CSSA

CATHOLIC SECONDARY SCHOOLS
ASSOCIATION OF NSW

Centre Number

Student Number

2017
TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 1

Afternoon Session
Friday, 11 August 2017

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black pen
- NESA-approved calculators may be used
- A Formula Reference Sheet is provided
- In Questions 11 – 14, show relevant mathematical reasoning and/or calculations
- Write your Centre Number and Student Number at the top of this page

Total marks – 70

Section I Pages 2 – 6

10 marks
- Attempt Questions 1 – 10
- Allow 15 minutes for this section

Section II Pages 7 – 11

60 marks
- Attempt Questions 11 – 14
- Allow about 1 hour and 45 minutes for this section

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the Multiple-Choice Answer Sheet for Questions 1–10.

1 What are the coordinates of the point P that divides the interval joining the points $A(1,2)$ and $B(7,5)$ internally in the ratio 2:1?

(A) $(3,3)$

(B) $(3,4)$

(C) $(5,4)$

(D) $(5,3)$

2 What is the size of the acute angle between the lines whose equations are $2x-y-2=0$ and $3x-y+1=0$, correct to the nearest degree?

(A) 8°

(B) 11°

(C) 36°

(D) 45°

3 Which group of three numbers could be the roots of the polynomial equation $x^3+px^2-26x+24=0$?

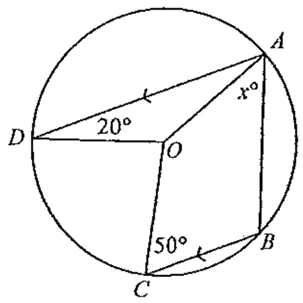
(A) $2,3,4$

(B) $1,-6,4$

(C) $-1,-2,12$

(D) $-1,-3,-8$

4 A, B, C and D are points on a circle with centre O. BC is parallel to AD.
$\angle ADO = 20°$ and $\angle BCO = 50°$.
Let $\angle BAO = x°$.



Not to scale

What is the value of x?

(A) 15

(B) 35

(C) 40

(D) 55

5 Which expression is equal to $\int \cos^2 \frac{2x}{5}\,dx$?

(A) $\frac{x}{2}+\frac{5}{8}\sin\frac{4x}{5}+C$

(B) $\frac{x}{2}-\frac{5}{8}\sin\frac{4x}{5}+C$

(C) $\frac{x}{2}+\frac{2}{5}\sin\frac{4x}{5}+C$

(D) $\frac{x}{2}-\frac{2}{5}\sin\frac{4x}{5}+C$

6 What are the solutions to the inequality $\frac{x^2-4}{2x}>0$?

(A) $-2<x<0$ and $x>2$

(B) $-2<x<0$ and $x>4$

(C) $-4<x<0$ and $x>2$

(D) $-4<x<0$ and $x>4$

7 A team of six students is to be formed from a class of ten students.
How many different teams can be formed if two particular students cannot both be selected for the team?

(A) 252

(B) 210

(C) 168

(D) 140

8 A particle is moving in simple harmonic motion along a straight line according to the equation $v^2=-x^2+2x+8$, where v is its velocity and x its displacement. What is the amplitude of the motion?

(A) 2π metres

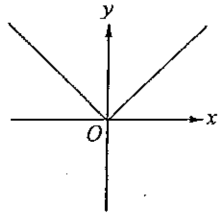
(B) 3 metres

(C) 8 metres

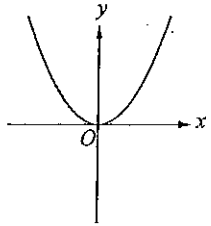
(D) 9 metres

9 What is the best representation of the graph of $y = x\tan^{-1}x$?

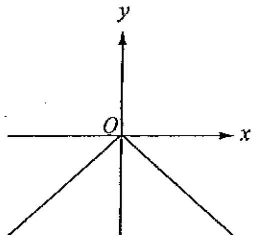
(A)



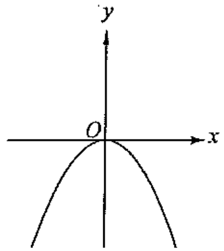
(B)



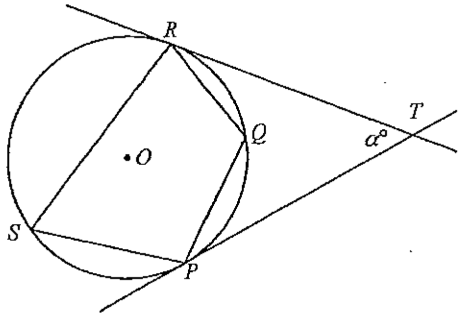
(C)



(D)



10 The points P, Q, R and S lie on a circle with centre at O.
The tangents at P and R meet at the point T and $\angle RTP = \alpha°$.



What is the size of $\angle PQR$ in terms of α?

(A) $(180 - \frac{\alpha}{2})°$

(B) $(180 - \alpha)°$

(C) $(90 + \frac{\alpha}{2})°$

(D) $(90 + \alpha)°$

End of Section I

Section II

60 marks
Attempt Questions 11 - 14
Allow about 1 hour and 45 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11-14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet.

(a) Solve $|x-3|>2x$. **2**

(b) A bag contains a large number of coloured marbles. The ratio of red marbles to other colours is 1:4.

Five marbles are selected at random from the bag.

(i) What is the probability that at least one of the marbles is red? **1**

(ii) What is the probability that exactly four marbles are red? **1**

(c) The equation $\cos 2x = e^{-x}$ has a root near $x = 0.4$. **2**
Taking $x = 0.4$ as a first approximation, use Newton's method to find a second approximation to the root of the equation.
Give your answer correct to two significant figures.

