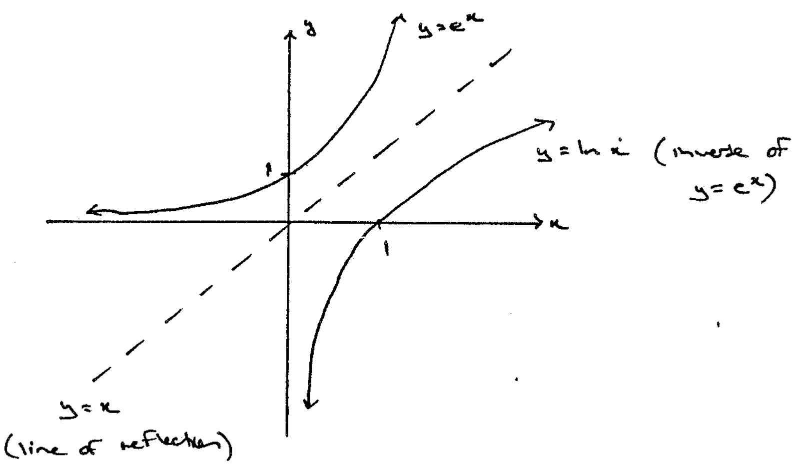
$x = (y-1)^2$

$y - 1 = \pm\sqrt{x}$

$y = 1 \pm \sqrt{x}$

$\therefore$ Range of inverse : $y \in \mathbb{R}$.

d)



Question 4

a) i) In the Δ's PRT & QPT

angle T is common

$\angle RPT = \angle RQP$ (angle between tangent & chord = angle in alternate segment.)

$\therefore \Delta PRT \,|||\, \Delta QPT$ (Equiangular)

ii) $\frac{QT}{PT} = \frac{PT}{RT}$ (corresponding sides proportional in similar Δ's)

$\therefore PT^2 = QT \cdot RT$

b) Using the chain rule:

$$\frac{d}{dx}\left(\tfrac{1}{2}v^2\right) = \frac{d}{dv}\left(\tfrac{1}{2}v^2\right) \times \frac{dv}{dx}$$

$$= v\frac{dv}{dx}$$

$$= \frac{dx}{dt} \cdot \frac{dv}{dx}$$

$$= \frac{dv}{dt}$$

c) i) $a = \frac{d}{dx}\left(\tfrac{1}{2}v^2\right)$

$= \frac{d}{dx}\left[\tfrac{1}{2}\{(1-x)^2\}^2\right]$

$= \frac{d}{dx}\left(\tfrac{1}{2}(1-x)^4\right)$

$= \frac{d}{dx}\left(\tfrac{1}{2}(1-x)^4\right)$

$= 2(1-x)^3 \cdot -1$

$\boxed{a = -2(1-x)^3}$

ii) $v = (1-x)^2$

$\frac{dx}{dt} = (1-x)^2$, $\frac{dt}{dx} = \frac{1}{(1-x)^2}$

$\int dt = \int \frac{dx}{(1-x)^2}$

$t = \frac{1}{1-x} + c$

when $t=0, x=0$ $\therefore c=-1$.

$t = \frac{1}{1-x} - 1$ or $t+1 = \frac{1}{1-x}$

$1-x = \frac{1}{t+1}$ $\therefore x = 1 - \frac{1}{t+1}$

or $\boxed{x = \frac{t}{t+1}}$

c (iii) Velocity to go from 1m/s to 0.01m/s

Velocity $= (1-x)^2$

$(1-x)^2 = 0.01$

$x^2 - 2x + 1 = 0.01$

$x^2 - 2x + 0.99 = 0$

$\therefore (x-1.1)(x-0.9) = 0$

$\therefore x = 0.9$m or $x = 1.1$ m

$= x = \frac{t}{t+1}$ / $t = \frac{x}{1-x}$

Note: $x \ngeq 1$ as $x = \frac{t}{t+1}$

$\therefore x \neq 1.1$

when $x = 0.9$m

$t = \frac{0.9}{1-0.9}$

$= \frac{0.9}{0.1}$

$= 9$ seconds

Time taken to decrease to 1% of initial speed is 9 sec

(9)

Question 5

a) Let $P(n) = 13 \times 6^n + 2$ for every positive integer n.

To prove: $13 \times 6^n + 2 = 5m$, where m is an integer.

Prove true for $n=1$

$P(1) = 13 \times 6 + 2 = 80$

which is divisible by 5 $\left(\frac{80}{5} = 16\right)$

$\therefore$ statement true for $n=1$.

