Student Number

2017
TRIAL HIGHER SCHOOL CERTIFICATE
EXAMINATION

Mathematics

Morning Session
Thursday, 3 August 2017

General Instructions

- Reading time – 5 mins
- Working time – 3 hours
- Write using black pen
- Use Multiple Choice Answer Sheet provided
- NESA-approved calculators may be used
- A reference sheet is provided on a SEPARATE sheet
- In Questions 11-16, show relevant mathematical reasoning and/or calculations
- Write your Centre Number and Student Number at the top of this page

Total marks - 100

Section I	Pages 2 - 5

10 marks
- Attempt Questions 1 - 10
- Allow about 15 minutes for this section

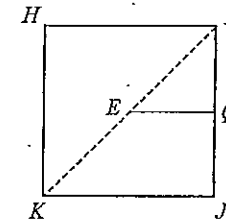| Section II	Pages 6 - 15

90 marks
- Attempt Questions 11 - 16
- Allow about 2 hours and 45 minutes for this section

6200-1

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the Multiple-Choice Answer Sheet for Questions 1–10.

---

1   *HIJK* is a square with diagonal *IK*. The line *EQ* is the perpendicular bisector of the line *IJ*.

What is the value of $\angle KEQ$?

(A)   45°

(B)   120°

(C)   135°

(D)   150°

2   Which expression is the complete factorisation of $2x^2 - 32$?

(A) $2(x^2 - 16)$

(B) $2x(x - 16)$

(C) $(2x - 16)(2x + 16)$

(D) $2(x + 4)(x - 4)$

3 A table of values is constructed to help sketch the curve of $y = f(x)$.

x	0	2	4	6	8
$f(x)$	7	9	14	4	−3

Given that $f(x)$ is continuous over the domain $0 \le x \le 8$, using Simpson's Rule with

five function values, what is an estimate of $\int_0^8 f(x)\ dx$?

(A) 28

(B) 40

(C) 56

(D) 80

4 What is the domain of $f(x) = \log_e(1+x)$?

(A) $x > -1$

(B) $x \ge -1$

(C) $x > 0$

(D) $x \ge 0$

5 The area of the minor segment of the circle pictured below is 100π m^2.
What is the exact radius of this circle?
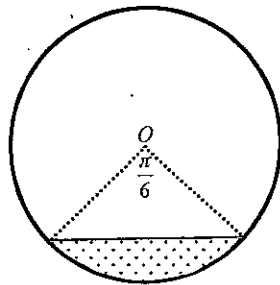
(A) $\sqrt{\dfrac{200\pi(\pi-3)}{6}}$

(B) $\sqrt{\dfrac{1200}{\pi-3}}$

(C) $\sqrt{\dfrac{200(\pi-3)}{\pi}}$

(D) $\sqrt{\dfrac{1200\pi}{\pi-3}}$

6 If the probability that an event is $\dfrac{1}{m}$, which of the following is the probability of the complementary event?

(A) $\dfrac{m}{m-1}$

(B) $\dfrac{m-1}{m}$

(C) $\dfrac{m+1}{m}$

(D) $\dfrac{1-m}{m}$

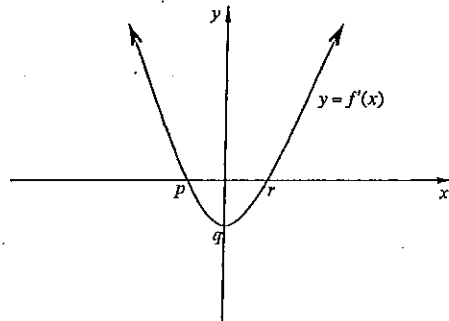7 Consider $f(x) = \left|\dfrac{x}{2}\right| - \dfrac{x}{2}$.
What are the coordinates of the midpoint of the line joining the points $H\left[f(4), f(-4)\right]$ and $K\left[f(6), f(-6)\right]$?

(A) $(0,0)$

(B) $(0,5)$

(C) $(4,6)$

(D) $(5,5)$

8 What is the limiting sum of the geometric series whose n^{th} term is $\dfrac{1}{5^{n-2}}$?

(A) $-\dfrac{5}{2}$

(B) $\dfrac{5}{4}$

(C) $\dfrac{25}{4}$

(D) $\dfrac{25}{2}$

9   The diagram below shows the graph of $y = f'(x)$ for a curve, $y = f(x)$.

Which of the following statements could be true?

(A) $y = f(x)$ could have a maximum turning point at p and a minimum at r.

(B) $y = f(x)$ could have a minimum turning point at p and a maximum at r.

(C) $y = f(x)$ could have a minimum turning point at q only.

(D) $y = f(x)$ could have a maximum turning point at q only.

10   The diagram shows the area under the curve $y = e^x$ from $x = 0$ to $x = a$.
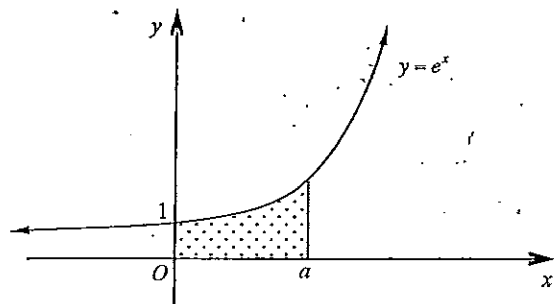
What value of a makes the shaded area equal to 3?

(A) e^3

(B) e^2

(C) $\ln 3$

(D) $2\ln 2$

-5-

90 marks
Attempt Questions 11 - 16
Allow about 2 hours and 45 minutes for this section

Answer each question in a separate writing booklet. Extra writing booklets are available.

In Questions 11 – 16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet.