(d) Let $f(x)=\dfrac{1}{\sqrt{1+x^2}}$ for $x \le 0$. **3**
Find an expression for the inverse function $f^{-1}(x)$ in terms of x.

(e) Find all the solutions to the equation $\cos\theta = \sin 2\theta$ for $0 \le \theta \le 2\pi$. **3**

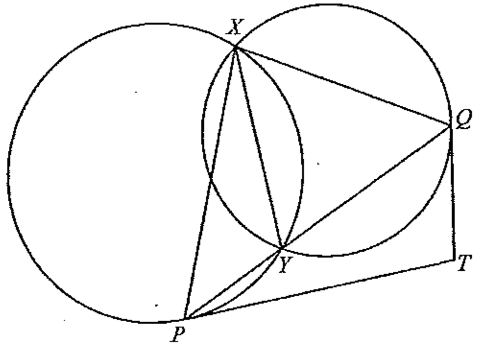
(f) Using $\cos 2\theta = \cos^2\theta - \sin^2\theta$, show that $\sin\dfrac{\pi}{8} = \dfrac{1}{2}\sqrt{2-\sqrt{2}}$. **3**

End of Question 11

Question 12 (15 marks) Use a SEPARATE writing booklet.

(a) Use mathematical induction to prove that $5^n - 2^n$ is divisible by 3 for all integers $n \ge 1$. **3**

(b) Two circles intersect at X and Y. **3**
The chord PY is produced to meet the second circle at Q.
Tangents at P and Q intersect at T.



Copy or trace the diagram to your answer booklet.

Prove that $PXQT$ is a cyclic quadrilateral.

(c) Use the substitution $u = t^2 + 2$ to find $\int t^3\sqrt{t^2+2}\,dt$. **3**

(d) Consider the function $f(x)=\dfrac{x}{1-x^2}$.

(i) Show that the function is increasing for all values of x in its domain. **2**

(ii) Hence sketch the graph of $y = f(x)$ showing the intercepts on the axes and the equations of any asymptotes. **2**

(iii) Find the value(s) of k such that the equation $\dfrac{x}{1-x^2} = kx$ has three real, distinct roots. **2**

End of Question 12

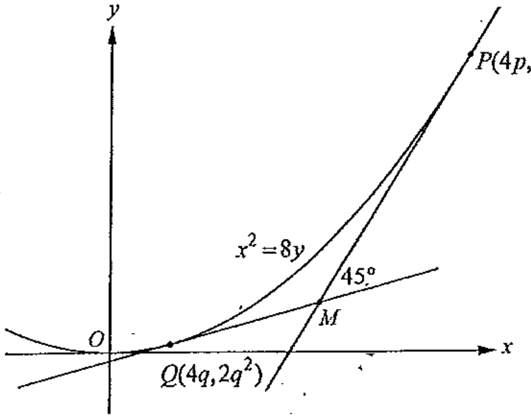
Question 13 (15 marks) Use a SEPARATE writing booklet.

(a) An oil slick in the shape of a circle is spreading across a lake, such that its radius is increasing at a rate of 0.1 m/s. **2**
Find the radius of the oil slick when its area is increasing at a rate of $2\pi \text{ m}^2/\text{s}$.

(b) (i) Find the derivative of $x\tan^{-1}x$. **1**

(ii) Hence find $\int_0^1 \tan^{-1}x\,dx$. **3**

(c) The points $P(4p,2p^2)$ and $Q(4q,2q^2)$ lie on the parabola $x^2=8y$.
The tangents at P and Q intersect at M. The acute angle between these tangents is $45°$.



(i) Find the coordinates of M. **2**

(ii) Show that $p-q=1+pq$. **2**

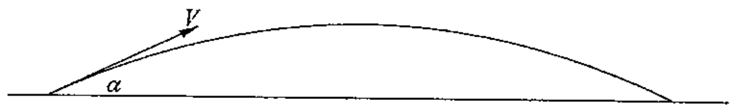
(iii) Show that the equation of the locus of M is $x^2-y^2-12y-4=0$. **3**

(d) Four couples, each consisting of a male and a female, sit around a circular table. **2**
In how many different ways is it possible to seat the four couples so that males and females sit in alternating positions and nobody sits next to his or her partner?

End of Question 13

Question 14 (15 marks) Use a SEPARATE writing booklet.

(a) A particle is projected from horizontal ground with an angle of projection α with a speed of $V\text{ms}^{-1}$. Assume that there is no air resistance and acceleration due to gravity is $g\text{ ms}^{-2}$.



(i) Show that the equation for the path of the particle is $y=x\tan\alpha-\frac{gx^2}{2V^2}\sec^2\alpha$. **3**

(ii) The particle hits a target on the same horizontal ground at a distance R m from the point of projection. **2**

Show that $\tan^2\alpha-(\frac{2V^2}{Rg})\tan\alpha+1=0$.

(iii) Suppose that two particles are projected at different angles α_1 and α_2 from the same point at $V\text{ ms}^{-1}$ and hit the same target R m away on the ground. **2**

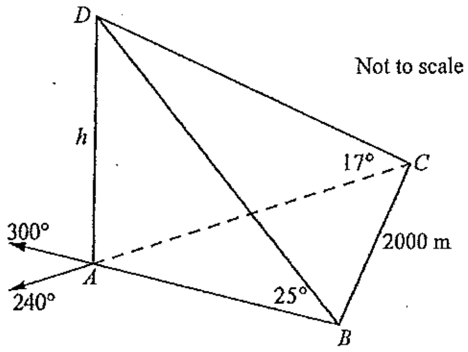
Show that the two angles α_1 and α_2 are complementary.

