

Student Number : .....

2000  
TRIAL EXAMINATION  
MATHEMATICS  
2/3 UNIT (COMMON)

*Time allowed : Three (3) hours  
plus 5 minutes reading time*

**Directions to Candidates**

- Attempt ALL questions.
- ALL questions are of equal value.
- All necessary working must be shown in every question.
- Marks may be deducted for careless or badly arranged work.
- Standard integrals are printed on page 12.
- Board-approved calculators may be used.
- Each question is to be started on a new page.
- This examination paper must NOT be removed from the examination room.

**Directions to School or College**

To ensure maximum confidentiality and security, examination papers must NOT be removed from the examination room. Examination papers may not be returned to students or used for revision purposes till September 2000. These examination papers are supplied Copyright Free, as such the purchaser may photocopy and/or cut and paste same for educational purposes within the confines of the school or college.

All care has been taken to ensure that this examination paper is error free and that it follows the style, format and material content of previous Higher School Certificate Examination. Candidates are advised that authors of this examination paper cannot in any way guarantee that the 2000 HSC Examination will have a similar content or format.

QUESTION 1.	(Start a new booklet)	MARKS
(a)	Calculate $\frac{432.5}{18.9 \times 4.6}$ correct to two decimal places.	2
(b)	Factorise $3x^2 - x - 10$ .	2
(c)	Find a primitive of $x^7 - 5$ .	2
(d)	Find the discriminant of the equation $3x^2 + 4x + 12 = 0$ . and state whether the roots are real.	2
(e)	Evaluate $\log_6 279936$ .	2
(f)	Solve and mark on a number line the values of $x$ for which $ x - 2  \leq 8$ .	2

**QUESTION 2.**     *(Start a new booklet)*

Let  $A$ ,  $B$  and  $C$  be the points  $(0,13)$ ,  $(8,7)$  and  $(1,6)$  respectively.

- (a) Show that  $AC$  and  $BC$  are perpendicular. 2
- (b) Show that  $AC = BC$ . 2
- (c) Find the area of the triangle  $ABC$ . 2
- (d) Write equation of the circle passing through  $A$  and  $B$  with the centre at  $C$ . 2
- (e) This circle cuts the  $y$ -axis in points  $A$  and  $K$ . Find the coordinates of point  $K$ . 2
- (f) Find the area of the sector  $ACB$ . 2

**QUESTION 3.** (Start a new booklet)

**MARKS**

(a) Differentiate the following functions:

**6**

(i)  $\sqrt{1 + \ln x}$

(ii)  $xe^x$

(iii)  $\frac{\tan x}{1 - x}$

(b) Evaluate the following integrals :

**4**

(i)  $\int_1^6 (8x + 5)^3 dx$

(ii)  $\int_0^{\pi/9} \sec^2 3x dx$

(c) Consider the parabola with equation  $(x - 3)^2 = 12(y + 2)$ .  
Find the coordinates of the focus of the parabola.

**2**

**QUESTION 4.**     *(Start a new booklet)*

**MARKS**

(a)     A red, a blue and a green die are thrown. Find the probability that 7

- (i)     the red die shows a four, the blue die a six and the green die a three ;
- (ii)    a four, a six and a three turn up ;
- (iii)   a five does not turn up on any face ;
- (iv)    all three faces show the same number ;
- (v)    the total score on the three dice is exactly 4 ;

(b)     (i)     Find the equation of the locus of a point  $P(x, y)$  that moves so that 5  
the line  $PA$  is perpendicular to line  $PB$  where  $A = (-1, 4)$  and  $B = (5, 12)$ .

(ii)    Describe all main features of the locus.

**QUESTION 5.**     *(Start a new booklet)*

**MARKS**

(a) Solve for  $x$  :  $25^x \times 125^{2-x} = 1$  .

**2**

(b) Consider the curve given by  $y = 6x^4 - 8x^3 + 3$ .

**10**

(i) Find  $\frac{dy}{dx}$  .

(ii) Find the coordinates of the two stationary points.

(iii) Find all values of  $x$  for which  $\frac{d^2y}{dx^2} = 0$ .

(iv) Determine the nature of the stationary points.