Assume true for $n=k$

$P(k) = 13 \times 6^k + 2 = 5m_1$, where m_1 is an integer

Prove true for $n=k+1$.

$P(k+1) = 13 \times 6^{k+1} + 2 = 5m_2$, where m_2 is an integer.

$= 13 \times 6^k \times 6 + 2$

$= (5m_1 - 2) \times 6 + 2$

$= 30m_1 - 12 + 2$

$= 30m_1 - 10$

$= 5(6m_1 - 2)$

$= 5m_2$

$\therefore$ statement is true for $n=k$, also true for $n=k+1$

$\therefore$ $P(n)$ is divisible by 5 for every integer by M.I.

b) i) $x^2 = 4ay$

$y = \frac{x^2}{4a}$

$\frac{dy}{dx} = \frac{2x}{4a}$

$= \frac{x}{2a}$

$\therefore m_T = \frac{x}{2a}$

$= \frac{2ap}{2a} = p$.

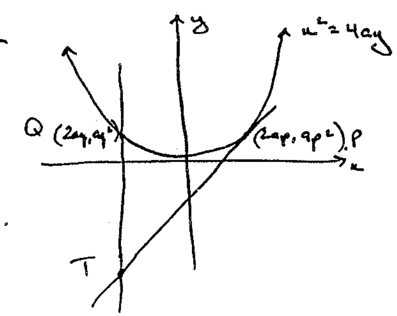
Touches at $(2ap, ap^2)$

$y - ap^2 = p(x - 2ap)$

$y - ap^2 = px - 2ap^2$

$y = px - ap^2$

ii) Diagram



Equation of QT is $x = 2aq$

Co-ordinates of T are $x = 2aq$

$y = p(2aq) - ap^2$ (from (i))

$y = 2apq - ap^2$

$\therefore$ T is $(2aq, 2apq - ap^2)$.

iii) Midpoint of PT

$x = \frac{2ap + 2aq}{2}$

$= ap + aq$

$= a(p+q)$.

$y = \frac{ap^2 + (2apq - ap^2)}{2}$

$= \frac{2apq}{2}$

$= apq$

Midpoint $M = (a(p+q), apq)$

iv) If $pq = -1$ & using $y = apq$,

then $y = a(-1)$

$y = -a$

$\therefore$ The locus of M is the directrix of the given parabola.

c) $\frac{dx}{dt} = \frac{1}{x+2}$

$\frac{dt}{dx} = x + 2$

$dt = (x+2)\, dx$

$t = \frac{x^2}{2} + 2x + c$

$x = 0,\ t = 0\ \therefore\ c = 0$

$t = \frac{x^2}{2} + 2x$

when $x = 4$

$t = \frac{4^2}{2} + 2(4)$.

$\boxed{t = 16}$

Question 6

a) i) $x = 5\cos \pi t$

$T = \frac{2\pi}{n}$

$= \frac{2\pi}{\pi}$

$= 2$ $\boxed{\therefore \text{ Period of oscillation} = 2 \text{ sec.}}$

ii) $v = \frac{d}{dt}(5\cos \pi t)$

$= -5\pi \sin \pi t$

$\therefore$ at equilibrium pos^n

$x = 0$

$5\cos \pi t = 0$

$\pi t = \frac{\pi}{2}$ (first time)

$t = \frac{1}{2}$ sec.

at $t = \frac{1}{2}$ s $v = -5\pi \sin \pi \cdot \frac{1}{2}$

$= -5\pi$

$\boxed{\text{Speed} = 5\pi \text{ m/s.}}$

Speed must be positive.

b) i) $SA = 4\pi r^2$

$\frac{dA}{dr} = 8\pi r$ $\frac{dA}{dt} = 0.01$ cm/s^2

Find $\frac{dr}{dt}$

$\frac{dr}{dt} = \frac{dA}{dt} \cdot \frac{dr}{dA}$

$= 0.01 \cdot \frac{1}{8\pi r}$ when $r = 5$

$\frac{dr}{dt} = 0.01 \cdot \frac{1}{8 \cdot \pi \cdot 5}$

$= \frac{0.01}{40\pi}$

$\boxed{\frac{dr}{dt} = 7.9577 \times 10^{-5} \text{ cm/sec.}}$

ii) $\frac{dV}{dt}$ when $r = 5$ $V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = \frac{dV}{dr} \cdot \frac{dr}{dt}$ $\frac{dV}{dr} = 4\pi r^2$

$= 4\pi r^2 \times 7.9577 \times 10^{-5}$

when $r = 5$

$\frac{dV}{dt} = 4 \cdot \pi \cdot 25 \times 7.9577 \times 10^{-5}$

$\boxed{\frac{dV}{dt} = 0.025 \text{ cm}^3\text{/sec.}}$

(c) $v^2 = 15 + 2x - x^2$

i) endpts $\rightarrow v = 0$.