Question 14 continues on page 11

Question 14 continued

(b) The diagram below shows Belinda standing at B on level ground, whilst Carrie is standing 2000 m away at C on the same level ground. They both take the bearing and elevation of an aeroplane D at the same instant.

Belinda finds the bearing is $300°$ and the angle of elevation $25°$, whilst Carrie finds the bearing to be $240°$ and the angle of elevation $17°$.



(i) Show that if the height DA of the aeroplane is h metres, then **3**

$$h = \frac{2000}{\sqrt{\cot^2 25° + \cot^2 17° - \cot 25° \cot 17°}}$$

(ii) Hence find the value of h correct to 3 significant figures. **1**

(c) Consider the expansion of $(1+x)^{2n} = {}^{2n}C_0 + {}^{2n}C_1 x + {}^{2n}C_2 x^2 + \ldots + {}^{2n}C_{2n} x^{2n}$.

(i) Show that ${}^{2n}C_0 + {}^{2n}C_1 + {}^{2n}C_2 + \ldots + {}^{2n}C_{2n} = 4^n$. **1**

(ii) Hence, show that ${}^{2n}C_0 + {}^{2n}C_1 + {}^{2n}C_2 + \ldots + {}^{2n}C_n = 2^{2n-1} + \dfrac{(2n)!}{2(n!)^2}$. **3**

End of Paper

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CATHOLIC SECONDARY SCHOOLS ASSOCIATION OF NSW
2017 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION
MATHEMATICS EXTENSION 1 - MARKING GUIDELINES

Section I
10 marks
Questions 1-10 (1 mark each)
Question 1 (1 mark)
Outcomes Assessed: PE 3
Targeted Performance Bands: E2

Solution	Mark
Using $x = \dfrac{mx_2 + nx_1}{m+n}$ and $y = \dfrac{my_2 + ny_1}{m+n}$, $x = \dfrac{2\times7+1\times1}{3}, y = \dfrac{2\times5+1\times2}{3}$ $x = 5, y = 4$ Hence (C)	1

Question 2 (1 mark)
Outcomes Assessed: PE3
Targeted Performance Bands: E2

Solution	Mark
$2x - y - 2 = 0$ gradient $m_1 = 2$ $3x - y + 1 = 0$ gradient $m_2 = 3$ Let θ be the angle between the lines $\tan\theta = \left\|\dfrac{m_1 - m_2}{1 + m_1 m_2}\right\|$ $= \left\|\dfrac{2-3}{1+2\times3}\right\|$ $= \dfrac{1}{7}$ $\theta = 8.13°$ Hence (A)	1

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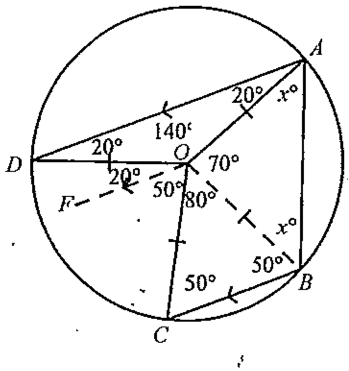
Question 3 (1 mark)
Outcomes Assessed: PE3
Targeted Performance Bands: E2/3

Solution	Mark
Product of roots is $-\frac{d}{a}=-24$ which eliminates A and C. Sum of roots two at a time is $\frac{c}{a}=-26$ which is for (B) $1\times-6+1\times4+-6\times4=-26$ Hence (B)	1

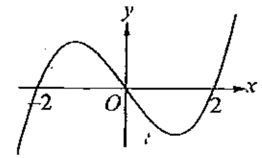
Question 4 (1 mark)
Outcomes Assessed: PE3
Targeted Performance Bands: E2

Solution	Mark
Construct $OF \parallel AD$ Join O to B $\angle DOC = 20° + 50° = 70°$ $\angle DOA = 180° - 20° - 20° = 140°$ $\angle COB = 180° - 50° - 50° = 80°$ $\angle BOA = 360° - 80° - 70° - 140° = 70°$ $2x° + 70° = 180°$ $2x° = 110°$ $x = 55$ Hence (D) 	1

Question 5 (1 mark)
Outcomes Assessed: HE4
Targeted Performance Bands: E3

Solution	Mark
$\cos 2A = \cos^2 A - \sin^2 A$ $= 2\cos^2 A - 1$ $\cos^2 A = \frac{1}{2}(1+\cos 2A)$ $\therefore \cos^2 \frac{2x}{5} = \frac{1}{2}(1+\cos\frac{4x}{5})$ $\int \cos^2 \frac{2x}{5}\,dx = \frac{1}{2}\int (1+\cos\frac{4x}{5})\,dx$ $= \frac{1}{2}(x+\frac{5}{4}\sin\frac{4x}{5})+C$ $= \frac{x}{2}+\frac{5}{8}\sin\frac{4x}{5}+C$ Hence (A)	1

Question 6 (1 mark)
Outcomes Assessed: PE3
Targeted Performance Bands: E2/3

Solution	Mark
$\frac{x^2-4}{2x} > 0,\quad x \neq 0$ Multiply both sides by x^2 $\frac{x(x-2)(x+2)}{2} > 0$ From the graph, solutions are $-2 < x < 0$ and $x > 2$ Hence (A) 	1

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Question 7 (1 mark)
Outcomes Assessed: PE3
Targeted Performance Bands: E3

