SK Copy

TRINITY GRAMMAR SCHOOL
MATHEMATICS DEPARTMENT

YEAR 11 2005 ASSESSMENT TASK 3

MATHEMATICS

(EXTENSION 1)

HALF YEARLY EXAMINATION

Time Allowed – *2 hours plus 5 minutes reading*

MONDAY 2nd MAY 2005

WEIGHTING 25% towards final result

Outcomes referred to: P1, P2, P3, P4, P5, PE1, PE2, PE6.

INSTRUCTIONS:

1. Attempt **ALL** questions.
2. There are 8 questions of equal value.
3. Each question is worth 12 marks.
4. Total marks available are 96.
5. Show all necessary working.
6. **Begin** each question on a **new page.**
7. **Write your name, your teacher's name and class on the top of each question**
8. Mark values are shown beside each part.
9. Non-programmable silent Board of Studies approved calculators are permitted.
10. If requested, additional writing sheets may be obtained from teacher upon request.

Name ______________________________

Start each question on a new page

Question 1

a) Find the surface area of a sphere that has a diameter of 10cm. Answer correct to 3 significant figures. 1

b) If $a = 2\cdot5\times10^{17}$, express in scientific notation the value of $\sqrt{a}$. 1

c) Write without a negative fractional index $(3a+1)^{-3/2}$. 1

d) Solve $\frac{a+1}{4} = \frac{2a}{3} - 1$ 1

e) Express $1\cdot2\dot{4}$ in simplest fraction form. 2

f) Simplify $\frac{1}{3x^3-24} \div \frac{1}{x^2-4}$. 2

g) Solve $|x-1| = 2x-5$. 2

h) Find the **exact** value of x and y if $\frac{\sqrt{3}-4}{2+3\sqrt{3}} = x + y\sqrt{3}$. 2

Start each question on a new page

Question 2

a) Solve $12 + 4m - m^2 > 0$. 2

b) Solve $4^{3-a} = 8^a$. 2

c) In an isosceles right-angled triangle the two equal sides each have length w cm and the hypotenuse is 1cm longer than each of these sides. 4

i) Show that $w^2 - 2w - 1 = 0$.

ii) Find the dimensions of the triangle in simplest **exact** form.

d) Solve simultaneously: 4

$$a - b - c = 1$$
$$2a + b - c = -9$$
$$2a - 3b - 2c = 7$$

Start each question on a new page

Question 3

a) Find the **exact** value of $\sin 240^\circ . \tan 30^\circ - \cos 135^\circ$. 2

b) Simplify $3 + 3\tan^2\theta$. 2

c) Solve $2\sin\theta + \sqrt{3} = 0$ for $-180^\circ \le \theta \le 180^\circ$. 2

d) Solve $\cos 2\theta = 0\cdot5$ for $0^\circ \le \theta \le 360^\circ$. 2

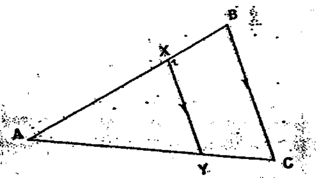
e) Sketch $y = \sec x$ for $0^\circ \le x \le 360^\circ$. 2

f) Show that $\tan x + \cot x = \operatorname{cosec} x . \sec x$. 2

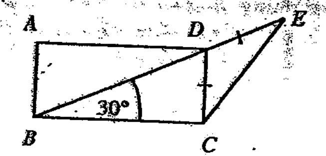
Start each question on a new page

Question 4

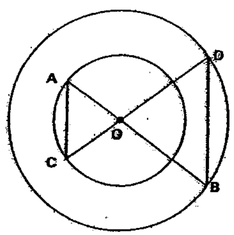
a) In the figure AB = 15cm, AC = 12cm and XB = 3cm. Find AY **2**



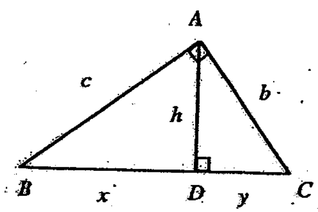
b) ABCD is a rectangle. Show CE = CB. **2**



c) In the diagram, O is the common centre of two concentric circles. **4**
i) Prove that the triangles AOC and BOD are similar.
ii) Prove AC is parallel to BD.



d) Use Pythagoras' Theorem in each of $\Delta ABD, \Delta ACD$ and ΔBCA to show that $h^2 = xy$. **4**



