TUTORIAL PROBLEMS (2) – WEEK 1

For the exercises below, do only every second question (the even numbered questions).
The answers to all these questions are at the end of this handout – so if you are having difficulty getting correct answers for a particular exercise, go on and do all the questions in that exercise.

1. Basic Algebra

Algebraic Factors

1. Write down the expansion of:

(a) $(x+5)(x+1)$	(b) $(x-2)(x-3)$	(c) $(a-3)(a+4)$
(d) $(p-7)(p-3)$	(e) $(2x+3)(x+5)$	(f) $(3x-4)(x-2)$
(g) $(3y-2)(4y+3)$	(h) $(x-14)(6x-1)$	(i) $(3x+4)^2$
(j) $(4p-5)^2$	(k) $(3x^2-5x+2)(2x-4)$	(l) $(2a+b)^3$
(m) $(x^2+5)(x^2-2x-3)$	(n) $(x-2)(x+2)(x+2)$	(o) $(5m-2n)^3$

2. Factorise

(a) $bx+by$	(b) $3n+3m$	(c) $6y+12$
(d) $18a-6b$	(e) y^2-4y	(f) a^2-ax
(g) $12xy+16yz$	(h) $17a^2x^5+2a^4x^3$	(i) $bx+by-bz$
(j) $3x^3-x^2+x$	(k) $am^2+2am+3m$	(l) $3xy-3yz+21y$
(m) $a(x+y)-az$	(n) $10a^2+a(c+b)$	(o) $a(x+y)+b(x+y)$

3. Factorise

(a) $xy+xz+yw+zw$	(b) $5a+5b+xa+xb$	(c) $3a+12c+ab+4bc$
(d) $2xy-2xz+7y-7z$	(e) $a^2+ab+ac+bc$	(f) $5a+ab+5b+b^2$
(g) $3ax-bx-3ay+by$	(h) $15ac-10ad+3bc-2bd$	(i) ab^2+ac-b^2d-cd
(j) $a(x-y)+b(y-x)$	(k) $2a-2b+ax-xb$	(l) x^4+x^3+2x+2
(m) $x^2(x+2y)-y(x+2y)$	(n) $xz+z^2-x-z$	(o) $2x^3-x^2-12x+6$

4. Factorise

(a) x^2-y^2	(b) a^2-36	(c) x^2-48
(d) $25-n^2$	(e) x^2-1	(f) $9-4a^2$
(g) x^2-16y^2	(h) $49a^2-121b^2$	(i) $25a^2b^2-4c^2$
(j) $a^2x^2-16b^2y^2$	(k) $1-(a-b)^2$	(l) $49x^2-64y^2z^2$
(m) $100-x^4$	(n) a^4-y^4	(o) $(x+y)^2-x^2$

5. Factorise

(a) x^2+3x+2	(b) $x^2+7x+12$	(c) $x^2+13x+40$
(d) $x^2-7x+12$	(e) $x^2-13x+36$	(f) $x^2-18x+56$
(g) x^2-2x-3	(h) $x^2-10x-11$	(i) $x^2+15x+36$
(j) $x^2-10x+25$	(k) x^2+6x+9	(l) x^2+2x+1
(m) x^2+6x+5	(n) $x^2-22x+120$	(o) x^2-x-42

6. Factorise

(a) $3x^2+7x+4$ (b) $4x^2-11x+6$ (c) $9x^2+24x+16$

(d) $4-3x-x^2$ (e) $12x^2-7xy+y^2$ (f) $2x^2-9x-18$

(g) $18x^2-9x-2$ (h) $3x^2-13x+14$ (i) $2+x-10x^2$

7. Factorise

(a) x^3-y^3 (b) a^3+27 (c) x^3-64

(d) b^3-1 (e) x^3+125 (f) a^3-1000

(g) $4a^3+32$ (h) $5m^3-5$ (i) $64+y^3$

(j) x^6-y^6 (k) $8x^3+27$ (l) $x^7y^4-x^4y^7$

(m) $5y^3-5000$ (n) a^3+b^3+a+b (o) $(x+5)^3+(x-2)^3$

Algebraic Fractions

1. Simplify the following fractions.

(a) $\dfrac{2ab}{3ac}$ (b) $\dfrac{5a^2}{15ab}$ (c) $\dfrac{2x+4}{3x+6}$ (d) $\dfrac{a^2-a}{a^2+a}$

