NSW INDEPENDENT SCHOOLS

2017
Higher School Certificate
Trial Examination

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Board approved calculators may be used.
- Write using black or blue pen
- A table of standard integrals is provided at the back of the paper
- All necessary working should be shown in Question 11 – 16
- Write your student number and/or name at the top of every page

Total marks – 100

Section I - Pages 2 – 5

10 marks

Attempt Questions 1 - 10

Allow about 15 minutes for this section

Section II - Pages 6 – 11

90 marks

Attempt Questions 11 – 16

Allow about 2 hours 45 minutes for this section

This paper MUST NOT be removed from the examination room

STUDENT NUMBER/NAME:.................................

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Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Select the alternative A, B, C or D that best answers the question and indicate your choice with a cross (X) in the appropriate space on the grid below.

	A	B	C	D
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				

Student name / number ____________________

Marks

Section I

10 Marks
Attempt Questions 1-10.
Allow about 15 minutes for this section.

Use the multiple-choice answer sheet for questions 1-10.

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1 What is the value of $\dfrac{\sqrt{8\cdot 9}+2\cdot 1^2}{\sqrt{8\cdot 9+2\cdot 1^2}}$ correct to 3 significant figures? **1**

(A) 2·02
(B) 2·03
(C) 2·026
(D) 2·027

2 What is the exact value of $\sec 30^\circ + \tan 30^\circ$? **1**

(A) $\frac{5\sqrt{3}}{6}$

(B) $\frac{3\sqrt{3}}{2}$

(C) $\frac{5\sqrt{3}}{3}$

(D) $\sqrt{3}$

3 The quadratic equation $2x^2+4x+1=0$ has roots α and β. What is the value of $\alpha^2\beta+\alpha\beta^2$? **1**

(A) -2
(B) -1
(C) 1
(D) 2

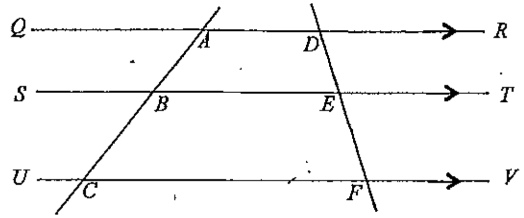
4 The function $f(x)$ is defined by $f(x)=\begin{cases} 4^x, & x\le 1 \\ \frac{4}{x}, & x>1 \end{cases}$. What is the value of $f(0\cdot 5)+f(2)$? **1**

(A) 4
(B) 10
(C) 18
(D) 24

Student name / number ____________________

Marks

5 In the diagram below $QR \parallel ST \parallel UV$, ABC and DEF are transversals such that $AB=4$ cm, $BC=6$ cm and $DF=8$ cm. What is the length of DE ? **1**



NOT TO SCALE

(A) 2·8 cm
(B) 3 cm
(C) 3·2 cm
(D) 3·4 cm

6 Two cards are chosen at random without replacement from a standard pack of 52 playing cards. What is the probability both cards are hearts ? **1**

(A) $\frac{1}{17}$

(B) $\frac{1}{16}$

(C) $\frac{33}{68}$

(D) $\frac{1}{2}$

7 What is the solution of the equation $2^x=5$? **1**

(A) $x=\log_e 5+\log_e 2$

(B) $x=\log_e 5-\log_e 2$

(C) $x=\log_e 5\times \log_e 2$

(D) $x=\dfrac{\log_e 5}{\log_e 2}$

Student name / number ____________________

Marks

8 For $k \neq 0$, what is the limiting sum of the geometric series 1

$$k+\frac{k}{1+k^2}+\frac{k}{\left(1+k^2\right)^2}+\frac{k}{\left(1+k^2\right)^3}+\dots \text{ ?}$$

(A) $\frac{1}{1+k^2}$

(B) $\frac{k^2}{1+k^2}$

(C) $\frac{1+k^2}{k}$

(D) $\frac{1+k^2}{k^2}$

9 After t hours the number $N(t)$ of individuals in a population is given by $N(t)=100e^{kt}$ for some constant $k>0$. After 1 hour there are x individuals in the population. What is the number of individuals in the population after 2 hours? 1

(A) $\frac{x}{100}$

(B) $\frac{x^2}{100}$

(C) $100x$

(D) $100x^2$

10 A sector of a circle of radius r cm contains an angle of θ radians at the centre of the circle. The sector has area 50 cm^2. Which of the following is NOT an expression for the perimeter P cm of the circle ? 1

(A) $P=r(2+\theta)$

(B) $P=2r+\frac{100}{r}$

(C) $P=\frac{20}{\sqrt{\theta}}+10\sqrt{\theta}$

(D) $P=\frac{50(2+\theta)}{r\theta}$

Student name / number ____________________

Marks

Section II

90 Marks
Attempt Questions 11-16
Allow about 2 hours and 45 minutes for this section.

Answer the questions in writing booklets provided. Use a separate writing booklet for each question. In Questions 11-16 your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) **Use a separate writing booklet.**

(a) Express in simplest form with rational denominator $\frac{\sqrt{5}}{3+\sqrt{5}}$. 2

