Maths Extension 1 Year 11 Task 3 H-Yearly 2006 Term 2
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TRINITY GRAMMAR SCHOOL
MATHEMATICS DEPARTMENT

YEAR 11 2006 ASSESSMENT TASK 3

MATHEMATICS

(EXTENSION 1)

HALF YEARLY EXAMINATION

Time Allowed – *2 hours plus 5 minutes reading*

Tuesday 9th MAY 2006

WEIGHTING 25% towards final result

Outcomes referred to: P1, P2, P3, P4, P5, P6, P7, P8.

INSTRUCTIONS:

1. Attempt **ALL** questions.
2. There are 8 questions of equal value.
3. Each question is worth 12 marks.
4. Total marks available are 96.
5. Show all necessary working.
6. **Begin each question on a new page.**
7. **Write your name, your teacher's name and class on the top of each question**
8. Mark values are shown beside each part.
9. Non-programmable silent Board of Studies approved calculators are permitted.
10. If requested, additional writing sheets may be obtained from teacher upon request.

Name ____________________________________

Start each question on a new page

Question 1

a) Write $(2w-1)^{\frac{-5}{2}}$ without a negative fractional index. **1**

b) Solve $\frac{3x-4}{2}-5=\frac{x}{3}$. **2**

c) Express $0\cdot 13\dot{4}$ in simplest fraction form. **2**

d) Solve $3x-2=|x+2|$ **2**

e) Solve $\frac{6}{x^2}+\frac{1}{x}-1=0$ **2**

f) Find the exact value of a and b if $\frac{4+\sqrt{5}}{2-\sqrt{5}}=a+b\sqrt{5}$. **3**

Start each question on a new page

Question 2

a) Simplify $\frac{4}{3a^2-27} \div \frac{6}{3a-9}$. **3**

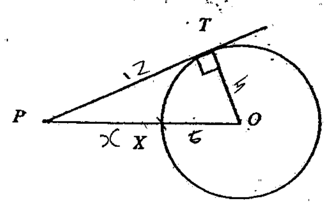
b) Solve $2+c-c^2>0$. **2**

c) Solve simultaneously: $x-y+z=7$, $x+2y-z=-4$ and $3x-y-z=3$. **4**

d) A circle with centre O has a radius of 5cm. From a point P outside the circle the tangent PT has length 12cm and the shortest distance PX from P to the circle has length x cm. **3**

i) Show that $x^2+10x-144=0$

ii) Find the shortest distance from P to the circle.



Start each question on a new page

Question 3

a) Find the exact value of $\sin 210^\circ + \sec 30^\circ . \tan 30^\circ$ **2**

$x - x \times y^2$

b) Simplify $\cot\theta - \cot\theta . \cos^2\theta$ **2**

c) If $\cos\theta = \frac{1}{4}$ and $270^\circ < \theta < 360^\circ$, find the exact value of $\sin\theta$. **2**

d) Solve $\sqrt{3}\tan\theta + 1 = 0$ for $-180^\circ \le \theta \le 180^\circ$. **3**

e) Sketch $y = 3\sin 2x$ over the domain $0^\circ \le x \le 360^\circ$. **3**

Start each question on a new page

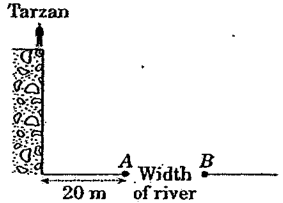
Question 4

a) Solve $\cos 2x = \frac{1}{2}$ for $0^\circ \le x \le 360^\circ$. 3

b) Sketch $y = \operatorname{cosec}\theta$ for $0^\circ \le \theta \le 360^\circ$. 3

c) Show that $\dfrac{\cos\theta(\sin\theta+\cos\theta)}{(1+\sin\theta)(1-\sin\theta)} = 1+\tan\theta$. 3

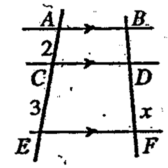
d) In the diagram, Tarzan stands at the top of a cliff above a river. 20m from the cliff is a river flowing between two boulders, A and B. 3
If Tarzan sees boulder A at an angle of depression of $36^\circ 20'$ and boulder B $18^\circ 40'$, find the width of the river.