Solution	Mark
Number of teams without restriction = $^{10}C_6$ $=210$ Number of teams containing both students = $^{8}C_4$ $=70$ $\therefore$ Required number of teams = $210-70$ $=140$ Hence (D)	1

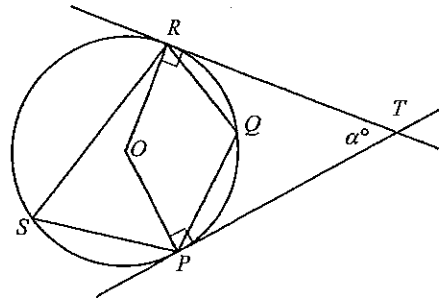
Question 8 (1 mark)
Outcomes Assessed: HE3
Targeted Performance Bands: E3

Solution	Mark
$v^2=-x^2+2x+8$ At endpoints, $v=0$ $x^2-2x-8=0$ $(x-4)(x+2)=0$ $\therefore$ Endpoints are at $x=-2$ and $x=4$. Centre is at $x=1$. $\therefore$ Amplitude is 3 Hence (B)	1

Question 9(1 mark)
Outcomes Assessed: PE3
Targeted Performance Bands: E3

Solution	Mark
When $x\geq 0, y\geq 0$ and when $x<0, y>0$ which eliminates (C) and (D) When $x>0$, y does not increase at a constant rate. Hence (B)	1

Question 10 (1 mark)
Outcomes Assessed: PE3
Targeted Performance Bands: E3

Solution	Mark
 $\angle TRO=\angle TPO=90°$ (Tangent perpendicular to radius at point of contact) $\therefore TROP$ is a cyclic quadrilateral (opposite angles supplementary) $\angle ROP+\alpha=180$ (opposite angles of cyclic quadrilateral are supplementary) $\therefore \angle ROP=180-\alpha$ $\angle ROP=2\times\angle RSP$ (angle at centre is twice angle at circumference on the same arc) $\therefore \angle RSP=\frac{1}{2}(180-\alpha)$ $=90-\frac{\alpha}{2}$ $SRQP$ is a cyclic quadrilateral. $\therefore \angle RQP+\angle RSP=180$ $\therefore \angle RQP=180-(90-\frac{\alpha}{2})$ $=90+\frac{\alpha}{2}$ Hence (C)	1

Section II
60 marks

Question 11 (15 marks)
(a) (2 marks)
Outcomes Assessed: PE3
Targeted Performance Bands: E2/3

Criteria	Marks
* Correct solution	2
* Obtains the solution of $x=1$	1

Sample Answer:

From the graph, $|x-3|>2x$ when $x<a$.

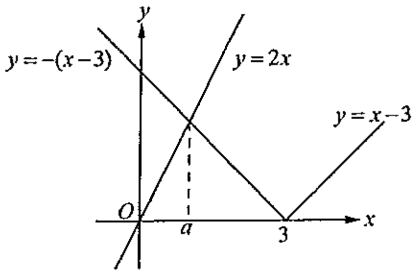
Solve $-(x-3)=2x$ to find a.

$-x+3=2x$

$3x=3$

$x=1$

$\therefore$ Solution is $x<1$.



11 (b) (i) (1 mark)
Outcomes assessed: PE3
Targeted Performance Bands: E2/3

Criteria	Marks
* Obtains correct answer	1

Sample Answer:

$P(\geq 1 \text{ red marble}) = 1 - P(\text{all other colours})$

$$= 1-\left(\frac{4}{5}\right)^5$$

$$= \frac{2105}{3125}$$

11 (b) (ii) (1 mark)
Outcomes assessed: PE3
Targeted Performance Bands: E2/3

Criteria	Marks
* Obtains correct answer	1

Sample Answer:

$$P(\text{exactly 4 red}) = {}^5C_4\left(\frac{4}{5}\right)^1\left(\frac{1}{5}\right)^4$$

$$= \frac{4}{625}$$

Question 11 (continued)
(c) (2 marks)
Outcomes assessed: HE1
Targeted Performance Bands: E2/3

Criteria	Marks
* Correct solution	2
* Obtains correct derivative of $f(x)$	1

Sample Answer:

$f(x)=\cos 2x - e^{-x}$

$f'(x) = -2\sin 2x + e^{-x}$

$x_0 = 0.4$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 0.4 - \frac{(\cos .8 - e^{-.4})}{(e^{-.4} - 2\sin .8)}$$

$= 0.43452..$

$\therefore$ Second approximation is 0.43 to two significant figures.

Question 11 (continued)
11 (d) (3 marks)
Outcomes Assessed: HE4
Targeted Performance Bands: E3/4

Criteria	Marks
* Correct solution	3
* Obtains $y = \pm\sqrt{\frac{1-x^2}{x^2}}$	2
* Interchanges x and y in the function	1

Sample Answer:

$f(x) = \frac{1}{\sqrt{1+x^2}}$ for $x \le 0$

Put $y = \frac{1}{\sqrt{1+x^2}}, x \le 0, y \ge 1$

Inverse is $x = \frac{1}{\sqrt{1+y^2}}, x \ge 1, y \le 0$ (1)

$\sqrt{1+y^2} = \frac{1}{x}$

$1+y^2 = \frac{1}{x^2}$

$y^2 = \frac{1}{x^2} - 1$

$y^2 = \frac{1-x^2}{x^2}$

$y = \pm\sqrt{\frac{1-x^2}{x^2}}$

From (1), range is negative

Hence $f^{-1}(x) = -\sqrt{\frac{1-x^2}{x^2}}$

Question 11 (continued)
11 (e) (3 marks)
Outcomes Assessed: HE4
Targeted Performance Bands: E3