(b) Solve the inequality $|2x-1|>3$. 2

(c) Find $\lim_{x\to 2}\frac{x^2-4}{2x-4}$. 2

(d)(i) If $y=\tan 2x$ find $\frac{dy}{dx}$. 1

(ii) If $y=e^{x^3}+4\sqrt{1-x}$ find $\frac{dy}{dx}$. 2

(e) Find in simplest exact form the equation of the tangent to the curve $y=x^2\log_e x$ at the point (e, e^2) on the curve. 3

(f) The region bounded by the curve $y=\frac{1}{2x+1}$ and the x axis between $x=0$ and $x=2$ is rotated through one revolution about the x axis. Find in simplest exact form the volume of the solid formed. 3

Student name / number ______________

Marks

Question 12 (15 marks) **Use a separate writing booklet.**

(a) Find the focus and the directrix of the parabola $(x-3)^2=8(y-1)$. 2

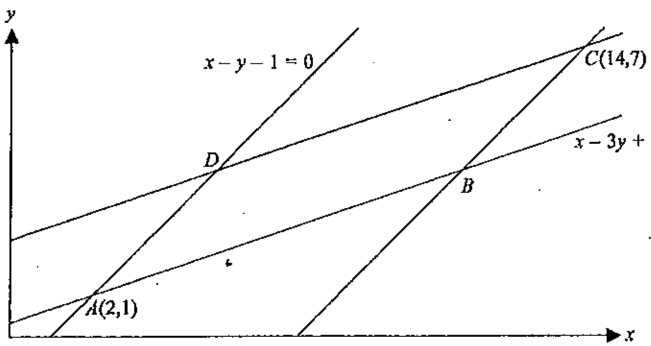
(b) Find in simplest form $\dfrac{d}{dx}\left(\dfrac{\cos x}{1-\sin x}\right)$. 2

(c) Sketch the graph of the function $f(x)=\sqrt{x}-2$ showing the intercepts on the axes. 2

(d) A curve has gradient function $\dfrac{dy}{dx}=\dfrac{x^2}{3}+\dfrac{3}{x^2}$ and passes through the point $(3,3)$. 3

Find the equation of the curve.

(e)



In the diagram, $A(2,1)$ and $C(14,7)$ are two vertices of a parallelogram $ABCD$.
The side AB has equation $x-3y+1=0$ and the side AD has equation $x-y-1=0$.

(i) Find the equation of the side BC. 1

(ii) Find the coordinates of the point B. 2

(iii) Find in simplest exact form the area of the parallelogram $ABCD$. 3

Student name / number ______________

Marks

Question 13 (15 marks) **Use a separate writing booklet.**

(a) Two fair dice are rolled. Find the probability that the highest number showing on either die is 5. 2

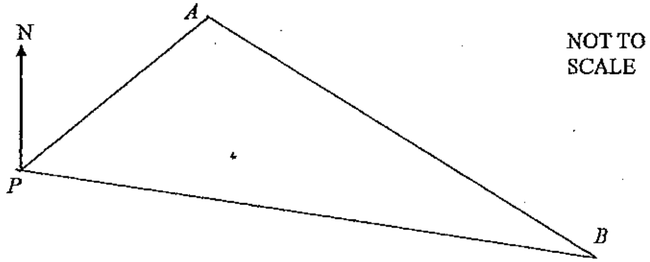
(b) Find $\int \tan^2 x\, dx$. 2

(c) Find any values of k such that 1, $\log_e k$, 4 are the first three terms of a geometric progression. 3

(d)(i) Show that $\dfrac{d}{dx}(\log_e \tan x)=\dfrac{1}{\cos x \sin x}$. 2

(ii) Hence find in simplest exact form the value of $\displaystyle\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{2}{\cos x \sin x}\,dx$. 2

(e)



In the diagram a yacht sails 640 metres from point P to point A on a bearing of 050°.
It then sails 960 metres from point A to point B on a bearing of 120°.

(i) Find the distance of point B from point P correct to the nearest metre. 2

(ii) Find the bearing of point B from point P correct to the nearest degree. 2

Student name / number ______________

Question 14 (15 marks) **Use a separate writing booklet.** Marks

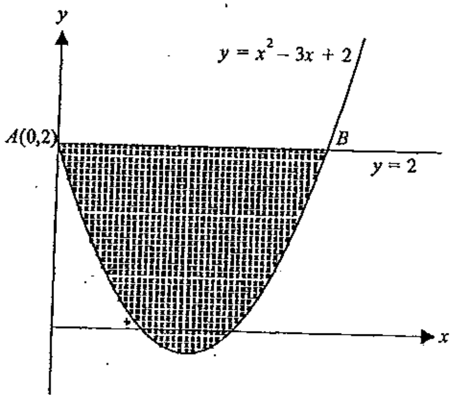
(a) Find the set of values of x for which the curve $y = 3x^2 - x^3$ is concave up. 2

(b) Find in simplest exact form the value of $\int_0^{\log_e 3} \frac{e^x}{e^x + 1}\,dx$. 3

(c) Find the coordinates and the nature of the stationary point on the curve $y = x + \frac{4}{x^2}$. 3

(d) Blaise has 3 tickets and Pierre has 4 tickets in a raffle. The total number of tickets is 25 and there are 2 prizes (a 1^{st} prize and a 2^{nd} prize). Find in simplest fraction form the probability that Pierre wins more prizes than Blaise. 3

(e)



In the diagram the parabola $y = x^2 - 3x + 2$ and the line $y = 2$ intersect at the points $A(0,2)$ and B.