Start each question on a new page

Question 5

a) Find x if BF = 6. 1

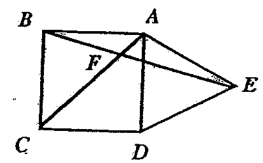


b) For a regular 9-sided polygon find: 2

i) the size of each exterior angle

ii) the size of each interior angle

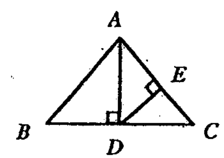
c) ABCD is a square. ADE is an equilateral triangle. Find the size of $\angle$BFA. 3



d) Given AB = AC, 3

i) show that $\Delta ABD\ |||\ \Delta DCE$.

ii) If AB = 5 and BD = CD = 3, find the length of AE



e) i) Draw the graph of $y = |x|$ and $y = x + 4$ on the same set of axes. 2

ii) Find the co-ordinates of the point of intersection of these two graphs. 1

Start each question on a new page

Question 6

a) What is the domain and range of $y=\sqrt{4-x^2}$? 2

b) Show whether the function $f(x)=\dfrac{x^5}{x^2-x^4}$ is even, odd or neither. 2

c) Sketch the graph of $x^2+y^2-6x+2y+6=0$. 3

d) Sketch the curve $y=4+3x-x^2$ showing all its essential features, including the x and y intercepts and vertex. 3

e) Find $\lim\limits_{x\to 2}\dfrac{x^2-4}{x-2}$ 2

Start each question on a new page

Question 7

a) Find the equation of a line passing through (3,1) and through the point of intersection of the lines $3x+2y-10=0$ and $2x+3y-5=0$. 3

b) Find the equation of a line that is the perpendicular bisector of the interval joining (1,4) and (-3, -2). 3

c) The perpendicular distance of (3, -2) to the line $5x-12y+c=0$ is 2 units. Find two possible values of c. 3

d) Differentiate $f(x)=x^2+2x$ from first principles. 3

Start each question on a new page

Question 8

a) Differentiate with respect to x:

i) $y=3x^4-x-5$ **1**

ii) $y=\dfrac{1}{x^2}$ **2**

iii) $y=\dfrac{x^2}{x-2}$ **2**

iv) $y=x^3(x+1)^3$ **3**

b) Given the curve $y=x^3-x^2+2x+6$: **4**

i) Find the equation of the normal at the point P (-1,1).

ii) Find the co-ordinates of the point Q where this normal meets the x axis.

iii) Calculate the **exact** length of PQ.

End of Test

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Maths Extension 1 YEAR 11 TASK 3 H-Yearly 2006 Term 2

Solutions

Y11 Ext 1 2006 Half Yearly

96 marks

solutions

① a) $(2w-1)^{-\frac{5}{2}} = \dfrac{1}{\sqrt{(2w-1)^5}}$ — ①

b) $6\times\left(\dfrac{3x-4}{2}\right) - 5\times 6 = \left(\dfrac{x}{3}\right)\times 6$