(v) Hence sketch the curve  $y = 6x^4 - 8x^3 + 3$ .

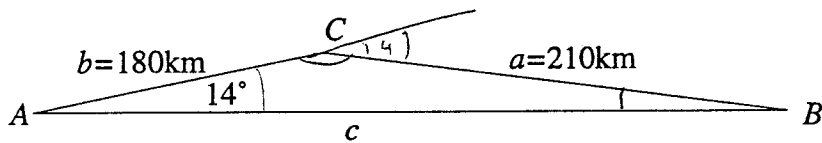
**QUESTION 6.** (Start a new booklet)

**MARKS**

- (a) A satellite has a radioisotope power supply. The power output in watts is given by the equation  $P = 70e^{-0.004t}$  where  $t$  is the time in days. 4

- (i) How much power will be available at the end of the year ?
- (ii) If the equipment aboard the satellite requires 10 watts of power to operate properly, what is the operational life of the satellite ?

- (b) 8



A plane flew 180 km when the pilot discovered a course error of  $14^\circ$ . The pilot immediately corrected his course and after flying another 210 km reached the destination.

- (i) What would have been the total distance if the course error had not been made ?
- (ii) At what time did the aircraft arrive, if it was originally expected to land at 2 p.m., given that the plane flew at 300 km/h ?

**QUESTION 7.**     *(Start a new booklet)*

**MARKS**

- (a)     Use the trapezoidal rule with 4 function values to approximate the definite integral  $\int_2^8 (-x^2 + 13x - 12)dx$ . 3
- (b)     Find the volume of the solid formed when an area between  $y = \ln(2x)$  and the  $y$ -axis is rotated about the  $y$ -axis from  $y = 1$  to  $y = 6$ . 4
- (c)     (i)     Sketch  $y = \frac{4}{x+1}$ . 5
- (ii)    An area between  $y = \frac{4}{x+1}$  and the  $x$ -axis from  $x = 0$  to  $x = 8$  is cut into two parts of equal areas by a vertical line  $x = k$ .  
                 Calculate the value of  $k$ .



**QUESTION 8.**     *(Start a new booklet)*

**MARKS**

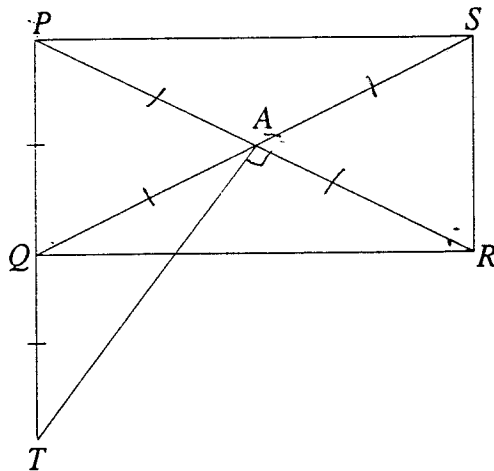
- (a)     Upon arrival at the campsite, a group of students began setting up their 8 tents, one at a time. With practice, they found that each tent took only 90% of time to assemble as the previous one. That is, the first took 20 minutes, the second takes 18 minutes and so on. **4**
- (i)     Find how long the 8<sup>th</sup> tent took to assemble.
- (ii)    Find the total time taken to assemble all 8 tents.
- 
- (b)     For the last phase of preparations before the Olympic Games, swimmers started a new training schedule. On the first day they had to complete 26 laps of the pool. Each succeeding day they increased their training by 6 laps, until their daily schedule reached 200 laps. They then continued swimming 200 laps daily for a total of 15 days to fully complete their training schedule. **5**
- (Note : Length of the pool = 50 m.)
- (i)     On which day did they first complete 200 laps ?
- (ii)    Find the total length (in km) completed by the swimmers.
- 
- (c)     How much should parents deposit into a savings account on the day their daughter is born, so that when she reaches the age of 18, she can collect \$10 000 to help pay her University studies, given that the investment is compounded monthly at a rate of 4% per annum ? **3**

QUESTION 9. (Start a new booklet)

MARKS

(a)

6



NOT TO SCALE

Copy the diagram into your booklet.