(i) Find the x coordinate of the point B. 1

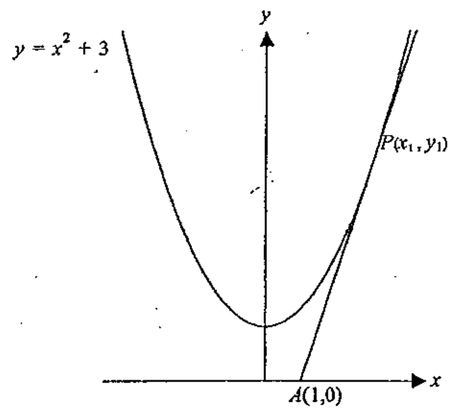
(ii) Find in simplest form the area of the shaded region between the parabola $y = x^2 - 3x + 2$ and the line $y = 2$. 3

Student name / number ______________

Marks

Question 15 (15 marks) **Use a separate writing booklet.**

(a)



In the diagram $P(x_1, y_1)$, where $x_1 > 1$, is a point on the parabola $y = x^2 + 3$. 3
The tangent to the parabola at the point P passes through the point $A(1,0)$.
By finding the gradient of AP in two different ways, find the value of x_1.

(b)(i) Solve the equation $1 + 2\sin x = 0$ for $0 \le x \le 2\pi$. 2

(ii) Sketch the curve $y = 1 + 2\sin x$ for $0 \le x \le 2\pi$ showing clearly the coordinates of the endpoints and the maximum and minimum points. 2

(c) *Oztown* had a 25 year house building program starting at the beginning of 1991 and finishing at the end of 2015. The number of houses built each calendar year follows an arithmetic progression with first term a and common difference d. 1900 houses were built in the year 2000 and 1100 houses were built in the year 2010.

(i) Find the values of a and d. 3

(ii) Find the total number of houses built over the 25 years. 1

(d) A particle is moving in a horizontal straight line. At time t seconds it has displacement x metres to the right of a fixed point O on the line given by $x = t(t-3)^2$, velocity v ms^{-1} and acceleration a ms^{-2}.

(i) Find expressions for v and a in terms of t. 2

(ii) Find when the particle is moving towards O. 1

(iii) Find when the particle is moving towards O and slowing down. 1

Student name / number ____________________

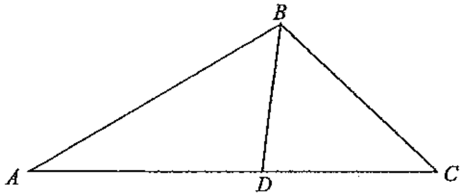
Question 16 (15 marks) **Use a separate writing booklet.** **Marks**

(a) A water tank containing 10 000 litres of water is being emptied. At time t minutes after it starts to empty the rate R litres/minute at which it is emptying is given by $R = 100e^{-0.01t}$.

(i) Show that the quantity Q litres of water remaining in the tank at time t minutes after it starts to empty is given by $Q = 10000e^{-0.01t}$. **2**

(ii) Find in simplest exact form the time taken for the tank to half empty and the rate at which the tank is emptying then. **2**

(b) In the diagram $\angle DBC = \angle DBA = x°$, $AB = c$, $AC = b$, $BC = 2a$ and $DB = DC = d$.

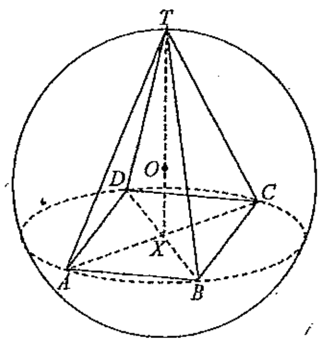


NOT TO SCALE

(i) Show that $\Delta ABD \parallel\!| \Delta ACB$. **2**

(ii) Hence show that $(a+c)^2 = a^2 + b^2$. **3**

(c)



In the diagram the square $ABCD$, whose diagonals AC and BD meet at X, is the base of a right, square-based pyramid with apex T which is inscribed in a sphere of radius 1 metre with centre O, so that the vertices of the pyramid touch the inside of the sphere, and $OX = x$ metres. Given that the volume of a pyramid is $\frac{1}{3}$ *area of base* $\times$ *height*:

(i) Show that the volume V m^3 of the inscribed pyramid is given by $V = \frac{2}{3}(1 + x - x^2 - x^3)$. **3**