(e) $\dfrac{x^3+8}{x^2-2x+4}$ (f) $\dfrac{x^2+2x+1}{x^2+3x+2}$ (g) $\dfrac{3x^2y-6xy}{2x^2y-4xy^2}$ (h) $\dfrac{a^2-5a}{a^2-4a-5}$

(i) $\dfrac{3x^2-12}{10-5x}$ (j) $\dfrac{6x^2-150}{3x+15}$ (k) $\dfrac{8a^2-8}{4a^2+8a+4}$ (l) $\dfrac{a^3-27}{a^2-9}$

(m) $\dfrac{(a+b)^2-c^2}{3a+3b-3c}$ (n) $\dfrac{1-\dfrac{y^2}{x^2}}{1+\dfrac{y^2}{x^2}}$ (o) $\dfrac{\dfrac{1}{x}-\dfrac{2}{x^2}-\dfrac{3}{x^3}}{\dfrac{9}{x}-x}$ (p) $\dfrac{2x^2-x-6}{\dfrac{4}{x^2}-1}$

2. Simplify the following fractions.

(a) $\dfrac{5}{x}\times\dfrac{15}{y}$ (b) $\dfrac{3x}{5}\times\dfrac{1}{6x}$ (c) $\dfrac{2m}{3n}\times\dfrac{6}{m^2}$

(d) $\dfrac{18}{7x}\div\dfrac{9}{28x}$ (e) $\dfrac{3ab}{4a}\times\dfrac{8a^2}{12b}$ (f) $\dfrac{10x-15}{6}\times\dfrac{1}{8x-12}$

(g) $\dfrac{3m-6}{4}\times\dfrac{8m}{m^2-2m}$ (h) $\dfrac{4a-6}{5}\div\dfrac{6a-9}{15}$ (i) $\dfrac{x-y}{4}\times\dfrac{8}{x^2-y^2}$

(j) $\dfrac{m^2-mn}{n^2-n}\times\dfrac{n-1}{m-n}$ (k) $\dfrac{1-a}{b+b^2}\div\dfrac{1-a^2}{1-b^2}$ (l) $\dfrac{x^2-4}{2x-4}\times\dfrac{2}{x+2}$

(m) $\dfrac{a^2+5a+6}{a^2-25}\div\dfrac{a+3}{a-5}$ (n) $\dfrac{x-3y}{x^3y}\div\dfrac{3y-x}{xy^3}$ (o) $\dfrac{x^2-y^2}{x^2-2xy+y^2}\times\dfrac{xy-y^2}{xy+y^2}$