$9x - 12 - 30 = 2x$ — ①

$7x = 42$

$\boxed{x = 6}$ — ①

c) let $x = 0\cdot134444\ldots$

$100x = 13\cdot4444\ldots$

$1000x = 134\cdot444\ldots$ — ①

$\therefore 900x = 121$

$\boxed{x = \dfrac{121}{900}}$ — ①

d) $3x-2 = x+2$ or $3x-2 = -(x+2)$

$2x = 4$ $\quad$ $3x-2 = -x-2$

$\boxed{x=2}$ ① $\quad$ $4x = 0$

$\boxed{x=0}$

test: $x=2$, $4=|4|$ ✓

$x=0$, $-2=|2|$ ✗

$\therefore \boxed{x=2 \text{ only}}$ — ①

e) $\dfrac{6}{x^2} + \dfrac{1}{x} - 1 = 0$

$6 + x - x^2 = 0$

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$\boxed{x=3,\ x=-2}$

① ①

f) $\dfrac{(4+\sqrt5)}{(2-\sqrt5)} \times \dfrac{(2+\sqrt5)}{(2+\sqrt5)}$ — ①

$= \dfrac{8+4\sqrt5+2\sqrt5+5}{4-5}$

$= \dfrac{13+6\sqrt5}{-1}$

$= -13-6\sqrt5$

$\therefore \boxed{a=-13},\ \boxed{b=-6}$

① ①

12

② a) $\dfrac{4}{3a^2-27} \div \dfrac{6}{3a-9}$

$= \dfrac{4}{3(a^2-9)} \times \dfrac{3a-9}{6}$ — ①

$= \dfrac{4}{3(a-3)(a+3)} \times \dfrac{3(a-3)}{6}$ — ①

$= \dfrac{2}{3(a+3)}$ — ①

b) $2+c-c^2 \geq 0$

let $2+c-c^2 = 0$

$\therefore (2-c)(1+c)$

$\therefore c=2,\ c=-1$ — ①

sol: $\therefore -1 \leq c \leq 2$ — ①

c) $x - y + z = 7$ ①

$x + 2y - z = -4$ ②

$3x - y - z = 3$ ③

①+② $2x + y = 3$ ④

③−② $2x - 3y = 7$ ⑤ — ①

④−⑤ $4y = -4$

$y = -1$

$\therefore 2x - 1 = 3$

$2x = 4$

$x = 2$

$\therefore 2-(-1)+z = 7$

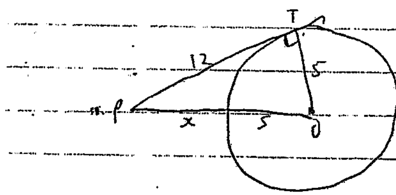
$3 + z = 7$

$z = 4$

$\therefore x=2,\ y=-1,\ z=4$

① ① ①

d) i) $(x+5)^2 = 12^2 + 5^2$ (Pythagoras' Theorem)



$x^2 + 10x + 25 = 144 + 25$

$x^2 + 10x + 25 = 169$

$\therefore x^2 + 10x - 144 = 0$ — ①

ii) $PO = \sqrt{5^2+12^2}$

$= \sqrt{169}$

$= 13$ — ①

$\therefore x = 13 - 5$

$= 8$ cm — ①

12

3

a) $-\sin 30 + \frac{1}{\cos 30} \times \tan 30$

$= -\frac{1}{2} + \frac{1}{\frac{\sqrt{3}}{2}} \times \frac{1}{\sqrt{3}}$ — ①

$= -\frac{1}{2} + \frac{2}{\sqrt{3}} \times \frac{1}{\sqrt{3}}$

$= -\frac{1}{2} + \frac{2}{3}$

$= \frac{1}{6}$ — ①

b) $\cot\theta - \cot\theta\cos^2\theta$

$= \cot\theta(1 - \cos^2\theta)$ — ①

$= \frac{\cos\theta}{\sin\theta} \times \sin^2\theta$

$= \cos\theta\sin\theta$ — ①

c) $\cos\theta = \frac{1}{4}$

θ in IV

$y = \sqrt{4^2 - 1^2}$

$= \sqrt{15}$ — ①

$\therefore \sin\theta = \frac{-\sqrt{15}}{4}$ — ①

d) $\sqrt{3}\tan\theta + 1 = 0$ $\quad -180 \le \theta \le 180$

$\tan\theta = -\frac{1}{\sqrt{3}}$ — ①

acute ∠θ is 30°

in II, (IV).

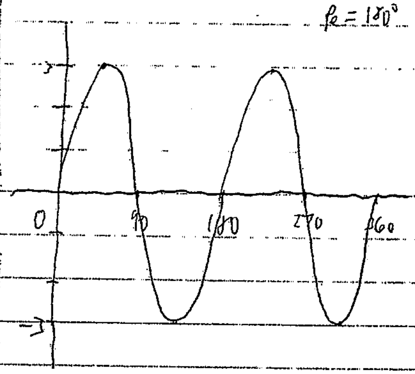
$\therefore \theta = 150°$ — ①

and $\theta = -30°$ — ①

e) $y = 3\sin 2x$

Amp = 3

Pe = 180°



① mark for basic sine curve shape

① mark for correct Amp. 3.