(ii) Hence find the maximum volume of the pyramid. **3**

Section 1 **Questions 1-10 (1 mark each)**

Question	Answer	Solution	Outcomes
1	B	$\dfrac{\sqrt{8.9}+2.1^2}{\sqrt{8.9+2.1^2}} \approx \dfrac{7.3933}{3.6483} \approx 2.03$ to 3 sig. fig.	P3
2	D	$\sec 30° + \tan 30° = \frac{2}{\sqrt{3}} + \frac{1}{\sqrt{3}} = \frac{3}{\sqrt{3}} = \sqrt{3}$	P3
3	B	$\alpha^2\beta + \alpha\beta^2 = \alpha\beta(\alpha+\beta) = \frac{1}{2}\times(-2) = -1$	P4
4	A	$f(0.5) + f(2) = 4^{\frac{1}{2}} + \frac{4}{2} = 2 + 2 = 4$	P5
5	C	$\dfrac{AB}{AC} = \dfrac{DE}{DF}$ (*a family of 3 or more parallel lines makes intercepts in proportion on any 2 transversals*) $\therefore \dfrac{4}{10} = \dfrac{DE}{8}$ $\therefore DE = 3.2\,cm$	P4
6	A	$P(\text{both hearts}) = \frac{13}{52} \times \frac{12}{51} = \frac{1}{17}$	H5
7	D	$2^x = 5$ $x\log_e 2 = \log_e 5$ $\therefore x = \dfrac{\log_e 5}{\log_e 2}$	H3
8	C	G.P with $a = k$, $r = \dfrac{1}{1+k^2}$ where $\|r\| < 1$. Limiting sum s exists and is given by $s = k + \left(1 - \dfrac{1}{1+k^2}\right) = k \times \dfrac{1+k^2}{k^2} = \dfrac{1+k^2}{k}$	H5
9	B	$N(1) = x \Rightarrow x = 100e^{-k}$ $\therefore N(2) = 100e^{-2k} = 100(e^{-k})^2 = \dfrac{(100e^{-k})^2}{100} = \dfrac{x^2}{100}$	H3
10	D	$\frac{1}{2}r^2\theta = 50$ $\therefore r\theta = \dfrac{100}{r}$ and $r^2 = \dfrac{100}{\theta}$. Also $P = 2r + r\theta$ $\therefore P = r(2+\theta)$, or $P = 2r + \dfrac{100}{r}$, or $P = \sqrt{\dfrac{100}{\theta}}(2+\theta) = \dfrac{20}{\sqrt{\theta}} + 10\sqrt{\theta}$	H5

Section II

Question 11

a. Outcomes assessed: P4

Marking Guidelines

Criteria	Marks
• applies correct procedure and evaluates the denominator	1
• simplifies the numerator	1

Answer

$$\frac{\sqrt{5}}{3+\sqrt{5}} = \frac{\sqrt{5}(3-\sqrt{5})}{(3+\sqrt{5})(3-\sqrt{5})} = \frac{3\sqrt{5}-5}{9-5} = \frac{3\sqrt{5}-5}{4}$$

Q11(cont)

b. Outcomes assessed: P4

Marking Guidelines

Criteria	Marks
• finds one inequality for x	1
• finds the second inequality for x and indicates how the two inequalities are to be combined	1

Answer

$$2x-1<-3 \quad or \quad 2x-1>3$$
$$2x<-2 \qquad 2x>4$$
$$\therefore \quad x<-1 \quad or \quad x>2$$

c. Outcomes assessed: P4

Marking Guidelines

Criteria	Marks
• factors both numerator and denominator	1
• completes calculation of the limiting value	1

Answer

$$\lim_{x\to 2}\frac{x^2-4}{2x-4}=\lim_{x\to 2}\frac{(x-2)(x+2)}{2(x-2)}=\lim_{x\to 2}\frac{(x+2)}{2}=\frac{2+2}{2}=2$$

d. Outcomes assessed: H5

Marking Guidelines

Criteria	Marks
i • uses the chain rule to derive the trigonometric function	1
ii • uses the chain rule to derive the exponential function	1
• uses the chain rule to derive an expression using fractional indices	1

Answer

i.

$$y=\tan 2x$$
$$\frac{dy}{dx}=2\sec^2 2x$$

ii.

$$y=e^{x^2}+4\sqrt{1-x}$$
$$\frac{dy}{dx}=2x\,e^{x^2}+4\left\{-\tfrac{1}{2}(1-x)^{-\frac{1}{2}}\right\}$$
$$=2x\,e^{x^2}-\frac{2}{\sqrt{1-x}}$$

e. Outcomes assessed: H5

Marking Guidelines

Criteria	Marks
• finds the gradient function of the curve by differentiation	1
• evaluates the gradient of the tangent at the required point	1
• completes the equation of the tangent in gradient/intercept or general form	1

Answer

$$y=x^2\log_e x$$
$$\frac{dy}{dx}=2x\log_e x+x^2.\frac{1}{x}$$
$$=x(2\log_e x+1)$$

$\therefore$ Tangent at (e,e^2) has gradient $3e$ and equation

$$y-e^2=3e(x-e)$$
$$y=3e\,x-2e^2$$

Q11(cont)

f. Outcomes assessed: H8

Marking Guidelines

Criteria	Marks
• writes a definite integral in terms of x for the volume	1
• finds the primitive function	1
• evaluates	1

Answer

The volume is V cu. units where $V=\pi\int_0^2 \frac{1}{(2x+1)^2}\,dx=-\frac{\pi}{2}\left[\frac{1}{2x+1}\right]_0^2=-\frac{\pi}{2}\left(\frac{1}{5}-1\right)=\frac{2\pi}{5}$

Question 12

a. Outcomes assessed: P4

Marking Guidelines

Criteria	Marks
• states coordinates of focus	1
• states equation of directrix	1

Answer

Parabola has vertical axis of symmetry and is concave up with vertex $(3,1)$ and focal length 2.