Criteria	Marks
* Correct solution	3
* Substantial progress towards solution	2
* Replaces $\sin 2\theta$ with $2\sin\theta\cos\theta$	1

Sample Answer:

$\cos\theta = \sin 2\theta$

$\cos\theta = 2\sin\theta\cos\theta$

$\cos\theta - 2\sin\theta\cos\theta = 0$

$\cos\theta(1 - 2\sin\theta) = 0$

$\cos\theta = 0,\ \sin\theta = \frac{1}{2}$

$\therefore \theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$

(f) (3 marks)
Outcomes Assessed: HE4
Targeted Performance Bands: E3/4

Criteria	Marks
* Correct solution	3
* Obtains $\sin\frac{\pi}{8} = \pm\frac{1}{2}\sqrt{2-\sqrt{2}}$	2
* Obtains correct expression for $\cos(\frac{\pi}{4})$	1

Sample Answer:

$\cos(\frac{\pi}{4}) = \cos^2\frac{\pi}{8} - \sin^2\frac{\pi}{8}$

$\frac{1}{\sqrt{2}} = 1 - 2\sin^2\frac{\pi}{8}$

$2\sin^2\frac{\pi}{8} = 1 - \frac{\sqrt{2}}{2}$

$\sin^2\frac{\pi}{8} = \frac{2-\sqrt{2}}{4}$

$\sin\frac{\pi}{8} = \pm\frac{1}{2}\sqrt{2-\sqrt{2}}$

$\frac{\pi}{8}$ is in first quadrant so $\sin\frac{\pi}{8} > 0$

Hence $\sin\frac{\pi}{8} = \frac{1}{2}\sqrt{2-\sqrt{2}}$

Question 12 (15 marks)

12 (a) (3 marks)
Outcomes Assessed: HE2
Targeted Performance Bands: E3/4

Criteria	Marks
* Correct solution	3
* Demonstrates significant progress towards solution	2
* Shows it is true for $n = 1$	1

Sample Answer:

Let $P(n)$ be the proposition that $5^n - 2^n$ is divisible by 3.

When $n = 1$, $5^n - 2^n = 5 - 2 = 3$

$\therefore P(n)$ is true for $n = 1$.

Assume that $P(n)$ is true for $n = k$.

ie assume that $5^k - 2^k = 3Q$, where Q is an integer.

Need to show that $P(n)$ is true for $n = k+1$.

ie Need to show that $5^{k+1} - 2^{k+1} = 3R$, where R is an integer.

$5^{k+1} - 2^{k+1} = 5 \times 5^k - 2^{k+1}$

From assumption, $5^k = 3Q + 2^k$

$$\begin{aligned}\therefore 5^{k+1} - 2^{k+1} &= 5 \times (3Q + 2^k) - 2^{k+1} \\ &= 15Q + 5 \times 2^k - 2 \times 2^k \\ &= 15Q + 3 \times 2^k \\ &= 3(5Q + 2^k), \text{ which is divisible by 3.}\end{aligned}$$

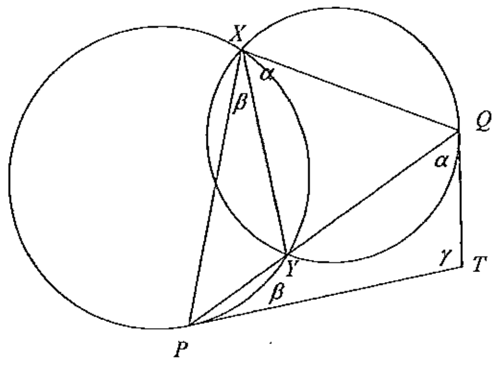
$\therefore$ If $P(n)$ is true for $n = k$ then $P(n)$ is true for $n = k+1$.

Hence by mathematical induction $P(n)$ is true for all integers $n \geq 1$.

Question 12 (continued)
12 (b) (3 marks)
Outcomes Assessed: PE3
Targeted Performance Bands: E3

Criteria	Marks
* Correct solution	3
* Demonstrates significant progress towards solution	2
* Explains why $\angle TQP = \angle QXY$	1

Sample Answer:



$\angle TQP = \angle QXY = \alpha$ (angle between tangent and chord equals angle in alternate segment)

$\angle TPQ = \angle PXY = \beta$ (same reason)

Let $\angle PTQ = \gamma$

$\alpha + \beta + \gamma = 180°$ (angle sum in ΔPTQ)

$\angle PXQ = \angle PXY + \angle QXY = \alpha + \beta$

$$\begin{aligned}\angle PXQ + \angle PTQ &= (\alpha + \beta) + \gamma \\ &= 180°\end{aligned}$$

$\therefore PXQT$ is a cyclic quadrilateral (opposite angles are supplementary)

Question 12 (continued)
12 (c) (3 marks)
Outcomes assessed: HE6
Targeted Performance Bands: E3/4

Criteria	Marks
* Correct solution	3
* Demonstrates significant progress towards solution	2
* Shows $du = 2t.dt$	1

Sample Answer:

$u = t^2 + 2$

$\frac{du}{dt} = 2t$

$du = 2t.dt$

$\frac{du}{2} = t.dt$

$\int t^3\sqrt{t^2+2}\ dt$

$= \int t^2\sqrt{t^2+2}\ t\,dt$

$= \frac{1}{2}\int (u-2)\sqrt{u}\ du$

$= \frac{1}{2}\int (u^{\frac{3}{2}} - 2u^{\frac{1}{2}})du$

$= \frac{1}{2}(\frac{2}{5}u^{\frac{5}{2}} - \frac{4}{3}u^{\frac{3}{2}})$

$= \frac{1}{5}(t^2+2)^{\frac{5}{2}} - \frac{2}{3}(t^2+2)^{\frac{3}{2}} + C$

Question 12 (continued)
12 (d) (i) (2 marks)
Outcomes Assessed: HE4
Targeted Performance Bands: E3/4

Criteria	Marks
* Correct solution	2
* Finds correct expression for the derivative of $f(x)$	1

Sample Answer:
Domain of $f(x)$ is all real numbers except ± 1.