(a)   Evaluate $\dfrac{e^5}{(-2)^2}$ correct to two significant figures.   1

(b)   Factorise $5x^2 - 2x - 3$.   1

(c)   Solve $|3x - 4| \le 8$ and sketch the solution on a number line.   3

(d)   If $f(x) = 3\sin 4x$ find the exact value of $f'\left(\dfrac{\pi}{6}\right)$.   2

(e)   Find $\displaystyle\int \dfrac{x^3}{x^4 + 7}\ dx$.   2

(f)   Evaluate $\displaystyle\lim_{x \to -1} \dfrac{3(x+1)}{x^2 - 1}$.   2

(g)   The equation $ax^2 - x + b = 0$ has roots -2 and 4.
       Find the value of b.   2

(h)   Solve for x: $5^{x+2} = \dfrac{1}{\sqrt[3]{125}}$.   2

-6-

Question 12 (15 marks) Use a SEPARATE writing booklet.

(a) The graph of $g(x) = \sqrt{2x+1}$ has a tangent, L, with equation $y = ax + b$ as shown in the diagram.

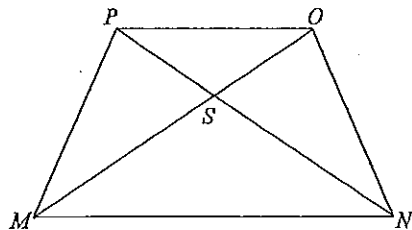The line L makes an angle of $30°$ with the positive x-axis.
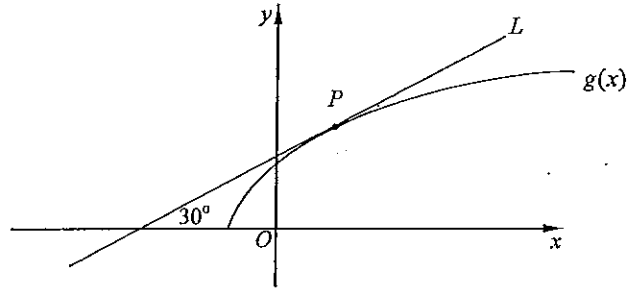
(i) Show that $a = \dfrac{1}{\sqrt{3}}$ 1

(ii) Hence, or otherwise, find the x-coordinate of the point of contact, P.   2

(b) $PONM$ is a quadrilateral with $PS = OS$ and $\angle PMS = \angle ONS$.
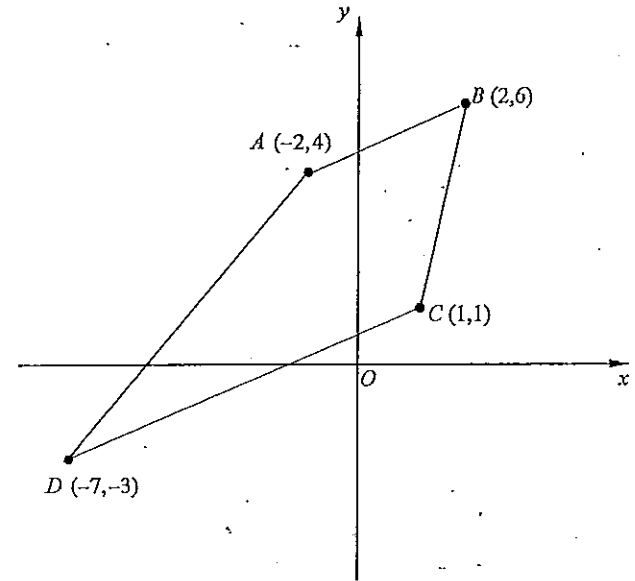

Prove $\triangle MPO \equiv \triangle NOP$.   3

Question 12 (continued)

(c) Solve $2\sin\theta \cot\theta + 1 = 0$ for $0 \le \theta \le 2\pi$.   3

(d) The points $A(-2,4)$, $B(2,6)$, $C(1,1)$ and $D(-7,-3)$ form a quadrilateral as shown. The length of DC is twice that of AB.

(i) Show that AB is parallel to DC.   1

(ii) Hence show the equation of AB is $x - 2y + 10 = 0$.   1

(iii) Show that the perpendicular distance from the line AB to the point C is $\dfrac{9}{\sqrt{5}}$.   2

(iv) Hence determine the area of the quadrilateral $ABCD$.   2

End of Question 12

Question 13 (15 marks) Use a SEPARATE writing booklet.

(a) Find the derivative of each of the following:

    (i) $y = e^x x$ **1**

    (ii) $y = \dfrac{x^2 + 5}{\ln x}$ **2**

(b) Find $\displaystyle\int \dfrac{1}{\sqrt{4x-7}}\,dx$.   **2**

(c) Consider the curve $y = 2x^3 + 3x^2 - 12x + 7$ for $-3 \le x \le 3$.

    (i)   Find the stationary points and determine their nature.   **3**

    (ii)   Find the point of inflexion.   **2**

    (iii) Hence sketch the graph in the given domain, indicating the stationary points and the point of inflexion.   **3**

    (iv) What is the maximum value in the given domain?   **1**

    (v)   Find the values of x for which both $\dfrac{dy}{dx} < 0$ and $\dfrac{d^2 y}{dx^2} > 0$.   **1**

Question 14 (15 marks) Use a SEPARATE writing booklet.