Hence focus has coordinates $(3,3)$ and directrix has equation $y=-1$.

b. Outcomes assessed: H5

Marking Guidelines

Criteria	Marks
• applies quotient rule	1
• simplifies as fully as possible	1

Answer

$$\frac{d}{dx}\left(\frac{\cos x}{1-\sin x}\right)=\frac{-\sin x(1-\sin x)-\cos x(-\cos x)}{(1-\sin x)^2}$$
$$=\frac{-\sin x+\sin^2+\cos^2 x}{(1-\sin x)^2}$$
$$=\frac{1-\sin x}{(1-\sin x)^2}$$
$$=\frac{1}{1-\sin x}$$

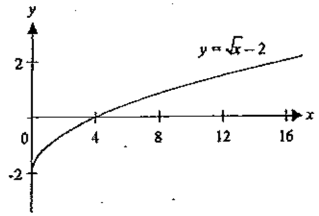
Q12(cont)

c. Outcomes assessed: P5

Marking Guidelines

Criteria	Marks
• semi-parabolic curve with correct position of endpoint at y intercept	1
• x intercept shown	1

Answer



d. Outcomes assessed: H8

Marking Guidelines

Criteria	Marks
• finds the primitive function of the first term in the gradient function	1
• finds the primitive function of the second term	1
• evaluates the constant of integration to complete the equation of the curve	1

Answer

$\frac{dy}{dx}=\frac{1}{3}x^2+3x^{-2}$

$y=\frac{1}{9}x^3-3x^{-1}+c$

$\left.\begin{array}{l}x=3\\ y=3\end{array}\right\}\Rightarrow \begin{array}{l}3=\frac{1}{9}\times 27-3\times\frac{1}{3}+c\\ \therefore c=1\end{array}$

$\therefore y=\frac{x^3}{9}-\frac{3}{x}+1$

e. Outcomes assessed: H5

Marking Guidelines

Criteria	Marks
i • uses gradient property of parallel lines to find the equation of BC	1
ii • solves simultaneous equations to find one coordinate of B	1
• finds the second coordinate of B	1
iii • finds the exact value of either the distance AB or its square	1
• finds the exact perpendicular distance from C to AB	1
• writes a numerical expression with surds for the area of the parallelogram and simplifies.	1

Answer

i. $BC \parallel AD$ $\therefore BC$ has equation $x-y+k=0$ for some constant k.

$(14,7)$ on $BC \Rightarrow 14-7+k=0$ $\therefore k=-7$. Hence BC has equation $x-y-7=0$.

ii. At B, $\begin{cases}x-y=7\\ x-3y=-1\end{cases}$ By subtraction, $2y=8$. Hence B has coordinates $(11,4)$.

iii. $AB^2=(11-2)^2+(4-1)^2=9^2+3^2=3^2(3^2+1)$ $\therefore AB=3\sqrt{10}$

⊥ distance from C to AB is $d=\frac{|14-3\times7+1|}{\sqrt{1^2+(-3)^2}}=\frac{6}{\sqrt{10}}$

Hence parallelogram $ABCD$ has area $3\sqrt{10}\times\frac{6}{\sqrt{10}}=18$ sq. units

Question 13

a. Outcomes assessed: H5

Marking Guidelines

Criteria	Marks
• counts the outcomes in the described event	1
• completes the calculation of the probability of this event	1

Answer

36 equally likely outcomes. Event comprises outcomes $(5,*)$, $(*,5)$, $(5,5)$ where $*=1,2,3,4$.

$\therefore P(\textit{highest number showing is } 5)=\frac{9}{36}=\frac{1}{4}$

b. Outcomes assessed: P4, H8

Marking Guidelines

Criteria	Marks
• uses an appropriate trigonometric identity	1
• finds primitive function	1

Answer

$\int \tan^2 x\,dx=\int(\sec^2 x-1)\,dx=\tan x-x+c$

c. Outcomes assessed: H3, H5

Marking Guidelines

Criteria	Marks
• uses the definition of geometric progression to write an equation for $\log_e k$	1
• finds one value for k	1
• finds the second value for k	1

Answer

1, $\log_e k$, 4 in G.P $\Rightarrow \frac{\log_e k}{1}=\frac{4}{\log_e k}$ $\therefore(\log_e k)^2=4$. Hence $\log_e k=\pm2$ $\therefore k=e^2$ or $k=e^{-2}$

d. Outcomes assessed: H5, H8

Marking Guidelines

Criteria	Marks
i • uses the chain rule to write an expression for the derivative	1
• simplifies into required form using appropriate trigonometric identities	1
ii • substitutes limits in the primitive function, evaluating in surd form	1
• uses log laws to express in simplest exact form	1

Answer

i.

$$\begin{aligned}\frac{d}{dx}(\log_e \tan x)&=\frac{\sec^2 x}{\tan x}\\ &=\frac{1}{\cos^2 x}\times\frac{\cos x}{\sin x}\\ &=\frac{1}{\cos x\sin x}\end{aligned}$$

ii.

$$\begin{aligned}\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}\frac{2}{\sin x\cos x}\,dx&=2\left[\log_e \tan x\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}}\\ &=2\left(\log_e\sqrt{3}-\log_e 1\right)\\ &=2\left(\tfrac{1}{2}\log_e 3-0\right)\\ &=\log_e 3\end{aligned}$$