$f(x) = \frac{x}{1-x^2}$

$f'(x) = \frac{(1-x^2)\times 1 - x\times -2x}{(1-x^2)^2}$

$= \frac{1-x^2+2x^2}{(1-x^2)^2}$

$= \frac{1+x^2}{(1-x^2)^2}$

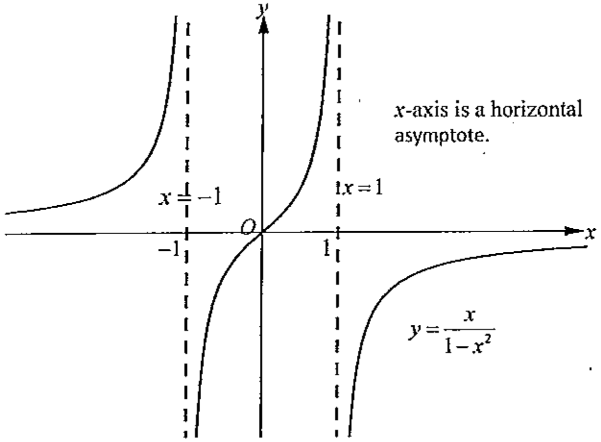
Since $x^2+1>0$ and $(1-x^2)^2>0$ for all x in the domain then $f(x)$ is increasing for all x in its domain.

Question 12 (continued)

12 (d) (ii) (2 marks)
Outcomes Assessed: HE4
Targeted Performance Bands: E3

Criteria	Marks
* Correct solution	2
* Sketches correctly $y = f(x)$ without correct labelling	1

Sample Answer:



Question 12 (continued)

12 (d) (iii) (2 marks)
Outcomes Assessed: HE4
Targeted Performance Bands: E3/4

Criteria	Marks
* Correct solution	2
* Finds either $k < 0$ or $k > 1$	1

Sample Answer:

$\frac{x}{1-x^2} = kx$

$x = kx(1-x^2)$

$kx^3 + (1-k)x = 0$

$x(kx^2 + 1 - k) = 0$

Since $x = 0$ is a root then $kx^2 + 1 - k = 0$ has 2 distinct real roots if $\Delta > 0$.

$\therefore$ Using $b^2 - 4ac > 0$,

$0 - 4 \times k \times (1-k) > 0$

$-4k(1-k) > 0$

which gives $k < 0$ or $k > 1$.

Question 13 (15 marks)

13 (a) (2 marks)
Outcomes Assessed: HE3
Targeted Performance Bands: E2/3

Criteria	Marks
* Correct solution	2
* Finds correctly $\frac{dA}{dr}$	1

$A = \pi r^2$.

Given $\frac{dr}{dt} = 0.1$

Need to find r when $\frac{dA}{dt} = 2\pi$.

$\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$

$2\pi = 2\pi r \times 0.1$

$0.1r = 1$

$r = 10$.

$\therefore$ Radius is 10 m when area is increasing at 2π m^2/s.

13 (b) (i) (1 mark)
Outcomes Assessed: HE4
Targeted Performance Bands: E2/3

Criteria	Marks
* Correct solution	1

Sample Answer:

$$\frac{d}{dx}(x\tan^{-1}x) = \tan^{-1}x + x.\frac{1}{1+x^2}$$
$$= \tan^{-1}x + \frac{x}{1+x^2}$$

Question 13 (continued)

13 (b) (ii) (3 marks)
Outcomes Assessed: HE4
Targeted Performance Bands: E3

Criteria	Marks
* Correct solution	3
* Demonstrates significant progress towards answer	2
* Finds correctly the primitive of $\frac{x}{1+x^2}$	1

Sample Answer:

$$\frac{d}{dx}(x\tan^{-1}x) = \tan^{-1}x + \frac{x}{1+x^2}$$

$$\therefore x\tan^{-1}x = \int (\tan^{-1}x + \frac{x}{1+x^2})dx$$

$$\therefore \int_0^1 \tan^{-1}x\,dx = \left[x\tan^{-1}x\right]_0^1 - \int_0^1 \frac{x}{1+x^2}\,dx$$
$$= \left[x\tan^{-1}x\right]_0^1 - \frac{1}{2}\int_0^1 \frac{2x}{1+x^2}\,dx$$
$$= \left[x\tan^{-1}x - \frac{1}{2}\log_e(1+x^2)\right]_0^1$$
$$= (\tan^{-1}1 - \frac{1}{2}\log_e 2)$$
$$= \frac{\pi}{4} - \frac{1}{2}\log_e 2$$

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Question 13 (continued)

13 (c) (i) (2 marks)
Outcomes Assessed: PE3
Targeted Performance Bands: E3

Criteria	Marks
* Correct solution	2
* Finds correctly x or y co-ordinate of M	1

Sample Answer:

Tangents at P and Q are $y = px - 2p^2$ ......(1)

$y = qx - 2q^2$ ......(2)