(a) A train wheel of radius 40 cm and centre O rolls along a horizontal track as shown in the diagram below. M is a point on the wheel where the wheel touches the track before it starts to roll.
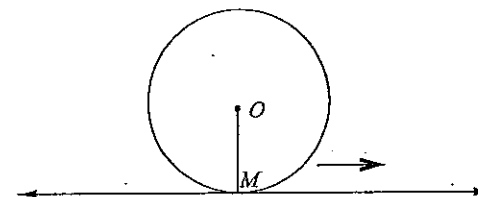
    (i)   Through what angle does M rotate about O in radians after the wheel rolls 1 metre?   **1**

    (ii)   What would be the vertical height of M above the track after the wheel rolls 1 metre?   **3**

Question 14 (continued)

(b) In the diagram, the shaded region is bounded by the x-axis and the curves $y = 2 - x^2$ and $y = x^3$. The curves intersect at the point $R(1,1)$.
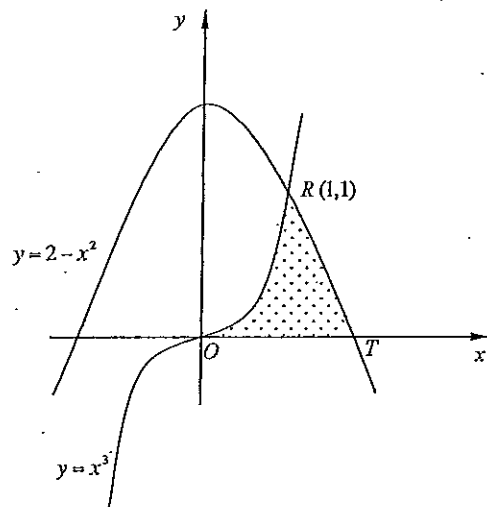
(i) Find the coordinates of T, where the curve $y = 2 - x^2$ meets the x-axis. **1**

(ii) The shaded area is rotated about the x-axis. By considering the sum of two regions find the volume of the solid formed. **3**

(c) Helen has just retired with $500 000 in her account and wishes to start an annuity. She intends to withdraw M at the end of each year for 30 years. Reducible interest is calculated at 6% p.a., compounded annually just before each withdrawal.

(i) Write down an expression for the amount remaining in the annuity after one year. **1**

(ii) Calculate M, the amount of each withdrawal. **3**

(iii) After her 15th withdrawal, the interest rate on the annuity reduces to 4% p.a. Helen is informed that to continue to withdraw M, the life of the annuity must also reduce.

For how many more years will Helen be able to withdraw money from this annuity? **3**

End of Question 14

Question 15 (15 marks) Use a SEPARATE writing booklet.

(a) In the diagram below the shaded region is bounded by the curves $y = \sin 2x$, $y = \cos 2x$ and the line $y = 1$.
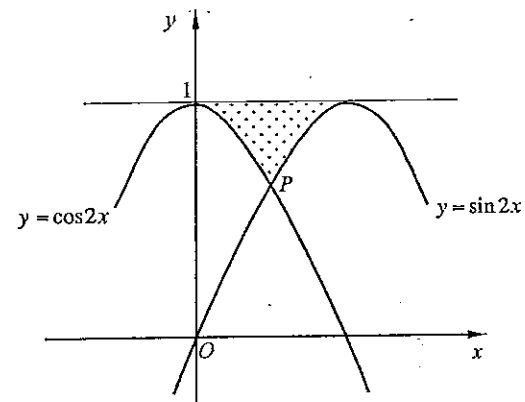
(i) Find the coordinates of the point of intersection P. **2**

(ii) Show that the shaded area is given by $A = \frac{1}{4}\left(\pi - 2\sqrt{2}\right)$ units^2. **3**

(b) Two students are selected at random to lead a discussion.

One person is chosen from class A, which has 2 females and 12 males, and the other is chosen from class B, which has 8 females and 6 males.

(i) Find the probability of choosing two females to lead a discussion. **1**

(ii) Find the probability of choosing one person of each gender to lead a discussion. **2**

Question 15 (continued)

(c)   A drug is used to control a medical condition for a human body.
The graph below shows the amount M (in milligrams) of the drug remaining in the body after t hours.
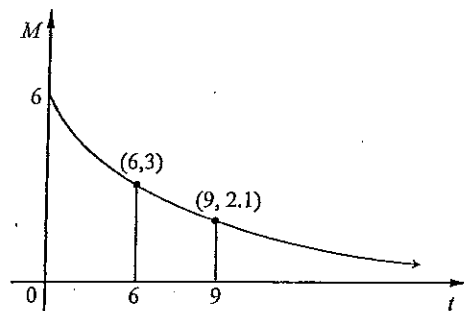
The graph above is modelled by the equation $M = Pe^{Kt}$.

Use the information given in the graph to answer the following.

(i)   Find the value of P.   1

(ii)   Hence find the value of K, correct to 4 decimal places.   2

(iii) When will one-fifth of the initial dose remain? Write your answer correct to the   2
       nearest hour.

(iv)   At what rate is the drug being removed from the human body when $t = 9$ hours?   2

End of Question 15

Question 16 (15 marks) Use a SEPARATE writing booklet.

(a)   A particle moves in a straight line so that its displacement, x metres, at time t seconds is given by:

$$x = 6t + 3\ln(3t^2 + 1).$$

(i)   Find the initial displacement.   1

(ii)   Prove that the particle is never at rest.   3

(iii) Hence find the limiting velocity.   1

(b)   Consider the circle $(x - x_1)^2 + (y + 4)^2 = 25$.