Start each question on a new page

Question 5

a) On a number plane shade in the region given by $x^2 + y^2 \le 4$ and $x + y \ge 1$. **3**

b) Show whether the function $f(x) = \dfrac{x^3}{x^4 - x^2}$ is even, odd or neither. **2**

c) Solve graphically $|2x+1| = 3$. **2**

d) Sketch the graph of this split domain function. **2**

$$f(x) = \begin{cases} \sqrt{25 - x^2}, & \text{for } -5 \le x \le 3 \\ 4, & \text{for } 3 < x \le 5 \end{cases}$$

e) Sketch the curve $y = x^2 - 5x + 6$ showing all its essential features. ie, the x and y intercepts and vertex. **3**

Start each question on a new page

Question 6

a) Find the gradient of a line that makes an angle of 135° with the positive direction of the x axis. **1**

b) Prove that the points A(-1,1), B(0,3) and C(2,7) are collinear. **2**

c) Find the equation of a line which passes through the origin and the intersection of the lines $5x + 2y = 12$ and $3x - 2y = 4$. **3**

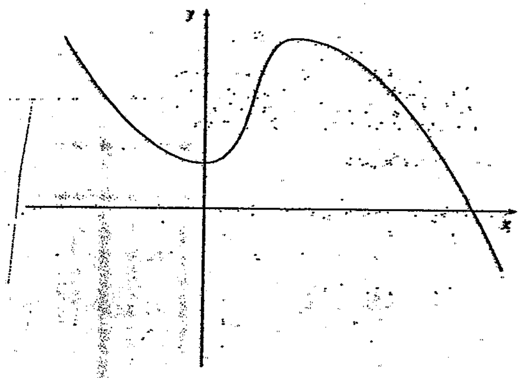
d) Find the equation of a line which passes through the midpoint of A(-3,1) and B(4,-2) and is perpendicular to $2x - y - 1 = 0$. **3**

e) Prove that the line $5x - 12y + 10 = 0$ is a tangent to the circle $(x-1)^2 + (y+2)^2 = 9$ **3**

Start each question on a new page

Question 7

a) Sketch the gradient function for this graph. **2**



b) Evaluate $\lim\limits_{x \to -4} \left(\dfrac{x^2 + 2x - 8}{x + 4} \right)$ **2**

c) Differentiate from first principles $f(x) = 3x^2 - x$. **2**

d) Differentiate with respect to x:

i) $y = 3x^4 - \dfrac{1}{2}x^2 + x - 1$. **2**

ii) $y = x\sqrt{x}$. **2**

e) Find the gradient of the tangent to the curve $y = (2x-1)^2$ when $x = 3$. **2**

Start each question on a new page

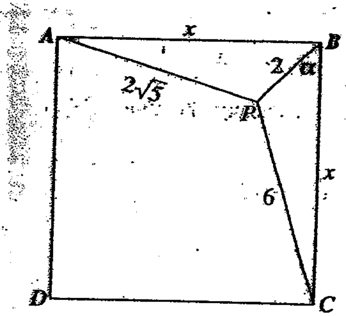
Question 8

a) From a lighthouse L a ship S bears $053^\circ T$ and is at a distance of 8 nautical miles. From L a boat B bears $293^\circ T$ and is at a distance of 6 nautical miles. **4**

i) Find the distance of the ship S from the boat B. Leave your answer as a surd.

ii) Find the bearing of the ship S from the boat B. Answer to the nearest degree.

b) **4**



The diagram shows a square ABCD of side x cm, with a point P within the square, such that PC = 6cm, PB = 2cm and AP = $2\sqrt{5}$ cm.
Let $\angle PBC = \alpha$

i) Using the cosine rule in triangle PBC, show that $\cos\alpha = \dfrac{x^2 - 32}{4x}$.

ii) By considering triangle PBA, show that $\sin\alpha = \dfrac{x^2 - 16}{4x}$.

iii) Hence, or otherwise, show that the value of x is a solution of $x^4 - 56x^2 + 640 = 0$.

c) **4**

i) Show that $\dfrac{x+3}{x+1} = 1 + \dfrac{2}{x+1}$.

ii) Hence or otherwise, sketch a graph for $y = \dfrac{x+3}{x+1}$.