3. Express each one of the following as single fraction.

(a) $\dfrac{x}{3}+\dfrac{x}{5}$ (b) $\dfrac{m}{2}+\dfrac{2m}{3}$

(c) $\dfrac{x+7}{3}+\dfrac{2x-1}{2}$ (d) $\dfrac{3x-1}{3}-\dfrac{x+2}{6}$

(e) $\dfrac{1}{x}+\dfrac{2}{x+y}$ (f) $\dfrac{2}{a+1}+\dfrac{1}{a-3}$

(g) $\dfrac{a}{b-c}-\dfrac{3a}{5b-5c}$ (h) $\dfrac{5c}{a^2+ab}-\dfrac{c}{a+b}$

(i) $\dfrac{x^2}{x^2+3x+2}-\dfrac{2x}{x+2}$

(j) $\dfrac{m}{m^2+mn}+\dfrac{n}{n^2+mn}$

(k) $\dfrac{x-1}{2}+\dfrac{x+3}{5}+\dfrac{x+7}{10}$

(l) $\dfrac{2a^2-b^2}{a^2}-\dfrac{b^2-c^2}{b^2}-\dfrac{c^2-a^2}{c^2}$

(m) $\dfrac{1}{x+2}+\dfrac{1}{x+3}$

(n) $\dfrac{1}{x^2-9x+20}+\dfrac{1}{x^2-11x+30}$

(o) $\dfrac{1}{2x^2-x-1}-\dfrac{3}{6x^2-x-2}$

(p) $\dfrac{x-3}{x+2}-\dfrac{x-2}{x+3}+\dfrac{1}{x-1}$

Surds

1. Express each of the surds in the simplest form.

(a) $\sqrt{8}$ (b) $\sqrt{12}$ (c) $\sqrt{32}$ (d) $3\sqrt{63}$

(e) $\sqrt{72}$ (f) $\sqrt{96}$ (g) $\sqrt{288}$ (h) $\dfrac{1}{6}\sqrt{18}$

(i) $\dfrac{3}{2}\sqrt{192}$ (j) $\sqrt{320}$ (k) $\sqrt{\dfrac{10}{25}}$ (l) $\sqrt{\dfrac{72}{25}}$

2. Simplify each of the following surds.

(a) $4\sqrt{3}+3\sqrt{3}-2\sqrt{3}$ (b) $3\sqrt{5}+5\sqrt{5}-\sqrt{5}$ (c) $3\sqrt{45}-\sqrt{20}+7\sqrt{5}$

(d) $\sqrt{12}+\sqrt{27}$ (e) $\sqrt{18}+\sqrt{50}$ (f) $4\sqrt{48}-5\sqrt{27}$

(g) $\sqrt{128}+3\sqrt{18}-\sqrt{162}$ (h) $3\sqrt{32}+2\sqrt{75}-5\sqrt{162}$ (i) $2\sqrt{363}-5\sqrt{320}-\sqrt{192}$

(j) $-2\sqrt{a}-4\sqrt{a}+3\sqrt{a}$ (k) $5\sqrt{y}-\sqrt{x}-2\sqrt{y}$ (l) $2\sqrt{x}+2\sqrt{y}-3\sqrt{x}$

3. Express each of the surds in the simplest form.

(a) $\sqrt{3}\times\sqrt{3}$ (b) $4\sqrt{5}\times2\sqrt{2}$ (c) $(\sqrt{5})^2$

(d) $(2\sqrt{7})^2$ (e) $\sqrt{8}\times\sqrt{5}\times\sqrt{125}$ (f) $\sqrt{2}(\sqrt{3}+2\sqrt{2})$

(g) $(2\sqrt{7}-1)^2$ (h) $2\sqrt{5}(3\sqrt{3}-4\sqrt{2})$ (i) $\sqrt{2}(\sqrt{18}+\sqrt{8}-4\sqrt{2})$

(j) $(\sqrt{3}+4)(\sqrt{3}-2)$ (k) $(2\sqrt{5}-3)(2\sqrt{5}+3)$ (l) $(3\sqrt{5}-2\sqrt{2})(2\sqrt{5}+3\sqrt{2})$

(m) $(2\sqrt{x}+\sqrt{y})(\sqrt{x}-3\sqrt{y})$ (n) $(3\sqrt{a}-\sqrt{b})^2$ (o) $\sqrt{x}(6\sqrt{y}-3\sqrt{x})$

4. Multiply each surd by its conjugate and write down the answer.

(a) $\sqrt{2}-1$ (b) $\sqrt{11}-\sqrt{5}$ (c) $2\sqrt{5}+\sqrt{3}$ (d) $2\sqrt{11}-3\sqrt{5}$

5. Express each of the surds in the simplest form with a rational denominator.

(a) $\dfrac{1}{\sqrt{7}}$ (b) $\dfrac{3}{\sqrt{5}}$ (c) $\dfrac{4}{3\sqrt{7}}$ (d) $\dfrac{3\sqrt{2}}{\sqrt{8}}$