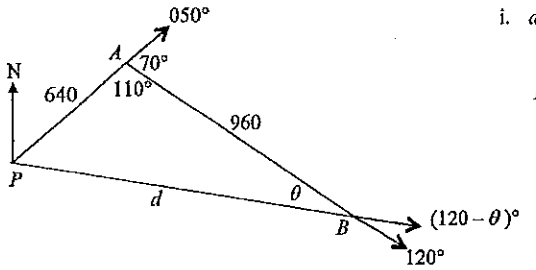
Q13(cont)

c. Outcomes assessed: P4

Marking Guidelines

Criteria	Marks
i • writes a numerical expression for the distance squared by applying the cosine rule	1
• calculates the distance to the required accuracy	1
ii • writes a numerical expression for the sine or cosine of angle at P or B in the triangle	1
• evaluates the angle and deduces the required bearing	1

Answer



i. $d^2 = 640^2 + 960^2 - 2\times 640\times 960\cos 110°$

$\approx 1751474\cdot 352$

$d \approx 1323\cdot 433$

$BP \approx 1323\text{ m}$ (to nearest m)

ii. $\dfrac{\sin\theta}{640} = \dfrac{\sin 110°}{d}$

$\sin\theta \approx \dfrac{640\sin 110°}{d}$

$\approx 0\cdot 4544$

$\therefore\ \theta \approx 27°$

Required bearing is $120° - \theta \approx 093°$

Question14

a. Outcomes assessed: H6

Marking Guidelines

Criteria	Marks
• finds second derivative	1
• deduces set of x values for curve to be concave up	1

Answer

$y = 3x^2 - x^3$ $\quad \dfrac{dy}{dx} = 6x - 3x^2$ $\quad \dfrac{d^2y}{dx^2} = 6 - 6x$ $\quad \dfrac{d^2y}{dx^2} > 0$ for $x < 1$

Curve is concave up for $\{x : x < 1\}$.

b. Outcomes assessed: H3, H8

Marking Guidelines

Criteria	Marks
• finds primitive function	1
• substitutes upper limit and simplifies	1
• completes evaluation in simplest form	1

Answer

$$\int_0^{\log_e 3} \frac{e^x}{e^x + 1}\,dx = \left[\log_e\left(e^x + 1\right)\right]_0^{\log_e 3} = \log_e(3+1) - \log_e 2 = \log_e 2$$

Q14(cont)

c. Outcomes assessed: H6

Marking Guidelines

Criteria	Marks
• finds value of x for which first derivative is 0.	1
• applies first or second derivative test to determine nature of the stationary point	1
• finds y coordinate	1

Answer

$y = x + \dfrac{4}{x^2}$ $\quad \dfrac{dy}{dx} = 1 - \dfrac{8}{x^3}$

$\dfrac{dy}{dx} = 0$ for $x = 2$

$\dfrac{dy}{dx} < 0$ for $0 < x < 2$, $\quad \dfrac{dy}{dx} > 0$ for $x > 2$

$(2, 3)$ is a minimum turning point.

d. Outcomes assessed: H5

Marking Guidelines

Criteria	Marks
• writes an expression for the probability Pierre wins both prizes	1
• identifies other cases where Pierre wins more prizes than Blaise, finding one such probability	1
• adds probabilities of all possible cases and simplifies	1

Answer

Pierre wins more prizes than Blaise if Pierre wins both prizes, or Pierre wins one prize and the other prize winner is neither Pierre nor Blaise. Hence required probability is $\dfrac{4}{25}\times\dfrac{3}{24} + \dfrac{4}{25}\times\dfrac{18}{24} + \dfrac{18}{25}\times\dfrac{4}{24} = \dfrac{13}{50}$

e. Outcomes assessed: P4, H8

Marking Guidelines

Criteria	Marks
i • finds the x coordinate of B	1
ii • writes a definite integral for the magnitude of the required area	1
• finds the primitive function	1
• evaluates in simplest form	1

Answer

i. At intersection $x^2 - 3x + 2 = 2 \Rightarrow x(x-3) = 0$. Hence $x = 3$ at B.

ii. Area of shaded region is given by $\displaystyle\int_0^3 \left\{2 - \left(x^2 - 3x + 2\right)\right\}dx = \int_0^3 \left(3x - x^2\right)dx$

$= \left[\tfrac{3}{2}x^2 - \tfrac{1}{3}x^3\right]_0^3$

$= \tfrac{3}{2}\times 9 - \tfrac{1}{3}\times 27$

Hence shaded area is $4\tfrac{1}{2}$ sq. units

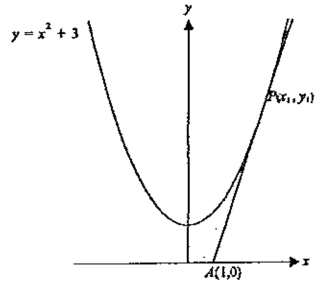
Question 15

a. Outcomes assessed: H5

Marking Guidelines

Criteria	Marks
• uses differentiation to find one expression for the gradient of the tangent at P	1
• finds the gradient of AP directly, then uses equation of the curve to express this in terms of x_1	1
• writes and solves an equation to deduce the value of x_1	1

Answer



$\frac{dy}{dx} = 2x \quad \therefore$ tangent at P has gradient $2x_1$.