End of Test

(SR copy) Solutions

1

a) $SA = 4\pi r^2$

$= 4 \times \pi \times 5^2$

$= 314.15926\ldots$ — ①

$= 314\text{ cm}^2$

b) $\sqrt{2.5 \times 10^{17}}$

$= 5 \times 10^{8}$ — ①

c) $\dfrac{1}{\sqrt{(3a+1)^3}}$ — ①

d) $12\left(\dfrac{a+1}{4}\right) = 12\left(\dfrac{2a}{3}\right) - 1 \times 12$

$3a + 3 = 8a - 12$

$5a = 15$

$\boxed{a = 3}$ — ①

e) let $x = 1.2444\ldots$

$10x = 12.444\ldots$

$100x = 124.4444\ldots$ — ①

$\therefore 90x = 112$

$x = \dfrac{112}{90}$

$= 1\dfrac{11}{45}$ — ①

take off 1 if not simplified.

f) $\dfrac{1}{3(x^3-8)} \times \dfrac{x^2-4}{1}$ ①

$= \dfrac{1}{3(x-2)(x^2+2x+4)} \times \dfrac{(x-2)(x+2)}{1}$

$= \dfrac{x+2}{3(x^2+2x+4)}$ — ①

g) $|x-1| = 2x-5$

$x - 1 = 2x - 5$ or $-(x-1) = 2x-5$

$\underline{x = 4}$

$-x + 1 = 2x - 5$

$-3x = -6$

$x = 2$

check $|4-1| = 2\times4-5$

$3 = 3$ ✓

check $|2-1| = 2\times2-5$

$|1| = -1$ ×

$\therefore x = 4$ only

× 1 mark for getting $x=4$ and $x=2$

1 mark for choosing $x=4$ only.

h) $\dfrac{\sqrt{3}-4}{2+3\sqrt{3}} \times \dfrac{2-3\sqrt{3}}{2-3\sqrt{3}}$

$= \dfrac{2\sqrt{3} - 3\sqrt{9} - 8 + 12\sqrt{3}}{4 - 9\sqrt{9}}$

$= \dfrac{14\sqrt{3} - 17}{-23}$ ① for correct rationalising

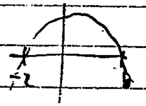
$= \dfrac{-17 + 14\sqrt{3}}{-23}$

$\therefore x = \dfrac{17}{23},\ y = \dfrac{-14}{23}$ ①

2

a) $12+4m-m^2>0$

$(2+m)(6-m)>0$ — 1



$\therefore -2<m<6$ — 1

b) $4^{3-a}=8^{a}$

$(2^2)^{3-a}=2^{3a}$

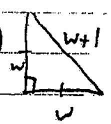
$2^{6-2a}=2^{3a}$ — 1

$\therefore 6-2a=3a$

$5a=6$

$a=1\frac{1}{5}$ — 1

c) i)



$(w+1)^2=w^2+w^2$ — 1

$w^2+2w+1=2w^2$

$w^2-2w-1=0$ — 1

ii) $w=\dfrac{2\pm\sqrt{4+4}}{2}$

$=\dfrac{2\pm 2\sqrt{2}}{2}$

$=1\pm\sqrt{2}$ — 1

$\therefore$ dimensions $(1+\sqrt{2}),(1+\sqrt{2})$ and $(2+\sqrt{2})$

ignoring −ve length. — 1

d) $a-b-c=1$ (1)

$2a+b-c=-9$ (2)

$2a-3b-2c=7$ (3)

(1) → $a=1+b+c$ — 1 for a start

sub (1) in (2) $2(1+b+c)+b-c=-9$

$2+2b+2c+b-c=-9$

$3b+c=-11$ (4)

sub (1) in (3) $2+2b+2c-3b-2c=7$

$-b=5$

$\boxed{b=-5}$ — 1

sub $b=-5$ in (4)