(e) $\dfrac{\sqrt{12}}{5\sqrt{3}}$ (f) $\dfrac{4\sqrt{2}}{3\sqrt{8}}$ (g) $\dfrac{\sqrt{2}+3}{\sqrt{5}}$ (h) $\dfrac{1-\sqrt{3}}{\sqrt{3}}$

(i) $\dfrac{\sqrt{5}-\sqrt{2}}{\sqrt{3}}$ (j) $\dfrac{4\sqrt{3}+2\sqrt{2}}{3\sqrt{2}}$ (k) $\dfrac{\sqrt{x}}{\sqrt{y}}$ (l) $\dfrac{2\sqrt{a}}{\sqrt{ab}}$

6. Simplify the following surds by rationalizing the denominator.

(a) $\dfrac{1}{\sqrt{3}+1}$ (b) $\dfrac{1}{\sqrt{3}+\sqrt{7}}$ (c) $\dfrac{1}{5+3\sqrt{2}}$ (d) $\dfrac{2}{2\sqrt{2}-1}$

(e) $\dfrac{3}{\sqrt{5}-2}$ (f) $\dfrac{\sqrt{3}}{4\sqrt{3}+5}$ (g) $\dfrac{6}{7+2\sqrt{5}}$ (h) $\dfrac{\sqrt{5}}{\sqrt{5}-1}$

(i) $\dfrac{\sqrt{7}-\sqrt{3}}{\sqrt{7}+\sqrt{3}}$ (j) $\dfrac{2\sqrt{3}-1}{2\sqrt{3}+1}$ (k) $\dfrac{\sqrt{5}+\sqrt{3}}{2\sqrt{10}-\sqrt{6}}$ (l) $\dfrac{5\sqrt{2}-1}{5\sqrt{2}+1}$

Answers to Exercises

1. Basic Algebra

Algebraic Factors

1. (a) x^2+6x+5 (b) x^2-5x+6 (c) a^2+a-12
   (d) $p^2-10p+21$ (e) $2x^2+13x+15$ (f) $3x^2-10x+8$
   (g) $12y^2+y-6$ (h) $6x^2-85x+14$ (i) $9x^2+24x+16$
   (j) $16p^2-40p+25$ (k) $6x^3-22x^2+24x-8$ (l) $8a^3+12a^2b+6ab^2+b^3$
   (m) $x^4-2x^3+2x^2-10x-15$ (n) x^3+2x^2-4x-8 (o) $125m^3-150m^2n+60mn^2-8n^3$

2. (a) $b(x+y)$ (b) $3(n+m)$ (c) $6(y+2)$
   (d) $6(3a-b)$ (e) $y(y-4)$ (f) $a(a-x)$
   (g) $4y(3x+4z)$ (h) $a^2x^3(17x^2+2a^2)$ (i) $b(x+y-z)$
   (j) $x(3x^2-x+1)$ (k) $m(am+2a+3)$ (l) $3y(x-z+7)$
   (m) $a(x+y-z)$ (n) $a(10a+c+b)$ (o) $(x+y)(a+b)$

3. (a) $(y+z)(x+w)$ (b) $(a+b)(5+x)$ (c) $(a+4c)(3+b)$
   (d) $(y-z)(2x+7)$ (e) $(a+b)(a+c)$ (f) $(5+b)(a+b)$
   (g) $(3a-b)(x-y)$ (h) $(3c-2d)(5a+b)$ (i) $(b^2+c)(a-d)$
   (j) $(x-y)(a-b)$ (k) $(a-b)(2+x)$ (l) $(x+1)(x^3+2)$
   (m) $(x+2y)(x^2-y)$ (n) $(x+z)(z-1)$ (o) $(2x-1)(x^2-6)$