(1) − (2) $\quad 0 = (p-q)x - 2(p^2 - q^2)$

$(p-q)x = 2(p-q)(p+q)$

$x = 2(p+q)$

Sub into (1) $\quad y = p \times 2(p+q) - 2p^2$

$= 2p^2 + 2pq - 2p^2$

$= 2pq$

Hence M is $(2(p+q), 2pq)$

13 (c) (ii) (2 marks)
Outcomes Assessed: PE3
Targeted Performance Bands: E3

Criteria	Marks
* Correct solution	2
* Obtains correctly $\tan 45° = \dfrac{p-q}{1+pq}$	1

Sample Answer:

Using $\tan\theta = \dfrac{m_1 - m_2}{1 + m_1 m_2}$ where $\theta = 45°$, $m_1 = p$, $m_2 = q$

then $\tan 45° = \dfrac{p-q}{1+pq}$

$\therefore 1 = \dfrac{p-q}{1+pq}$

$\therefore 1 + pq = p - q$ as required.

13 (c) (iii) (3 marks)
Outcomes Assessed: PE3
Targeted Performance Bands: E3/4

Criteria	Marks
* Correct solution	3
* Demonstrates significant progress towards the solution	2
* Demonstrates some correct working	1

Sample Answer:

At M, $x = 2(p+q) \ \rightarrow \ \dfrac{x}{2} = p + q$

$y = 2pq \ \rightarrow \ \dfrac{y}{2} = pq$

From $1 + pq = p - q$

$1 + \dfrac{y}{2} = p - q$

Squaring, $1 + y + \dfrac{y^2}{4} = p^2 - 2pq + q^2$

$1 + y + \dfrac{y^2}{4} = (p+q)^2 - 2pq - 2pq$

$1 + y + \dfrac{y^2}{4} = (\dfrac{x}{2})^2 - 4 \times \dfrac{y}{2}$

$1 + y + \dfrac{y^2}{4} = \dfrac{x^2}{4} - 2y$

$4 + 4y + y^2 = x^2 - 8y$

$\therefore$ Equation of locus of M is $x^2 - y^2 - 12y - 4 = 0$, as required.

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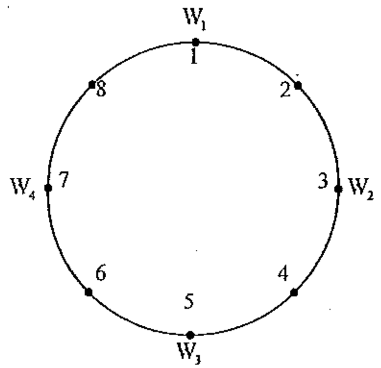
Question 13 (continued)
13 (d) (2 marks)
Outcomes Assessed: HE3
Targeted Performance Bands: E3-E4

Criteria	Marks
* Correct solution	2
* Demonstrates some correct working	1

Sample Answer:



Women can be seated in 3! ways.
Suppose that W_2 is seated in position 3.
The partner of W_2 has only 2 options (position 6 or position 8) since he cannot sit next to his partner.
Suppose that the partner of W_2 sits at position 8.
All other positions are now fixed.
Hence the number of seating arrangements is $3!\times 2 = 12$.

Question 14 (15 marks)

14 (a) (i) (3 marks)
Outcomes Assessed: HE3
Targeted Performance Bands: E3/4

Criteria	Marks
* Correct solution	3
* Demonstrates significant progress towards the solution	2
* Finds correct expressions for $\dot{x}$ and $\dot{y}$	1

Sample Answer:

Horizontal

$\ddot{x} = 0$
$\dot{x} = c_1$
When $t = 0,\ \dot{x} = V\cos\alpha$
$\therefore c_1 = V\cos\alpha$
$\dot{x} = V\cos\alpha$
$x = Vt\cos\alpha + c_2$
When $t = 0, x = 0$
$x = Vt\cos\alpha$

Vertical

$\ddot{y} = -g$
$\dot{y} = -gt + c_3$
When $t = 0, \dot{y} = V\sin\alpha$
$\dot{y} = -gt + V\sin\alpha$

$$y = Vt\sin\alpha - \frac{gt^2}{2} + c_4$$

When $t = 0, y = 0$

$$y = Vt\sin\alpha - \frac{gt^2}{2}$$

But $t = \dfrac{x}{V\cos\alpha}$

$$\text{So, } y = \frac{Vx\sin\alpha}{V\cos\alpha} - \frac{g}{2}\left(\frac{x}{V\cos\alpha}\right)^2$$

$$y = x\tan\alpha - \frac{gx^2}{2V^2}\sec^2\alpha$$

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Question 14 (continued)
14 (a) (ii) (2 marks)
Outcomes Assessed: HE3
Targeted Performance Bands: E3/4

Criteria	Marks
* Correct solution	2
* Substitutes correctly $x = R$ and $y = 0$	1

Sample Answer:

Substituting $x = R, y = 0$ into $y = x\tan\alpha - \dfrac{gx^2}{2V^2}\sec^2\alpha$ we obtain

$$R\tan\alpha - \frac{gR^2}{2V^2}\sec^2\alpha = 0$$

$$R\tan\alpha - \frac{gR^2}{2V^2}(1+\tan^2\alpha) = 0$$

$$2V^2R\tan\alpha - gR^2 - gR^2\tan^2\alpha = 0$$

Dividing by $(-gR^2)$ we get $\tan^2\alpha - \dfrac{2V^2}{Rg}\tan\alpha + 1 = 0$, as required.