$\therefore 15 + 2x - x^2 = 0$

$(5-x)(3+x) = 0$

$\therefore x = 5$ and $x = -3$

Between $x = 5$m & $x = -3$m the particle is oscillating

ii) amplitude $= \frac{5-(-3)}{2}$

$= 4$ m

iii) Acceleration:

$v^2 = 15 + 2x - x^2$

$\frac{1}{2}v^2 = \frac{15}{2} + x - \frac{x^2}{2}$

$\frac{d}{dx}\left(\frac{1}{2}v^2\right) = \frac{d}{dx}\left(\frac{15}{2} + x - \frac{x^2}{2}\right)$

$\frac{d^2x}{dt^2} = 1 - x$ or

$a = -(x-1)$

Question 7

a) i) $T = 20 + Ae^{-kt}$ — (1)

$\frac{dT}{dt} = -kAe^{-kt}$

But from (1)

$\frac{dT}{dt} = -k(T-20)$.

ii) $T = 20 + Ae^{-kt}$

$t=0$, $T = 90°$

$90 = 20 + Ae^{-k0}$

$Ae^{-k0} = 70$

$\therefore A = 70$

when $t=10$, $T = 70$

$\therefore \quad 70 = 20 + 70e^{-k.10}$

$50 = 70e^{-10k}$

$\frac{5}{7} = e^{-10k}$

$\ln\frac{5}{7} = \ln e^{-10k}$

$-10k = \ln\frac{5}{7}$

$k = \frac{\ln\frac{5}{7}}{-10}$

$k = 0.0336\ldots$

$t = 15$.

$T = 20 + 70e^{-0.0336\ldots \times 15}$

$= 62.2577\ldots$

Temperature after another 5 minutes $= 62°$. (Nearest degree)

Question 7 (cont.)

(b) i) At $t=0$, $x=0$, $\frac{dx}{dt} = 40\cos\alpha$

At $t=0$, $y=50$, $\frac{dy}{dt} = 40\sin\alpha$

Horizontal motion:

$\frac{d^2x}{dt^2} = 0$

$\frac{dx}{dt} = c$

at $t=0$, $\frac{dx}{dt} = 40\cos\alpha$

$\therefore c = 40\cos\alpha$

$x = 40t\cos\alpha + c_1$

at $t=0$, $x=0$ $\therefore c_1 = 0$

$x = 40t\cos\alpha$

Vertical Motion

$\frac{d^2y}{dt^2} = -10$

$\frac{dy}{dt} = -10t + c_2$

At $t=0$, $\frac{dy}{dt} = 40\sin\alpha$

$\therefore c_2 = 40\sin\alpha$

$\frac{dy}{dt} = -10t + 40\sin\alpha$

$y = -\frac{10t^2}{2} + 40t\sin\alpha + c_3$

at $t=0$, $y=50$, $c_3 = 50$

$y = -5t^2 + 40t\sin\alpha + 50$.

ii) $x = 40t\cos\alpha$

at $x = 200$, $200 = 40t\cos\alpha$

$5 = t\cos\alpha$

$t = \frac{5}{\cos\alpha}$

Now $y = -5t^2 + 40t\sin\alpha + 50$

at $x = 200$, $y = 0$, $t = \frac{5}{\cos\alpha}$

$\therefore 0 = -5\left(\frac{5}{\cos\alpha}\right)^2 + 40\left(\frac{5}{\cos\alpha}\right)\sin\alpha + 50$

$0 = \frac{-125}{\cos^2\alpha} + \frac{200\sin\alpha}{\cos\alpha} + 50$

$0 = -125\sec^2\alpha + 200\tan\alpha + 50$

$\therefore 0 = -5\sec^2\alpha + 8\tan\alpha + 2$

$5\sec^2\alpha - 8\tan\alpha - 2 = 0$

$5(1+\tan^2\alpha) - 8\tan\alpha - 2 = 0$

$5 + 5\tan^2\alpha - 8\tan\alpha - 2 = 0$

$5\tan^2\alpha - 8\tan\alpha + 3 = 0$

Quadratic.

$\tan\alpha = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(5)(3)}}{2(5)}$

$= \frac{8 \pm \sqrt{4}}{10}$

$= 1$ or $\frac{3}{5}$

$\therefore \boxed{\alpha = 45° \text{ or } 31°}$ to nearest degree.