Gradient of AP is $\frac{y_1}{x_1 - 1} = \frac{x_1^2 + 3}{x_1 - 1}$

$\therefore \quad 2x_1 = \frac{x_1^2 + 3}{x_1 - 1}$

$2x_1^2 - 2x_1 = x_1^2 + 3$

$x_1^2 - 2x_1 - 3 = 0$

$(x_1 - 3)(x_1 + 1) = 0 \qquad \therefore x_1 > 1 \Rightarrow x_1 = 3$

b. Outcomes assessed: H5

Marking Guidelines

Criteria	Marks
i • finds one solution	1
• finds the second solution	1
ii • sketches a sine curve with amplitude 2 translated upwards by 1 unit	1
• shows required detail about turning points and endpoints	1

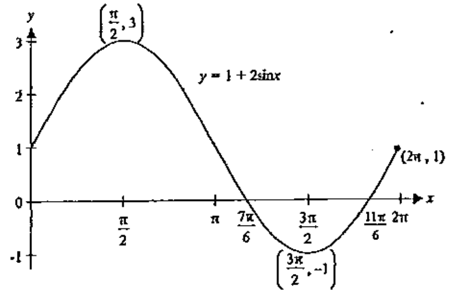
Answer

i. $1 + 2\sin x = 0, \quad 0 \le x \le 2\pi$

$\sin x = -\frac{1}{2}$

$x = \frac{7\pi}{6}, \frac{11\pi}{6}$

ii.



Q15(cont)

c. Outcomes assessed: H5

Marking Guidelines

Criteria	Marks
i • writes simultaneous equations for a and d	1
• finds the value of d	1
• finds the value of a	1
ii • finds the sum of the first 25 terms of the A.P.	1

Answer

i. $a + 9d = 1900$

$a + 19d = 1100$

$-10d = 800$

$\therefore d = -80, \; a = 2620$

ii. $S_{25} = \frac{25}{2}\{2 \times 2620 + 24(-80)\} = 41500$

41500 houses are built over the 25 years

d. Outcomes assessed: H5

Marking Guidelines

Criteria	Marks
i • applies appropriate rules of differentiation to express v as function of t	1
• differentiates a second time to express a as a function of t	1
ii • uses sign of v to deduce when particle is moving towards O	1
iii • uses signs of v and a to deduce when particle is moving towards O and slowing down	1

Answer

i. $x = t(t-3)^2$

$v = 1.(t-3)^2 + t.2(t-3)$

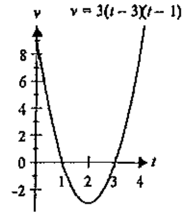
$= (t-3)(t-3+2t)$

$= 3(t-3)(t-1)$

$a = 3\{1.(t-1) + (t-3).1\}$

$= 6(t-2)$

ii. $t = 0 \Rightarrow x = 0$ and $v > 0$ Particle initially moving right from O.



$t > 0 \Rightarrow x > 0$

$\therefore$ particle moves towards O for $v < 0$

i.e. for $1 < t < 3$

iii. Particle moves left and slows down for $v < 0$ and $a > 0$. $\therefore 2 < t < 3$

Question 16

a. Outcomes assessed: H5

Marking Guidelines

Criteria	Marks
i • identifies $-R$ as the derivative of Q with respect to t and integrates	1
• evaluates the constant of integration to express Q as a function of t	1
ii • writes and solves an equation for t when tank is half empty	1
• evaluates R at this time	1

Answer

i. $\frac{dQ}{dt}=-100e^{-0.01t}$

$Q=\frac{-100}{-0.01}e^{-0.01t}+c$

$=10\,000\,e^{-0.01t}+c$

$\left.\begin{matrix}t=0\\Q=10\,000\end{matrix}\right\}\Rightarrow\begin{matrix}c=0\\Q=10\,000e^{-0.01t}\end{matrix}$

ii. $Q=5\,000\Rightarrow e^{-0.01t}=\frac{1}{2}$

$e^{0.01t}=2$

$0\cdot01t=\ln 2$

$\therefore t=100\ln 2$ and $R=100e^{-0.01t}=100\times\frac{1}{2}$

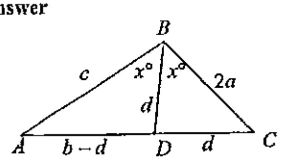
Tank is half empty after $(100\ln 2)$ minutes and is then emptying at a rate of 50 L/min.

b. Outcomes assessed: H5

Marking Guidelines

Criteria	Marks
i • deduces triangles each have one angle of size $x°$, giving reasons	1
• identifies the common angle and deduces triangles equiangular	1
ii • writes relations linking a, b, c, d using sides in proportion	1
• rearranges to find two expressions for bd in terms of a, b, c	1
• eliminates bd and deduces required result	1

Answer



i. In ΔBDC

$\angle DCB=x°$ (*∠'s opp. equal sides are equal*)

Then, in ΔABD, ΔACB

$\angle ABD=\angle ACB=x°$ (*∠ACB, ∠DCB same angle*)

$\angle BAD=\angle CAB$ (*common angle*)

$\therefore \Delta ABD|||\Delta ACB$ (*equiangular, since ∠ sum of each Δ is 180°*)

ii. $\frac{AD}{AB}=\frac{AB}{AC}=\frac{BD}{CB}$ (*sides of similar Δ's are in proportion*)