① mark for correct Period, 2 wavelength.

(12)

4

a) $\cos 2x = \frac{1}{2}$ $\quad 0° \le x \le 360°$

$\therefore 0° \le 2x \le 720°$ ← ①

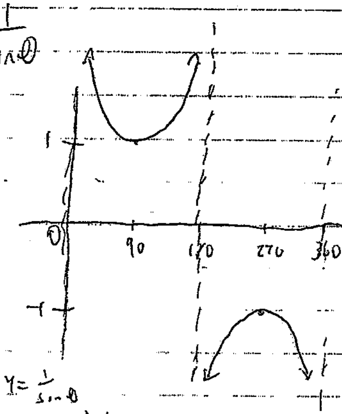
+ve ∴ in I, IV

$2x = 60°, 300°, 420°, 660°$ — ①

$\therefore x = 30°, 150°, 210°, 330°$ — ①

b) $y = \operatorname{cosec}\theta$

$y = \frac{1}{\sin\theta}$



① − for $y = \frac{1}{\sin\theta}$

① − for asymptotes

① − correct parts.

c) $\frac{\cos\theta(\sin\theta + \cos\theta)}{(1+\sin\theta)(1-\sin\theta)} = 1 + \tan\theta$

LHS $= \frac{\cos\theta(\sin\theta + \cos\theta)}{1 - \sin^2\theta}$

$= \frac{\cos\theta(\sin\theta + \cos\theta)}{\cos^2\theta}$ — ①

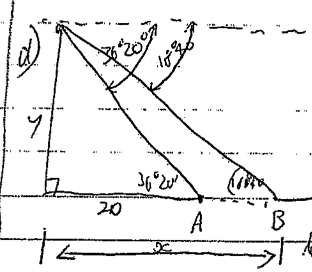
$= \frac{\sin\theta + \cos\theta}{\cos\theta}$ — ①

$= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\cos\theta}$

$= 1 + \tan\theta$ — ①

= rhs.

d)



let y be cliff length

let x be distance from cliff to B.

$\tan 36°20' = \frac{y}{20}$

$\therefore y = 14.71..$ (cliff length) → ①

$\therefore \tan 18°40' = \frac{y}{x}$

$x = \frac{14.71..}{\tan 18°40'}$

$= 43.54..$ — ①

∴ width of river AB is $43.54 - 20$

$= 23.54$ m (2dp). — ①

(12)

5 a) $\frac{x}{6} = \frac{3}{5}$

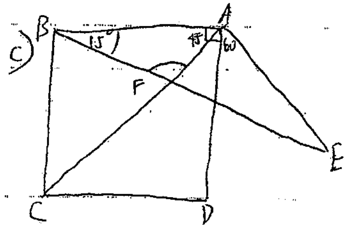
$x = 3\frac{3}{5} = \frac{18}{5}$ — 1

b) i) ext. $\angle = \frac{360°}{9}$

$= 40°$ — 1

ii) int. $\angle = \frac{(9-2)\times 180}{9}$ or $180° - 40° = 140°$

$= 140°$ — 1

c)


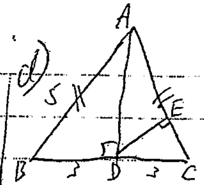
$\angle DAE = 60°$ (equil. $\triangle$) — 1

$\therefore \angle BAE = 150°$

$\angle ABE = 15°$ ($\triangle ABE$ is isos)

$\angle BAF = 45°$ (diag. of square) — 1

$\therefore \angle BFA = 120°$ ($\angle$ sum of $\triangle$) — 1

d)


i) $\angle ADB = \angle DEC = 90°$ (given)

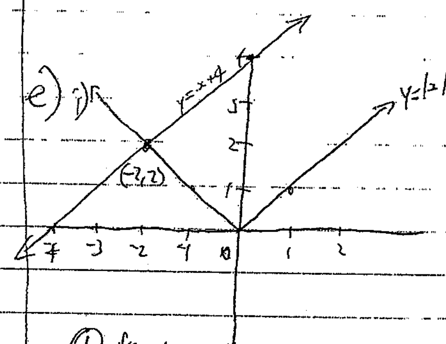
$\angle ABD = \angle ACD$ (base $\angle$s of isos $\triangle$)