4. (a) $(x+y)(x-y)$ (b) $(a+6)(a-6)$ (c) $(x+\sqrt{48})(x-\sqrt{48})$
   (d) $(5+n)(5-n)$ (e) $(x+1)(x-1)$ (f) $(3+2a)(3-2a)$
   (g) $(x+4y)(x-4y)$ (h) $(7a+11b)(7a-11b)$ (i) $(5ab+2c)(5ab-2c)$
   (j) $(ax+4by)(ax-4by)$ (k) $(1+a-b)(1-a+b)$ (l) $(7x+8yz)(7x-8yz)$
   (m) $(10+x^2)(10-x^2)$ (n) $(a^2+y^2)(a+y)(a-y)$ (o) $y(2x+y)$

5. (a) $(x+1)(x+2)$ (b) $(x+4)(x+3)$ (c) $(x+8)(x+5)$
   (d) $(x-3)(x-4)$ (e) $(x-4)(x-9)$ (f) $(x-4)(x-14)$
   (g) $(x+1)(x-3)$ (h) $(x-11)(x+1)$ (i) $(x+12)(x+3)$
   (j) $(x-5)^2$ (k) $(x+3)^2$ (l) $(x+1)^2$
   (m) $(x+5)(x+1)$ (n) $(x-10)(x-12)$ (o) $(x+6)(x-7)$

6. (a) $(3x+4)(x+1)$ (b) $(4x-3)(x-2)$ (c) $(3x+4)^2$
   (d) $(4+x)(1-x)$ (e) $(3x-y)(4x-y)$ (f) $(2x+3)(x-6)$
   (g) $(6x+1)(3x-2)$ (h) $(3x-7)(x-2)$ (i) $(2+5x)(1-2x)$

7. (a) $(x-y)(x^2+xy+y^2)$ (b) $(a+3)(a^2-3a+9)$ (c) $(x-4)(x^2+4x+16)$
   (d) $(b-1)(b^2+b+1)$ (e) $(x+5)(x^2-5x+25)$ (f) $(a-10)(a^2+10a+100)$
   (g) $4(a+2)(a^2-2a+4)$ (h) $5(m-1)(m^2+m+1)$ (i) $(4+y)(16-4y+y^2)$

(j) $(x+y)(x-y)(x^2+xy+y^2)(x^2-xy+y^2)$

(k) $(2x+3)(4x^2-6x+9)$

(l) $x^4y^4(x-y)(x^2+xy+y^2)$

(m) $5(y-10)(y^2+10y+100)$

(n) $(a+b)(a^2-ab+b^2+1)$

(o) $(2x-3)(x^2+3x+39)$

Algebraic Fractions

1. (a) $\frac{2b}{3c}$ (b) $\frac{a}{3b}$ (c) $\frac{2}{3}$ (d) $\frac{a-1}{a+1}$

(e) $x+2$ (f) $\frac{x+1}{x+2}$ (g) $\frac{3(x-2)}{2(x-2y)}$ (h) $\frac{a}{a+1}$

(i) $\frac{-3(x+2)}{5}$ (j) $2(x-5)$ (k) $\frac{2(a-1)}{a+1}$ (l) $\frac{a^2+3a+9}{a+3}$

(m) $\frac{a+b+c}{3}$ (n) $\frac{x^2-y^2}{x^2+y^2}$ (o) $-\frac{(x+1)}{x^2(3+x)}$ (p) $-\frac{x^2(2x+3)}{2+x}$