Find the values of x_1 for which the point $M(4, -1)$ will lie outside the circle.   3

(c) The diagram below is the hyperbola represented by the equation $y = \dfrac{6}{x}$.
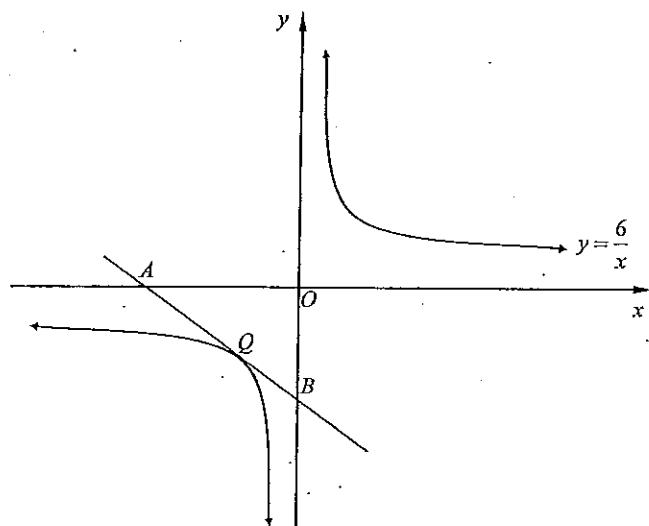
$Q\left(2q, \dfrac{3}{q}\right)$ is a variable point on the hyperbola in the third quadrant.

The tangent at Q cuts the x-axis at A and the y-axis at B.

(i) Show that the equation of the tangent at Q is given by $2q^2 y = 12q - 3x$. **2**

(ii) Find the coordinates of A and B. **2**

Let the square of the length of AB, i.e. AB^2 be denoted by L.

(iii) Find the value of q for which L is a minimum. **3**

End of Paper

CATHOLIC SECONDARY SCHOOLS ASSOCIATION
2017 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION
MATHEMATICS – MARKING GUIDELINES

These marking guidelines show the criteria to be applied to responses along with the marks to be awarded in line with the quality of responses. These guidelines are suggested and not prescriptive. This is not intended to be an exhaustive list but rather an indication of the considerations that students could include in their responses.

Section I
10 Marks

Questions 1-10 (1 mark each)

Question	Answer	Outcomes Assessed	Targeted Performance Bands
1	C	P4, H5	3
2	D	P4	3
3	C	H8	3
4	A	P5	3-4
5	D	P3, P4, H5	4
6	B	P4, H5	4
7	B	P4, H5	4
8	C	P3, H5	4-5
9	A	H6	5
10	D	H8	6

Section II
90 Marks

Question 11 (15 marks)
(a) (1 mark)
Outcomes Assessed: P3, H1
Targeted Performance Bands: 2

Criteria	Mark
• Correctly determines the value to 2 significant figures	1

Sample answer:

37

(b) (1 mark)
Outcomes Assessed: P4
Targeted Performance Bands: 3

Criteria	Mark
• Correctly factorises the expression	1

Sample answer:

$(5x+3)(x-1)$

(c) (3 marks)
Outcomes Assessed: P4
Targeted Performance Bands: 4

Criteria	Marks
• Correctly solves the inequality and correct sketching of inequality on number line	3
• Correctly solves the inequality	2
• Some progress towards solution	1

Sample answer:

$-8 \le 3x - 4 \le 8$

$-4 \le 3x \le 12$

$-\dfrac{4}{3} \le x \le 4$

6200-2

(d) (2 marks)
Outcomes Assessed: H5
Targeted Performance Bands: 4

Criteria	Marks
• Correct substitution into first derivative and simplification	2
• Correct differentiation of trigonometric function	1

Sample answer:

$f(x) = 3\sin 4x$

$f'(x) = 12\cos 4x$

$f'\left(\dfrac{\pi}{6}\right) = 12\cos\dfrac{4\pi}{6}$

$= 12 \times -\dfrac{1}{2}$

$= -6$

(e) (2 marks)
Outcomes Assessed: H3, H8
Targeted Performance Bands: 4

Criteria	Marks
• Correctly finds the primitive	2
• Progress towards finding the primitive	1

Sample answer:

$\dfrac{1}{4}\displaystyle\int \dfrac{4x^3}{x^4+7}\ dx$

$\dfrac{1}{4}\log_e (x^4+7) + C$

(f) (2 marks)
Outcomes Assessed: P3
Targeted Performance Bands: 3-4

Criteria	Marks
• Correct answer	2
• Progress towards answer	1

Sample answer:

$\displaystyle\lim_{x \to -1} \dfrac{3(x+1)}{(x+1)(x-1)}$

$= \displaystyle\lim_{x \to -1} \dfrac{3}{(x-1)}$

$= -\dfrac{3}{2}$

(g) (2 marks)
Outcomes Assessed: P3
Targeted Performance Bands: 3-4

Criteria	Marks
• Finds b	2
• Progress towards a solution	1

Sample answer:

$-2 + 4 = \dfrac{1}{a}$

$\therefore a = \dfrac{1}{2}$

$-2 \times 4 = \dfrac{b}{a}$

$\therefore b = -4$

(h) (2 marks)
Outcomes Assessed: P3
Targeted Performance Bands: 3-4

Criteria	Marks
• Correct answer	2
• Progress towards answer using index laws	1

Sample answer:

$$5^{x+2} = \frac{1}{5} = 5^{-1}$$
$$x + 2 = -1$$
$$\therefore x = -3$$

Question 12 (15 marks)
(a)(i) (1 marks)
Outcomes Assessed: P3
Targeted Performance Bands: 3

Criteria	Mark
• Obtains correct expression for a	1

Sample answer:

$$a = \tan 30°$$
$$= \frac{1}{\sqrt{3}}$$

(a) (ii) (2 marks)
Outcomes Assessed: P6
Targeted Performance Bands: 3-4

Criteria	Marks
• Correctly obtains the x-coordinate of P	2
• Substantial progress towards a solution	1

Sample answer:

$$g'(x) = \frac{1}{\sqrt{2x+1}}$$
$$\frac{1}{\sqrt{2x+1}} = \frac{1}{\sqrt{3}}$$
$$2x + 1 = 3$$
$$x = 1$$