$3(-5)+c=-11$

$\boxed{c=4}$ — 1

sub in (1) $a--5-4=1$

$\boxed{a=0}$ — 1

3

a) $-\dfrac{\sqrt{3}}{2}\times\dfrac{1}{\sqrt{3}}--\dfrac{1}{\sqrt{2}}$ — 1

$=-\dfrac{1}{2}+\dfrac{1}{\sqrt{2}}$ — 1 accept this

$=\boxed{\dfrac{-\sqrt{2}+2}{2\sqrt{2}}}\times\dfrac{\sqrt{2}}{\sqrt{2}}$

$=\dfrac{-2+2\sqrt{2}}{4}$

$=\dfrac{-1+\sqrt{2}}{2}$

b) $3(1+\tan^2\theta)$ — 1

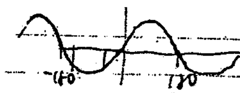
$=3\sec^2\theta$ — 1

c) $\sin\theta=-\dfrac{\sqrt{3}}{2}$ — 1

$-180\le\theta\le 180^\circ$

acute ∠ is 60°

$\therefore\theta=\underbrace{-60^\circ,-120^\circ}_{\text{1 for both.}}$

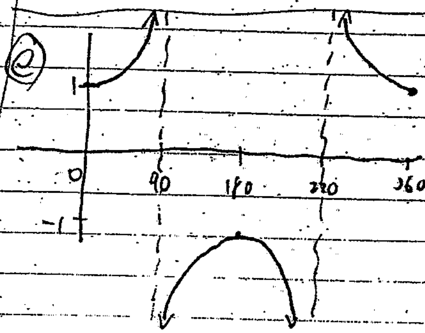


d) $\cos 2\theta=0.5$ $0^\circ\le 2\theta\le 720^\circ$

$2\theta=60,300,420,660$ — 1

$\therefore\theta=30^\circ,150^\circ,210^\circ,330^\circ$ — 1

e)



1 – for asymptotes

1 – for graph

f) $\tan x+\cot x=\operatorname{cosec}x\cdot\sec x$

LHS $=\dfrac{\sin x}{\cos x}+\dfrac{\cos x}{\sin x}$ — 1

$=\dfrac{\sin^2x+\cos^2x}{\cos x\sin x}$

$=\dfrac{1}{\cos x\sin x}$

$=\dfrac{1}{\cos x}\times\dfrac{1}{\sin x}$

$=\sec x\cdot\operatorname{cosec}x$

$=\operatorname{cosec}x\cdot\sec x$ — 1

$=$ rhs.

4

(a) $\frac{YC}{12} = \frac{3}{15}$

$15\,YC = 36$

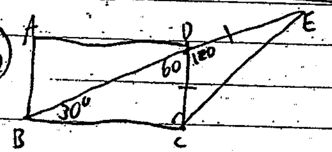
$YC = \frac{36}{15}$

$= 2.4$ — (1)

$\therefore AY = 12 - 2.4$

$= 9.6$ — (1)

(b)



$\angle BCD = 90^\circ$ (rect.)

$\angle BDC = 60^\circ$ ($\angle$ sum Δ)

$\therefore \angle CDE = 120^\circ$ (supp.) — (1)

$\therefore \angle DEC = 30^\circ$ (isos. ΔDEC)

$\therefore \Delta BCE$ is isos. — (1)

$\therefore CB = CE$

(c) i) $AO = CO$ (radii)

$BO = DO$ (radii)

$\therefore \frac{AO}{BO} = \frac{CO}{DO}$ — (1)

$\angle AOC = \angle BOD$ (vert. opp.)

$\therefore \Delta AOC \,|||\, \Delta BOD$ (two sides in prop. and inc. $\angle$ equal) — (1)

ii) $\angle CAO = \angle DBO$ (corr. $\angle$'s in similar Δ's are equal) — (1)

$\therefore AC \parallel BD$ (angles in alternate position are equal) — (1)

(d) $c^2 = h^2 + x^2$ — (1)

$b^2 = h^2 + y^2$ — (1)

$(x+y)^2 = b^2 + c^2$ — (1)

$x^2 + 2xy + y^2 = h^2 + y^2 + h^2 + x^2$

$2xy = 2h^2$

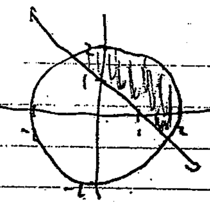
$\therefore h^2 = xy$ — (1)

5

a) $x^2 + y^2 \leq 4$

$x + y \geq 1$

(3)



(1) each for graph

(1) for region.

b) $f(x) = \frac{x^3}{x^4 - x^2}$

$f(-x) = \frac{(-x)^3}{(-x)^4 - (-x)^2}$ — (1) correct substitution

$= \frac{-x^3}{x^4 - x^2}$

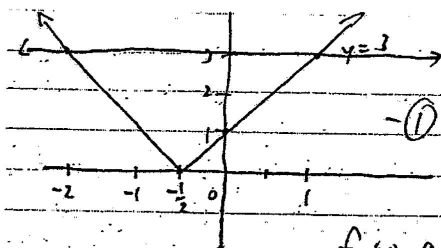
$= -\left[\frac{x^3}{x^4 - x^2}\right]$

$= -f(x)$

$\therefore$ odd. — (1)