$\therefore\frac{b-d}{c}=\frac{c}{b}=\frac{d}{2a}$

$\therefore\quad c^2=b^2-bd$

and $2ac=bd$

$\therefore a^2+2ac+c^2=a^2+b^2$

$(a+c)^2=a^2+b^2$

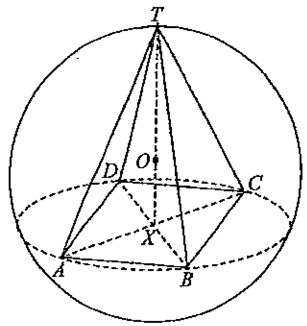
Q16 (cont)

c. Outcomes assessed: H4, H5

Marking Guidelines

Criteria	Marks
i • finds the square of the distance XC in terms of x	1
• finds the area of square $ABCD$ in terms of x	1
• finds the volume of the pyramid and rearranges into required form	1
ii • finds the derivative of V with respect to x in factored form	1
• finds the zero of this derivative within the domain and checks this gives maximum V	1
• substitutes to find the maximum value of V	1

Answer



i. $ABCD$ is a square with area $\frac{1}{2}AC\,.\,BD=\frac{1}{2}AC^2=\frac{1}{2}(2XC)^2$

(since a square is a rhombus with equal diagonals bisecting each other at 90°)

But $XC^2=OC^2-OX^2=1-x^2$ (by Pythagoras' theorem)

$\therefore ABCD$ has area $2(1-x^2)$ and $TX=TO+OX=1+x$

$\therefore$ pyramid has volume $V=\frac{1}{3}.2(1-x^2).(1+x)$

$=\frac{2}{3}(1+x-x^2-x^3)$

ii.

$\frac{dV}{dx}=\frac{2}{3}(1-2x-3x^2)$

$=-\frac{2}{3}(3x-1)(x+1)$

$\frac{d^2V}{dx^2}=\frac{2}{3}(-2-6x)$

$=-\frac{4}{3}(1+3x)$

Since $0<x<1$, $\frac{dV}{dx}=0\Rightarrow x=\frac{1}{3}$, and $\frac{d^2V}{dx^2}<0$.

Hence $V_{max}=\frac{2}{3}(1+\frac{1}{3}-\frac{1}{9}-\frac{1}{27})=\frac{64}{81}$

Maximum volume is $\frac{64}{81}$ m³

Independent HSC Trial Examinations 2017 Mathematics Mapping Grid

Question	Marks	Content	Syllabus Outcomes	Targeted Performance Bands
1	1	Basic arithmetic and algebra	P3	2-3
2	1	Trigonometric ratios	P3	2-3
3	1	Quadratic polynomial and parabola	P4	2-3
4	1	Functions	P5	2-3
5	1	Plane geometry	P4	3-4
6	1	Probability	H5	3-4
7	1	Exponential and logarithmic functions	H3	4-5
8	1	Series and applications	H5	4-5
9	1	Exponential growth and decay	H3	5-6
10	1	Trigonometric functions	H5	5-6
11 a	2	Basic arithmetic and algebra	P4	2-3
b	2	Basic arithmetic and algebra	P4	2-3
c	2	Differentiation	P4	2-3
d i	1	Trigonometric functions	H5	3-4
ii	2	Differentiation; Exponential and logarithmic functions	H5	3-4
e	3	Exponential and logarithmic functions	H5	3-4
f	3	Integration	H8	3-4
12 a	2	Quadratic polynomial and parabola	P4	2-3
b	2	Trigonometric functions	H5	3-4
c	2	Functions	P5	2-3
d	3	Integration	H8	3-4
e i	1	Linear function	H5	3-4
ii	2	Linear function	H5	3-4
iii	3	Linear function	H5	3-4
13 a	2	Probability	H5	2-3
b	2	Trigonometry ratios; Integration	P4, H8	2-3
c	3	Exponential and logarithmic functions; Series	H3, H5	3-4
d i	2	Trigonometric functions	H5	4-5
ii	2	Integration	H8	4-5
e i	2	Trigonometric ratios	P4	2-3
ii	2	Trigonometric ratios	P4	2-3
14 a	2	Geometrical applications of differentiation	H6	4-5
b	3	Exponential and logarithmic functions; Integration	H3, H8	4-5
c	3	Geometrical applications of differentiation	H6	4-5
d	3	Probability	H5	4-5
e i	1	Basic arithmetic and algebra	P4	2-3
ii	3	Integration	H8	4-5

Question	Marks	Content	Syllabus Outcomes	Targeted Performance Bands
15 a	3	Differentiation; Linear function	H5	5-6
b i	2	Trigonometric functions	H5	4-5
ii	2	Trigonometric functions	H5	4-5
c i	3	Series and applications	H5	4-5
ii	1	Series and applications	H5	2-3
d i	2	Kinematics	H5	5-6
ii	1	Kinematics	H5	5-6
d iii	1	Kinematics	H5	5-6
16 a i	2	Rates of change	H5	5-6
ii	2	Rates of change	H5	5-6
b i	2	Plane geometry	H5	5-6
ii	3	Plane geometry	H5	5-6
c i	3	Basic arithmetic and algebra	H4	5-6
ii	3	Geometrical applications of differentiation	H5	5-6