(b) (3 marks)
Outcomes Assessed: H2
Targeted Performance Bands: 3-4

Criteria	Marks
• Correct reasoning is used to establish $\Delta MPO \equiv \Delta OSN$	3
• Significant progress is made towards proving $\Delta MPO \equiv \Delta OSN$	2
• Correctly establishes that $\Delta PSM \equiv \Delta OSN$	1

Sample answer:

In Δ's PSM and OSN

$\angle PMS = \angle ONS$ (given)
$PS = OS$ (given)
$\angle PSM = \angle OSN$ (vertically opposite angles)
$\therefore \Delta PSM \equiv \Delta OSN$ (AAS)
In Δ's MPO and NOP
$PM = ON$ (corresponding sides of congruent triangles)
PO is common
$OM = PN$ (since $SM = SN$ and $PS = OS$)
$\therefore \Delta MPO \equiv \Delta OSN$ (SSS)

(c) (3 marks)
Outcomes Assessed: P3
Targeted Performance Bands: 4

Criteria	Marks
• Obtains the correct values of	3
• Significant progress is made towards correct solution	2
• Correctly establishes that $\cos\theta = -\dfrac{1}{2}$	1

Sample answer:
$$2\sin\theta \cdot \frac{\cos\theta}{\sin\theta} + 1 = 0$$
$$\cos\theta = -\frac{1}{2}$$
$$\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$$

(d) (i) (1 mark)
Outcomes Assessed: P4
Targeted Performance Bands: 3

Criteria	Mark
• Correctly shows that AB is parallel to DC	1

Sample answer:
$$m_{AB} = \frac{6-4}{2--2}$$
$$= \frac{1}{2}$$
$$m_{DC} = \frac{1--3}{1--7}$$
$$= \frac{1}{2}$$
$$\therefore AB \parallel DC$$

(d) (ii) (1 mark)
Outcomes Assessed: P4
Targeted Performance Bands: 3

Criteria	Mark
• Correctly shows the equation of AB is $x - 2y + 10 = 0$	1

Sample answer:
$$y - 4 = \frac{1}{2}(x - -2)$$
$$2y - 8 = x + 2$$
$$\therefore x - 2y + 10 = 0$$

(d) (iii) (2 marks)
Outcomes Assessed: P4
Targeted Performance Bands: 3

Criteria	Marks
• Correctly determines the perpendicular distance from AB to point C	2
• Significant progress is made towards correct answer	1

Sample answer:

$$d = \frac{|1(1) - 2(1) + 10|}{\sqrt{(1)^2 + (-2)^2}}$$
$$= \frac{9}{\sqrt{5}}$$

(d) (iv) (2 marks)
Outcomes Assessed: P3, P4
Targeted Performance Bands: 3-4

Criteria	Marks
• Correct area of trapezium is found	2
• The correct lengths of AB and DC are determined	1

Sample answer:

$$d_{AB} = \sqrt{(2--2)^2 + (6-4)^2}$$
$$= \sqrt{20}$$
$$= 2\sqrt{5}$$

$$d_{DC} = 4\sqrt{5} \ (DC = 2AB, \text{ given})$$

$$\text{Area} = \frac{9}{2\sqrt{5}}(2\sqrt{5} + 4\sqrt{5})$$
$$= 27 \text{ units}^2$$

Question 13 (15 marks)
(a) (i) (1 mark)
Outcomes Assessed: P7, H3
Targeted Performance Bands: 3-4

Criteria	Mark
• Find the correct derivative	1

Sample answer:

$$\frac{dy}{dx} = e^x$$

(a)(ii) (2 marks)
Outcomes Assessed: P7, H5
Targeted Performance Bands: 4

Criteria	Marks
• Obtains the correct expression for the derivative	2
• Progress towards the correct derivative	1

Sample answer:

$$\frac{dy}{dx} = \frac{(\ln x)(2x) - (x^2+5)\frac{1}{x}}{(\ln x)^2}$$

(b) (2 marks)
Outcomes Assessed: H5
Targeted Performance Bands: 4

Criteria	Marks
• Correct solution is obtained	2
• Progress is made towards correct solution	1

Sample answer:

$$\int \frac{1}{\sqrt{4x-7}} \, dx$$

$$= \int (4x-7)^{-\frac{1}{2}} \, dx$$

$$= \frac{(4x-7)^{\frac{1}{2}}}{\frac{1}{2} \times 4} + C$$

$$= \frac{\sqrt{4x-7}}{2} + C$$
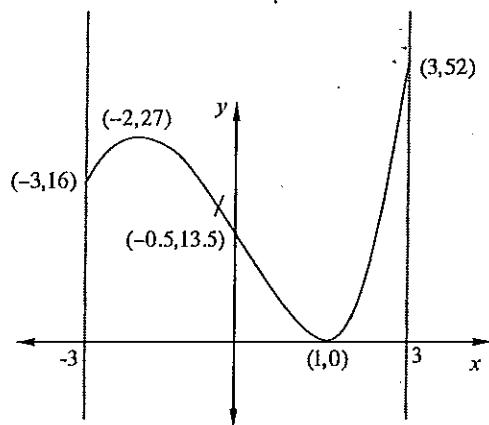
(c)(i) (3 marks)
Outcomes Assessed: H5, H6
Targeted Performance Bands: 3

Criteria	Marks
• Correctly determines both stationary points and their nature	3
• Some progress – e.g. determines the x values where the curve is stationary and their nature	2
• Correctly differentiates	1