* no marks if just say 'odd'.

c) $|2x + 1| = 3$



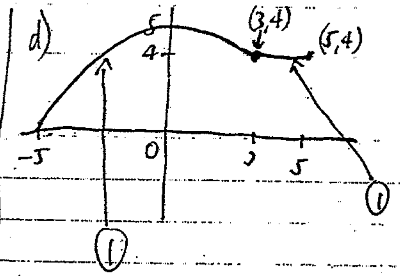
— (1) for graph

$x = -2$

$x = 1$ } — (1)

x, if no graph then no marks.

d)



(1)

(1)

e) $y = x^2 - 5x + 6$

$x = 0,\ y = 6$

$y = 0\quad x^2 - 5x + 6 = 0$

$(x-3)(x-2) = 0$

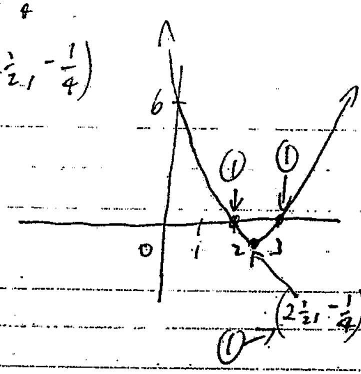
$\therefore x = 3,\ x = 2$

axis: $x = 2\frac{1}{2}$

$\therefore y = (2\frac{1}{2})^2 - 5(2\frac{1}{2}) + 6$

$y = -\frac{1}{4}$

$\therefore V(2\frac{1}{2}, -\frac{1}{4})$



6

a) $\tan\theta = m$

$m = \tan 135^\circ$

$m = -1$ — (1)

b) $m_{AB} = \dfrac{3-1}{0--1}$

$= 2$

$m_{BC} = \dfrac{7-3}{2-0}$

$= 2$ — (1)

$\therefore$ pts are collinear as each interval has same gradient. — (1)

c) $5x + 2y = 12$ (1)

$3x - 2y = 4$ (2)

(1)+(2) $8x = 16$

$\boxed{x = 2}$

$\therefore 6 - 2y = 4$

$2y = 2$

$\boxed{y = 1}$ — (1)

$\therefore P(0,0)$ and $(2,1)$

$m = \dfrac{1-0}{2-0}$

$= \dfrac{1}{2}$ — (1)

$\therefore$ eqn: $y - 0 = \frac{1}{2}(x - 0)$

$\boxed{y = \frac{1}{2}x}$ — (1)

or

$x - 2y = 0$

d) mid $= \left(\dfrac{-3+4}{2}, \dfrac{1+-2}{2}\right)$

$= \left(\dfrac{1}{2}, -\dfrac{1}{2}\right)$ — (1)

$2x - y - 1 = 0$

$y = 2x - 1$

$\therefore \perp m = -\frac{1}{2}$ — (1)

eqn: $y + \frac{1}{2} = -\frac{1}{2}\left(x - \frac{1}{2}\right)$

$2y + 1 = -x + \frac{1}{2}$

$4y + 2 = -2x + 1$

$2x + 4y + 1 = 0$ — (1)

e) For line to be a tangent, the $\perp$ dist to the centre = radius.

ie show $\perp d = 3$ $c(1,-2)$.

$\perp d = \dfrac{|5(1) + -12(-2) + 10|}{\sqrt{5^2 + (-12)^2}}$

$= \dfrac{5 + 24 + 10}{\sqrt{169}}$

$= \dfrac{39}{13}$

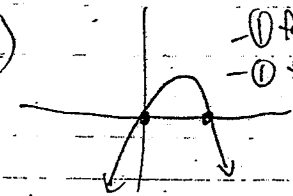
$= 3$

(1) – state why it is a tangent.

(2) for $\perp d = 3$.

7

a)



— (1) for intercepts

— (1) for shape

b) $\displaystyle\lim_{x \to 4} \frac{(x+4)(x-2)}{(x+4)}$ — (1)

$= -6$ — (1)

c) $f(x) = 3x^2 - x$

$f(x+h) = 3(x+h)^2 - (x+h)$

$f'(x) = \displaystyle\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \displaystyle\lim_{h \to 0} \frac{3(x+h)^2 - (x+h) - (3x^2 - x)}{h}$ — (1)

$= \displaystyle\lim_{h \to 0} \frac{3x^2 + 6xh + 3h^2 - x - h - 3x^2 + x}{h}$

$= \displaystyle\lim_{h \to 0} \frac{6xh + 3h^2 - h}{h}$

$= \displaystyle\lim_{h \to 0} \frac{h(6x + 3h - 1)}{h}$

$= 6x - 1$ — (1)