14 (a) (iii) (2 marks)
Outcomes Assessed: HE3
Targeted Performance Bands: E3/4

Criteria	Marks
* Correct solution	2
* Demonstrates significant progress towards solution	1

Sample Answer:

The roots of $\tan^2\alpha - \dfrac{2V^2}{Rg}\tan\alpha + 1 = 0$ are $\tan\alpha_1$ and $\tan\alpha_2$.

Using the product of the roots, $\tan\alpha_1\tan\alpha_2 = 1$.

Using $\tan(\alpha_1+\alpha_2) = \dfrac{\tan\alpha_1 + \tan\alpha_2}{1 - \tan\alpha_1\tan\alpha_2}$

$$= \frac{\tan\alpha_1 + \tan\alpha_2}{1-1}$$

Hence $\alpha_1 + \alpha_2 = 90°$ since $\tan 90°$ is undefined.

$\therefore \alpha_1$ and α_2 are complementary.

Question 14 (continued)
14 (b) (i) (3 marks)
Outcomes Assessed: HE3
Targeted Performance Bands: E3/4

Criteria	Marks
* Correct solution	3
* Applies cosine rule correctly	2
* Finds correctly either AB or AC	1

Sample Answer:

$\tan 25° = \dfrac{h}{AB} \rightarrow AB = \dfrac{h}{\tan 25°} = h\cot 25°$

$\tan 17° = \dfrac{h}{AC} \rightarrow AC = \dfrac{h}{\tan 17°} = h\cot 17°$

$\angle CAB = 60°$

Using the cosine rule in ΔBAC

$$2000^2 = h^2\cot^2 25° + h^2\cot^2 17° - 2\times h\cot 25° \times h\cot 17° \times \cos 60°$$

$$2000^2 = h^2\cot^2 25° + h^2\cot^2 17° - 2h^2\cot 25°\cot 17° \times \frac{1}{2}$$

$$2000^2 = h^2\cot^2 25° + h^2\cot^2 17° - h^2\cot 25°\cot 17°$$

$$2000^2 = h^2(\cot^2 25° + \cot^2 17° - \cot 25°\cot 17°)$$

$$h^2 = \frac{2000^2}{\cot^2 25° + \cot^2 17° - \cot 25°\cot 17°}$$

$$h = \frac{2000}{\sqrt{\cot^2 25° + \cot^2 17° - \cot 25°\cot 17°}}, \text{ as required.}$$

14 (b) (ii) (1 mark)
Outcomes Assessed: HE3
Targeted Performance Bands: E2-E3

Criteria	Marks
* Correct answer	1

Sample Answer:

$$h = \frac{2000}{\sqrt{\cot^2 25° + \cot^2 17° - \cot 25°\cot 17°}}$$

$$= \frac{2000}{\sqrt{\tan^2 65° + \tan^2 73° - \tan 65°\tan 73°}}$$

$= 695$ m, to 3 significant figures.

Question 14 (continued)

14 (c) (i) (1 mark)

Outcomes Assessed: HE3

Targeted Performance Bands: E3

Criteria	Marks
* Correct solution	1

Sample Answer:

Using the expansion and substitution $x = 1$,

$(1+1)^{2n} = {}^{2n}C_0 + {}^{2n}C_1(1) + {}^{2n}C_2(1)^2 + ... + {}^{2n}C_{2n}(1)^{2n}$

$2^{2n} = {}^{2n}C_0 + {}^{2n}C_1(1) + {}^{2n}C_2(1)^2 + ... + {}^{2n}C_{2n}(1)^{2n}$

or ${}^{2n}C_0 + {}^{2n}C_1(1) + {}^{2n}C_2(1)^2 + ... + {}^{2n}C_{2n}(1)^{2n} = 4^n$

14(c) (ii) (3 marks)

Outcomes Assessed: HE3

Targeted Performance Bands: E4

Criteria	Marks
* Correct solution	3
* Demonstrates significant progress towards solution	2
* Demonstrates some correct working	1

Sample Answer:

From $2^{2n} = {}^{2n}C_0 + {}^{2n}C_1 + {}^{2n}C_2 + ... + {}^{2n}C_{2n}$

$2^{2n} = ({}^{2n}C_0 + {}^{2n}C_1 + {}^{2n}C_2 + ... + {}^{2n}C_n) + {}^{2n}C_{n+1} + {}^{2n}C_{n+2} + ... + {}^{2n}C_{2n}$

Add ${}^{2n}C_n$ to both sides.

$2^{2n} + {}^{2n}C_n = ({}^{2n}C_0 + {}^{2n}C_1 + {}^{2n}C_2 + ... + {}^{2n}C_n) + {}^{2n}C_n + {}^{2n}C_{n+1} + {}^{2n}C_{n+2} + ... + {}^{2n}C_{2n}$

$= 2 \times ({}^{2n}C_0 + {}^{2n}C_1 + {}^{2n}C_2 + ... + {}^{2n}C_n)$, since ${}^{2n}C_r = {}^{2n}C_{n-r}$

$\therefore {}^{2n}C_0 + {}^{2n}C_1 + {}^{2n}C_2 + ... + {}^{2n}C_n = \frac{1}{2}(2^{2n} + {}^{2n}C_n)$

$= \frac{1}{2} \times 2^{2n} + \frac{1}{2}(\frac{(2n)!)}{n!(2n-n)!})$

$= 2^{2n-1} + \frac{(2n)!}{2n!n!}$

$\therefore {}^{2n}C_0 + {}^{3n}C_1 + {}^{2n}C_2 + ... + {}^{2n}C_n = 2^{2n-1} + \frac{(2n)!}{2(n!)^2}$