Sample answer:

$$y = 2x^3 + 3x^2 - 12x + 7$$

$$\frac{dy}{dx} = 6x^2 + 6x - 12$$

For stationary points, $\frac{dy}{dx} = 0$

$$6x^2 + 6x - 12 = 0$$

$$(x+2)(x-1) = 0$$

$$\therefore x = -2 \text{ and } x = 1$$

$f(-2) = 27$ and $f(1) = 0$, so stationary points occur at $(-2, 27)$ and $(1, 0)$

$$\frac{d^2y}{dx^2} = 12x + 6$$

at $x = -2$, $\frac{d^2y}{dx^2} = -18$ so $\cap$ i.e. a MAXIMUM turning point at $(-2, 27)$

at $x = 1$, $\frac{d^2y}{dx^2} = 18$ so $\cup$ i.e. a MINIMUM turning point at $(1, 0)$

(c)(ii) (2 marks)
Outcomes Assessed: H5, H6
Targeted Performance Bands: 3-4

Criteria	Marks
• Correctly determines point of inflexion with concavity change	2
• Correctly determines the point of inflexion	1

Sample answer:

Possible Point of inflexion at $\dfrac{d^2y}{dx^2} = 0$

$12x + 6 = 0$

$x = -0.5$, at this value of x the y value is 13.5

x	-1	-0.5	0
$\dfrac{d^2y}{dx^2}$	-6	0	6

A change of concavity has occurred.
∴ a point of inflexion occurs at $(-0.5, 13.5)$

(c)(iii) (3 marks)
Outcomes Assessed: H5, H6
Targeted Performance Bands: 4

Criteria	Marks
• Correctly draws graph, including all stationary points, points of inflexion & end points	3
• Significant progress is made towards correct graph	2
• Some attempt, but does not include all points	1

Sample answer:

(c) (iv) (1 mark)
Outcomes Assessed: H7
Targeted Performance Bands: 3

Criteria	Mark
• Correct answer obtained	1

Sample answer:

Maximum value of the function in the domain $-3 \le x \le 3$ is 52

(c) (v) (1 mark)
Outcomes Assessed: H7
Targeted Performance Bands: 5

Criteria	Mark
• Correct domain and inequality symbols given	1

Sample answer:

$-\dfrac{1}{2} < x < 1$

Question 14 (15 marks)
(a)(i) (1 mark)
Outcomes Assessed: H5
Targeted Performance Bands: 3

Criteria	Mark
• Correctly finds the value of θ in radians	1

Sample answer:

$l = r\theta$

$100 = 40\theta$

$\theta = \dfrac{5}{2}$ radians

(a)(ii) (3 marks)
Outcomes Assessed: H2, H5
Targeted Performance Bands: 3-5

Criteria	Marks
• Evaluates the height correctly	3
• Significant progress towards correct solution	2
• Establishes a right-angled triangle with the correct angle	1

Sample answer:

Let h be the height of M above the track after the wheel has rolled 1 metre.

The angle required for triangle is $\dfrac{5-\pi}{2}$.

$\sin\dfrac{5-\pi}{2} = \dfrac{x}{40}$

$x = 40\sin\dfrac{5-\pi}{2}$

$\cong 32$

So the height of M above the track is $32 + 40 = 72$cm.

(b)(i) (1 mark)
Outcomes Assessed: P3, P4
Targeted Performance Bands: 3

Criteria	Mark
• Gives the correct coordinate for T	1

Sample answer:

$y = 2 - x^2$, cuts the x-axis when $y=0$.

$0 = 2 - x^2$

$x = \pm\sqrt{2}, x > 0$

So T is the coordinate $\left(\sqrt{2}, 0\right)$.

(b)(ii) (3 marks)
Outcomes Assessed: H5, H8, H9
Targeted Performance Bands: 4-5

Criteria	Marks
• Evaluates the volume correctly	3
• Correctly integrates the expression	2
• Some progress towards the solution	1

Sample answer:

Let $y_1 = x^3$ and $y_2 = 2 - x^2$.

$V = \pi \displaystyle\int_0^1 (y_1)^2 \, dx + \pi \int_1^{\sqrt{2}} (y_2)^2 \, dx$

$= \left[\pi \displaystyle\int_0^1 x^6 \, dx + \pi \int_1^{\sqrt{2}} (2-x^2)^2 \, dx \right]$

$= \left[\pi \displaystyle\int_0^1 x^6 \, dx + \pi \int_1^{\sqrt{2}} (4 - 4x^2 + x^4) \, dx \right]$

$= \pi \left[\left[\dfrac{x^7}{7} \right]_0^1 + \left[4x - \dfrac{4x^3}{3} + \dfrac{x^5}{5} \right]_1^{\sqrt{2}} \right]$

$= \pi \left[\dfrac{1}{7} + \left(4\sqrt{2} - \dfrac{8\sqrt{2}}{3} + \dfrac{4\sqrt{2}}{5} \right) - \left(4 - \dfrac{4}{3} + \dfrac{1}{5} \right) \right] = \pi \left[\dfrac{32\sqrt{2}}{15} - \dfrac{286}{105} \right]$

$\cong 0.921 \ (3 \text{ decimal places})$

(c)(i) (1 mark)
Outcomes Assessed: H4, H5
Targeted Performance Bands: 3-4

Criteria	Mark
• Gives the correct expression for the annuity after one year	1

Sample answer:
Let the amount in her account at the start of Helen's annuity be \$500 000; let the rate $r = 0.06$ and let A_n be the amount remaining in the annuity after n years.

$$A_1 = 500\ 000(1.06) - M$$

(c) (ii)(3 marks)
Outcomes Assessed: H4, H5, H9
Targeted Performance Bands: 4-5

Criteria	Marks
• Correctly determines the value of \$M	3
• Progress towards finding \$M	2
• Forms an equation involving \$M	1

Sample answer:
$$A_2 = A_1(1.06) - M$$
$$= (500\ 000(1.06) - M)(1.06) - M$$
$$= 500\ 000(1.06)^2 - M(1+1.06)$$

so continuing the pattern
$$A_{30} = 500\ 000(1.06)^{30} - M(1+1.06+...+1.06^{29})$$