× if use short method, zero marks..

d) i) $y' = 12x^3 - x + 1$ — (1)

ii) $y = x^1 \cdot x^{\frac{1}{2}}$

$y = x^{\frac{3}{2}}$

$\therefore y' = \frac{3}{2}x^{\frac{1}{2}}$ — (1)

e) $y = (2x-1)^2$

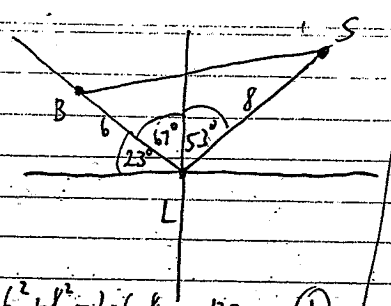
$y = 4x^2 - 4x + 1$

$y' = 8x - 4$ — (1)

$x = 3,\ y' = 20$

$\therefore m_T = 20$ — (1)

8 (a)



i) $d^2 = 6^2+8^2-2\times6\times8\times\cos 120$ — (1)

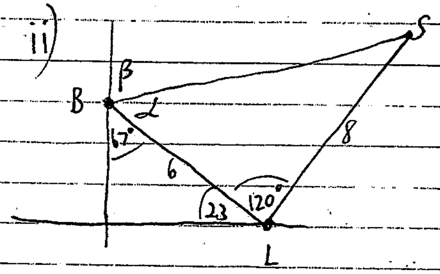
$= 36+64-96\cos120°$

$= 100 - 96\cos 120°$

$= 148$

$\therefore d = \sqrt{148}$
$= 2\sqrt{37}$ } — (1)

ii)



$\frac{\sin\alpha}{8} = \frac{\sin 120}{2\sqrt{37}}$

$\sin\alpha = \frac{8\times\sin 120}{2\sqrt{37}}$

$\doteqdot 34.715\ldots°$

$\doteqdot 35°$ (1) finding α

$\therefore \beta = 180-35-67$
$= 78°$ $\therefore$ bearing $078°T$ (1)

(b)

i) $\cos\alpha = \frac{x^2+2^2-6^2}{2\times x\times2}$ show.

$\cos\alpha = \frac{x^2-32}{4x}$ — (1)

ii) $\angle ABP = 90-\alpha$

$\therefore \cos(90-\alpha) = \frac{x^2+2^2-(2\sqrt5)^2}{2\times x\times 2}$

$= \frac{x^2+4-20}{4x}$

$= \frac{x^2-16}{4x}$ show

$\therefore \sin\alpha = \frac{x^2-16}{4x}$ — (1)

iii) Using $\sin^2\alpha+\cos^2\alpha=1$ — (1)

$\therefore \left(\frac{x^2-16}{4x}\right)^2 + \left(\frac{x^2-32}{4x}\right)^2 = 1$

$\frac{x^4-64x^2+1024}{16x^2} + \frac{x^2-32x^2+256}{16x^2} = 1$

$2x^4-96x^2+1280 = 16x^2$

$2x^4-112x^2+1280=0$

$x^4-56x^2+640=0$ — (1)

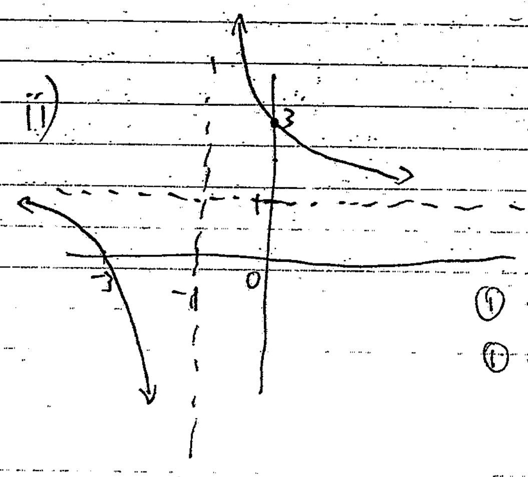
8 (c)

i) $\frac{x+3}{x+1} = \frac{x+1+2}{x+1}$ — (1)

$= \frac{x+1}{x+1} + \frac{2}{x+1}$

$= 1 + \frac{2}{x+1}$ — (1)

ii)



(1) for asymptotes

(1) shape.

[redacted]