2. (a) $\frac{75}{xy}$ (b) $\frac{1}{10}$ (c) $\frac{4}{mn}$

(d) 8 (e) $\frac{a^2}{2}$ (f) $\frac{5}{24}$

(g) 6 (h) 2 (i) $\frac{2}{x+y}$

(j) $\frac{m}{n}$ (k) $\frac{1-b}{b(1+a)}$ (l) 1

(m) $\frac{a+2}{a+5}$ (n) $-\frac{y^2}{x^2}$ (o) 1

3 (a) $\frac{8x}{15}$ (b) $\frac{7m}{6}$

(c) $\frac{8x+11}{6}$ (d) $\frac{5x-4}{6}$

(e) $\frac{3x+y}{x(x+y)}$ (f) $\frac{3a-5}{(a+1)(a-3)}$

(g) $\frac{2a}{5(b-c)}$ (h) $\frac{c(5-a)}{a(a+b)}$

(i) $\frac{-x}{x+1}$ (j) $\frac{2}{m+n}$

(k) $\frac{4(x+1)}{5}$ (l) $\frac{a^4b^2-b^4c^2+a^2c^4}{a^2b^2c^2}$

(m) $\frac{2x+5}{(x+2)(x+3)}$ (n) $\frac{2}{(x-4)(x-6)}$

(o) $\frac{1}{(x-1)(2x+1)(3x-2)}$ (p) $\frac{x^2+11}{(x-1)(x+2)(x+3)}$

Surds – ANSWERS

1. (a) $2\sqrt{2}$ (b) $2\sqrt{3}$ (c) $4\sqrt{2}$ (d) $9\sqrt{7}$
   (e) $6\sqrt{2}$ (f) $4\sqrt{6}$ (g) $12\sqrt{2}$ (h) $\frac{1}{2}\sqrt{2}$
   (i) $12\sqrt{3}$ (j) $8\sqrt{5}$ (k) $\frac{\sqrt{10}}{5}$ (l) $\frac{6\sqrt{2}}{5}$

2. (a) $5\sqrt{3}$ (b) $7\sqrt{5}$ (c) $14\sqrt{5}$
   (d) $5\sqrt{3}$ (e) $8\sqrt{2}$ (f) $\sqrt{3}$
   (g) $8\sqrt{2}$ (h) $10\sqrt{3}-33\sqrt{2}$ (i) $14\sqrt{3}-40\sqrt{5}$
   (j) $-3\sqrt{a}$ (k) $3\sqrt{y}-\sqrt{x}$ (l) $2\sqrt{y}-\sqrt{x}$

3. (a) 3 (b) $8\sqrt{10}$ (c) 5
   (d) 28 (e) $50\sqrt{2}$ (f) $\sqrt{6}+4$
   (g) $29-4\sqrt{7}$ (h) $6\sqrt{15}-8\sqrt{10}$ (i) 2
   (j) $2\sqrt{3}-5$ (k) 11 (l) $18+5\sqrt{10}$
   (m) $2x-5\sqrt{xy}-3y$ (n) $9a-6\sqrt{ab}+b$ (o) $6\sqrt{xy}-3x$

4. (a) 1 (b) 6 (c) 17 (d) -1

5. (a) $\frac{\sqrt{7}}{7}$ (b) $\frac{3\sqrt{5}}{5}$ (c) $\frac{4\sqrt{7}}{21}$ (d) $\frac{3}{2}$
   (e) $\frac{2}{5}$ (f) $\frac{2}{3}$ (g) $\frac{\sqrt{10}+3\sqrt{5}}{5}$ (h) $\frac{\sqrt{3}}{3}-1$
   (i) $\frac{\sqrt{15}-\sqrt{6}}{3}$ (j) $\frac{2\sqrt{6}+2}{3}$ (k) $\frac{\sqrt{xy}}{y}$ (l) $\frac{2\sqrt{b}}{b}$

6. (a) $\frac{\sqrt{3}-1}{2}$ (b) $\frac{\sqrt{7}-\sqrt{3}}{4}$ (c) $\frac{5-3\sqrt{2}}{7}$ (d) $\frac{4\sqrt{2}+2}{7}$
   (e) $3(\sqrt{5}+2)$ (f) $\frac{12-5\sqrt{3}}{23}$ (g) $\frac{42-12\sqrt{5}}{29}$ (h) $\frac{5+\sqrt{5}}{4}$
   (i) $\frac{5-\sqrt{21}}{2}$ (j) $\frac{13-4\sqrt{3}}{11}$ (k) $\frac{13\sqrt{2}+3\sqrt{30}}{34}$ (l) $\frac{51-10\sqrt{2}}{49}$