$\therefore \triangle ABD \ ||| \ \triangle DCE$ (equiangular) — 1

ii) $\frac{3}{5} = \frac{EC}{3}$

$EC = \frac{9}{5}$ — 1

$\therefore AE = 5 - \frac{9}{5} = 3\frac{1}{5}$ — 1

e) i)


1 for $y = x+4$

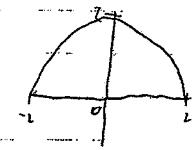
1 for $y = |x|$

ii) $(-2, 2)$ — 1

from graph

12

6 a) $y = \sqrt{4 - x^2}$



$\therefore$ D: $-2 \le x \le 2$ — 1

R: $0 \le y \le 2$ — 1

b) $f(x) = \frac{x^5}{x^2 - x^4}$

$\therefore f(-x) = \frac{(-x)^5}{(-x)^2 - (-x)^4}$ — 1 shows sub. of $-x$.

$= \frac{-x^5}{x^2 - x^4}$

$= -\left(\frac{x^5}{x^2 - x^4}\right)$

$= -f(x)$ — 1

$\therefore$ odd fn.

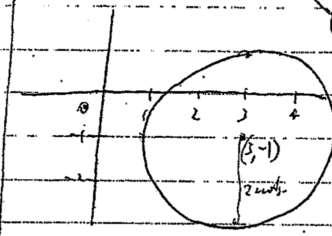
c) $x^2 - 6x + (-3)^2 + y^2 + 2y + 1^2 = -6 + 9 + 1$

$(x-3)^2 + (y+1)^2 = 4$ — 1 for eqn.

$\therefore$ circle centre $(3, -1)$ radius 2 units

1 centre

1 radius



must have sketch for full marks.

d) $y = 4 + 3x - x^2$

$x = 0, y = 4$

$y = 0$, $4 + 3x - x^2 = 0$

$\therefore x^2 - 3x - 4 = 0$

$(x-4)(x+1) = 0$

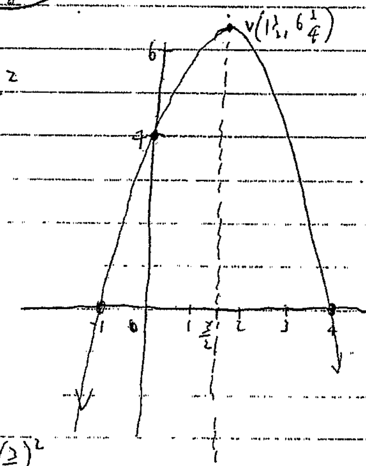
$\therefore x = 4, x = -1$

axis: $x = \frac{3}{2}$

$y = 4 + 3\left(\frac{3}{2}\right) - \left(\frac{3}{2}\right)^2$

$y = 6\frac{1}{4}$

$\therefore V\left(1\frac{1}{2}, 6\frac{1}{4}\right)$



1 for y int.

1 for x ints

1 for vertex

but must have correct graph to get full marks.

e) $\lim_{x \to 2} \frac{x^2 - 4}{x - 2}$

$= \lim_{x \to 2} \frac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}}$ — 1

$= 4$ — 1

12

7 a) $3x+2y-10=0$ ① $\times 2$

$2x+3y-5=0$ ② $\times 3$

$6x+4y-20=0$ ③

$6x+9y-15=0$ ④

④−③ $5y+5=0$

$5y=-5$

$\boxed{y=-1}$

$\therefore 3x-2-10=0$

$3x=12$

$\boxed{x=4}$

$\therefore P(3,1)$, $Q(4,-1)$ — ① pt of int

$m_{PQ}=\frac{1--1}{3-4}=\frac{2}{-1}=-2$ — ① for m.