But $A_{30} = 0$ and $(1+1.06+...+1.06^{29})$ is a GP where $n=30$, $a=1$ and $r=1.06$

So, $0 = 500\ 000(1.06)^{30} - M \times \dfrac{(1.06^{30}-1)}{1.06-1}$

$$M = \dfrac{0.06 \times 500\ 000 \times (1.06)^{30}}{(1.06^{30}-1)}$$

$$\cong \$36\ 324.46 \ (2 \text{ decimal places})$$

(c) (iii)(3 marks)
Outcomes Assessed: H4, H5, H9
Targeted Performance Bands: 4-6

Criteria	Marks
• Correctly determines how many more years Helen will be able to withdraw money	3
• Significant progress towards solving the problem	2
• Finding the amount remaining after 15 withdrawals	1

Sample answer:
$$A_{15} = 500\ 000(1.06)^{15} - 36\ 324.46(1+1.06+...+1.06^{14})$$
$$\approx 500\ 000(1.06)^{15} - 36\ 324.46 \times \dfrac{(1.06^{15}-1)}{0.06}$$
$$\cong \$352\ 792.06$$

So, after 15 years there is approximately \$352 792.06 remaining in Helen's annuity.
Let B_n be the amount remaining in the annuity n years after rate changes to 4% p.a.
$$B_1 = 352\ 792.06(1.04) - M$$
$$B_2 = B_1(1.04) - M$$
$$= 352\ 792.06(1.04)^2 - M(1+1.04)$$
$$...$$
$$\therefore B_n = 352\ 792.06(1.04)^n - M(1+1.04+...+1.04^{n-1})$$

When $B_n = 0$.
$$352\ 792.06 \times (1.04)^n = 36\ 324.46 \times \dfrac{(1.04^n-1)}{0.04}$$
$$(1.04)^n = 1.635293$$
$$n = \dfrac{\log(1.635293)}{\log(1.04)}$$
$$n \cong 12.539 \text{ years}$$

Therefore, Helen will be able to withdraw money from this annuity for 12 more years.

Question 15 (15 marks)
(a)(i) (2 marks)
Outcomes Assessed: H5, H9
Targeted Performance Bands: 4

Criteria	Marks
• Correctly finds the coordinate P	2
• Progress towards an answer	1

Sample answer:

$\sin 2x = \cos 2x$

$\tan 2x = 1$

$2x = \dfrac{\pi}{4}, \dfrac{5\pi}{4}$

$x = \dfrac{\pi}{8}, \dfrac{5\pi}{8}, \quad x$ is in first quadrant, so $x = \dfrac{\pi}{8}$.

When $x = \dfrac{\pi}{8}$, $y = \dfrac{1}{\sqrt{2}}$. So P is the coordinate $\left(\dfrac{\pi}{8}, \dfrac{1}{\sqrt{2}} \right)$.

(a)(ii) (3 marks)
Outcomes Assessed: H5, H8
Targeted Performance Bands: 3-5

Criteria	Marks
• Evaluates shaded area correctly	3
• Significant progress towards correct solution	2
• Correctly determines part integral of sin2x or cos2x correctly	1

Sample answer

$A = 2 \displaystyle\int_{0}^{\frac{\pi}{8}} (1 - \cos 2x)\,dx$

$= 2 \left[x - \dfrac{1}{2}\sin 2x \right]_{0}^{\frac{\pi}{8}}$

$= 2 \left[\dfrac{\pi}{8} - \dfrac{1}{2} \times \dfrac{1}{\sqrt{2}} \right] - \left[0 - \dfrac{1}{2} \times 0 \right]$

$= 2 \left[\dfrac{\pi}{8} - \dfrac{\sqrt{2}}{4} \right]$

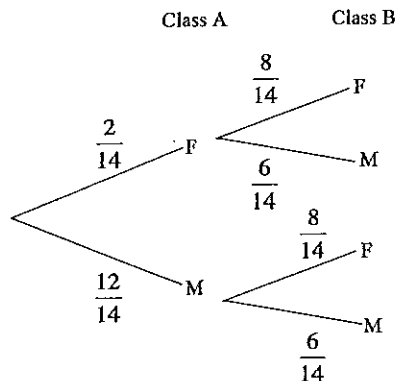$= \dfrac{1}{4}(\pi - 2\sqrt{2})$

(b)(i) (1 mark)
Outcomes Assessed: H4, H5
Targeted Performance Bands: 3-4

Criteria	Mark
• Correctly determines the probability	1

Sample answer:

Class A   Class B

$$P(FF) = \frac{2}{14} \times \frac{8}{14} = \frac{4}{49}$$

(b)(ii) (2 marks)
Outcomes Assessed: H4, H5
Targeted Performance Bands: 3-4

Criteria	Marks
• Correctly determines the probability	2
• Some progress towards the correct answer	1

Sample answer:

$$P(1\text{ female},1\text{ male}) = P(FM) + P(MF)$$

$$= \frac{2}{14} \times \frac{6}{14} + \frac{12}{14} \times \frac{8}{14}$$

$$= \frac{3}{49} + \frac{24}{49} = \frac{27}{49}$$

(c)(i) (1 mark)
Outcomes Assessed: H3, H9
Targeted Performance Bands: 3-4

Criteria	Mark
• Correctly writes down the value of P	1

Sample answer:
From the graph, $P = 6$.