$\therefore$ eqn PQ: $y-1=-2(x-3)$

$y-1=-2x+6$

$\boxed{y=-2x+7}$ — ① eqn.

or

$\boxed{2x+y-7=0}$

b) $P(1,4)$ and $Q(-3,-2)$

$m_{PQ}=\frac{4--2}{1--3}=\frac{6}{4}=\frac{3}{2}$

$\therefore \perp m=\frac{-2}{3}$ — ①

Mid PQ: $\left(\frac{1+-3}{2},\frac{4+-2}{2}\right)$

$=(-1,1)$ — ①

eqn: $y-1=-\frac{2}{3}(x+1)$

$3y-3=-2x-2$

$\boxed{2x+3y-1=0}$ — ①

or

$\boxed{y=-\frac{2}{3}x+\frac{1}{3}}$

7 c) $(3,-2)$ $5x-12y+c=0$

$\perp d=2$

$\perp d=\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$

$\therefore 2=\left|\frac{5(3)+-12(-2)+c}{\sqrt{5^2+12^2}}\right|$ — ①

$2=\left|\frac{15+24+c}{13}\right|$

$26=|39+c|$

$\therefore 26=39+c$ or $26=-(39+c)$

$\boxed{c=-13}$ ①

or $26=-39-c$

$\boxed{c=-65}$ ①

d) $f(x)=x^2+2x$

$f(x+h)=(x+h)^2+2(x+h)$ — ①

$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$=\lim_{h\to 0}\frac{(x+h)^2+2(x+h)-(x^2+2x)}{h}$

$=\lim_{h\to 0}\frac{x^2+2xh+h^2+2x+2h-x^2-2x}{h}$ — ①

$=\lim_{h\to 0}\frac{2xh+h^2+2h}{h}$

$=\lim_{h\to 0}\frac{\cancel{h}(2x+h+2)}{\cancel{h}}$

$=2x+2$ — ①

* zero if not done by 1st. principles

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8) a) i) $y = 3x^4 - x - 5$

$\frac{dy}{dx} = 12x^3 - 1$ — 1

ii) $y = \frac{1}{x^2}$

$\therefore y = x^{-2}$

$\frac{dy}{dx} = -2x^{-3}$ — 1

$\therefore \frac{dy}{dx} = \frac{-2}{x^3}$ — 1

iii) $y = \frac{x^2}{x-2}$

$u = x^2$
$du = 2x$
$v = x-2$
$dv = 1$

$\frac{dy}{dx} = \frac{(x-2)2x - x^2 \cdot 1}{(x-2)^2}$ — 1

$= \frac{2x^2 - 4x - x^2}{(x-2)^2}$

$= \frac{x^2 - 4x}{(x-2)^2}$ — 1

iv) $y = x^3(x+1)^3$

$u = x^3$
$du = 3x^2$
$v = (x+1)^3$
$dv = 3(x+1)^2 \cdot 1$
1

$\frac{dy}{dx} = x^3 \cdot 3(x+1)^2 + (x+1)^3 3x^2$ — 1

$= 3x^2(x+1)^2[x + (x+1)]$

$= 3x^2(x+1)^2(2x+1)$ — 1

b) $y = x^3 - x^2 + 2x + 6$

i) $y' = 3x^2 - 2x + 2$

$P(-1,1)$

$x = -1$, $y' = 3(-1^2) - 2(-1) + 2$

$= 3 + 2 + 2$

$= 7$

$\therefore m_T = 7$ — 1

$m_N = \frac{-1}{7}$

$\therefore$ eqn normal: $y - 1 = \frac{-1}{7}(x+1)$

$7y - 7 = -x - 1$

$\boxed{x + 7y - 6 = 0}$ — 1

or $y = \frac{-1}{7}x + \frac{6}{7}$

ii) $y = 0$

$\therefore x + 7(0) - 6 = 0$

$\therefore x = 6$

$\therefore \boxed{Q(6,0)}$ ← 1

iii) $PQ = \sqrt{(6-(-1))^2 + (0-1)^2}$

$= \sqrt{7^2 + (-1)^2}$

$= \boxed{\sqrt{50}\text{ u}}$ — 1

u

$5\sqrt{2}$

12