(c)(ii) (2 marks)
Outcomes Assessed: H3, H9
Targeted Performance Bands: 3-5

Criteria	Marks
• Correctly determines the value of k	2
• Progress towards finding the value of k	1

Sample answer:
When $t = 6$, $M = 3$ and $P = 6$:

$$3 = 6e^{6t}$$

$$\frac{1}{2} = e^{6k}$$

$$\ln\left(\frac{1}{2}\right) = 6k$$

$$k \cong -0.1155 \,(4\text{ decimal places})$$

(c)(iii) (2 marks)
Outcomes Assessed: H3, H9
Targeted Performance Bands: 3-5

Criteria	Marks
• Correct answer	2
• Progress towards the correct answer	1

Sample answer:

$$1.2 = 6e^{-0.1155t}$$

$$\frac{1.2}{6} = e^{-0.1155t}$$

$$\ln\left(\frac{1.2}{6}\right) = -0.1155 \times t$$

$$t \cong 13.934...$$

So one fifth of the initial dose will remain after approximately 14 hours.

(c)(iv) (2 marks)
Outcomes Assessed: H3, H5
Targeted Performance Bands: 3-5

Criteria	Marks
• Correct answer	2
• Progress towards the correct answer	1

Sample answer:

$$\frac{dM}{dt} = kPe^{kt}$$

$$= -0.1155 \times 6 \times e^{-0.1155 \times 9}$$

$$= -0.245 \text{ mg/hr}$$

∴ Drug removed at rate of 0.245 mg/hr.

Question 16 (15 marks)

(a) (i) (1 mark)
Outcomes Assessed: H3, H4
Targeted Performance Bands: 2-3

Criteria	Mark
• Solve for $t = 0$	1

Sample answer:
When $t = 0$ $x = 0 + 3\ln 1 = 0$

(a) (ii) (3 marks)
Outcomes Assessed: H4, H5
Targeted Performance Bands: 3-5

Criteria	Marks
• Gives the correct answer	3
• Substantial progress towards an answer	2
• Differentiates and equates to zero	1

Sample answer:

$$x = 6t + 3\ln(3t^2 + 1) \qquad \therefore \dot{x} = 6 + 3 \times \frac{6t}{3t^2 + 1} = 6 + \frac{18t}{3t^2 + 1} \quad \text{and at rest } \dot{x} = 0$$

$$\therefore 0 = 6 + \frac{18t}{3t^2 + 1} \qquad \therefore 18t^2 + 18t + 6 = 0 \qquad \therefore 3t^2 + 3t + 1 = 0$$

and since $\Delta = 9 - 12 = -3 < 0$ ∴ no solution i.e. the particle never come to rest.

(a) (iii) (1 mark)
Outcomes Assessed: P2, H5
Targeted Performance Bands: 3-4

Criteria	Mark
• Gives the correct answer	1

Sample answer:

The limiting velocity is 6 m/s.

(b) (3 marks)

Criteria	Marks
• Gives the correct values for x_1	3
• Makes substantial progress towards the solution including justification	2
• Gives the correct substitution into the correct formula	1

Sample answer:

$$(x-x_1)^2+(y+4)^2=25$$

Centre: $C\left(x_1,-4\right)$ and the radius $=5$.

The point $M(4,-1)$ will lie outside the circle if the distance $CM>5$

$$\therefore \sqrt{(x_1-4)^2+(-4+1)^2}>5$$

$$(x_1-4)^2+9>25$$

$$(x_1-4)^2>16 \quad \therefore x_1^2-8x_1+16>16 \qquad \therefore x_1(x_1-8)>0$$

$$x_1<0 \text{ or } x_1>8$$

(c) (i) (2 marks)

Criteria	Marks
• Gives the correct answer	2
• Gives the correct gradient of the tangent	1

Sample answer:

$$y=\frac{6}{x}=6x^{-1} \quad \therefore \frac{dy}{dx}=\frac{-6}{x^2} \quad \text{and at } Q, \quad x=2q \quad \therefore m=\frac{-6}{4q^2}=\frac{-3}{2q^2}$$

$\therefore$ the equation of the tangent at Q is : $\left(y-\frac{3}{q}\right)=\frac{-3}{2q^2}(x-2q)$

$$2q^2y-6q=-3x+6q$$

$$2q^2y=12q-3x$$

(c) (ii) (2 marks)

Criteria	Mark
• Gives the correct answer	2
• Correctly evaluates either A or B	1

Sample answer:
At A: let $y=0$ so $x=4q$ $\therefore A=(4q,0)$

At B: let $x=0$ so $y=\frac{12q}{2q^2}=\frac{6}{q}$ $\qquad \therefore B=\left(0,\frac{6}{q}\right)$

(c) (iii) (3marks)
Outcomes Assessed: H5, H6
Targeted Performance Bands: 4-6

Criteria	Marks
• Gives the correct answer which includes a correct test	3
• Correctly determines the derivative and recognises that q is in the third quadrant	2
• Differentiates a correct expression for L and equates to zero	1

Sample answer:

$$AB^2 = 16q^2 + \frac{36}{q^2} \qquad \therefore L = 16q^2 + 36q^{-2} \qquad \therefore \frac{dL}{dq} = 32q - \frac{72}{q^3}$$

For minimum length, $\quad \frac{dL}{dq} = 0 \qquad \therefore 0 = 32q - \frac{72}{q^3} \qquad \therefore 32q^4 = 72$

$$\therefore q = \pm\sqrt{\frac{3}{2}} \quad \text{and since } Q \text{ in the third quadrant} \quad \therefore q = -\sqrt{\frac{3}{2}}$$

$$\therefore \frac{d^2L}{dq^2} = 32 + \frac{216}{q^4} \quad \text{and when } q = -\sqrt{\frac{3}{2}} \qquad \frac{d^2L}{dq^2} = 32 + \frac{216}{9/4} = 128$$

$$\therefore \frac{d^2L}{dq^2} > 0 \quad \text{i.e. minimum length when } q = -\sqrt{\frac{3}{2}}$$