Given :  $PQRS$  is a rectangle. Diagonals  $PR$  and  $SQ$  meet at  $A$ .

$Q$  is the midpoint of  $PT$ .  $PT = PR$ .

Prove :  $AR \perp AT$ .

(b) A closed cylinder with radius  $r$  and height  $h$  will hold a volume of  $250\pi \text{ cm}^3$ .

6

(i) Show that the surface area  $S = 2\pi r^2 + \frac{500\pi}{r}$

(ii) Find dimensions of the cylinder so that it will have the least surface area.

**QUESTION 10.** (Start a new booklet)

**MARKS**

- (a) (i) Solve  $1 - 2\sin x = 0$  for  $0 \leq x \leq 2\pi$ . 4
- (ii) Sketch the graph of  $y = 1 - 2\sin x$  for  $0 \leq x \leq 2\pi$ .
- (b) A particle  $P$  moves along the  $x$ -axis. The velocity  $v$ , in cm/s is given by the equation  $v = 1 - 2\sin t$ ,  $t \geq 0$ , where  $t$  is the time in seconds. 8  
Initially the particle is 2 cm to the right of the origin.
- (i) Find an expression for the position of the particle at time  $t$ .
- (ii) In what direction is the particle moving when  $t = 0$  ?
- (iii) When does the particle change direction during the first  $\pi$  seconds ?
- (iv) Determine the distance travelled during the first  $\frac{\pi}{2}$  seconds.
- (v) Particle  $Q$  moves along the  $x$ -axis so that its position is given by the equation  $x = 6 + t + 2\cos t$ ,  $t \geq 0$ . Describe the motion of particle  $Q$  relative to particle  $P$ .

### STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + C, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} dx = \ln x + C, \quad x > 0$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + C, \quad a \neq 0$$

$$\int \cos ax dx = \frac{1}{a} \sin ax + C, \quad a \neq 0$$

$$\int \sin ax dx = -\frac{1}{a} \cos ax + C, \quad a \neq 0$$

$$\int \sec^2 ax dx = \frac{1}{a} \tan ax + C, \quad a \neq 0$$

$$\int \sec ax \tan ax dx = \frac{1}{a} \sec ax + C, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a} + C, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a} + C, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln(x + \sqrt{x^2 - a^2}) + C, \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln(x + \sqrt{x^2 + a^2}) + C$$

NOTE :  $\ln x = \log_e x, \quad x > 0$

### QUESTION 1.

(a)  $\frac{432.5}{18.9 \times 4.6} = \frac{432.5}{86.94}$   
 $= 4.974695$  Al. 1  
 $\approx 4.97$  (to 2d.p.) Aw. 2

(b)  $3x^2 - x - 10 = (3x + 5)(x - 2)$  Aw. 2

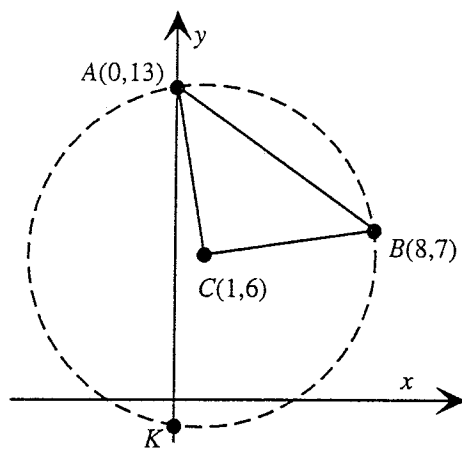
(c)  $\int (x^7 - 5) dx = \frac{x^8}{8} - 5x + C$  Aw. 2

(d)  $\Delta = b^2 - 4ac$   
 $= 16 - 144$   
 $= -144 < 0$  Al. 1

There are no real roots. Aw. 2

(e)  $\log_6 279936 = \frac{\ln 279936}{\ln 6}$  Al. 1  
 $= 7$  Aw. 2

(f)  $|x - 2| \leq 8$   
 $-8 \leq x - 2 \leq 8$   
 $-6 \leq x \leq 10$  Al. 1



### QUESTION 2.

(a)  $m_{AC} = \frac{13-6}{0-1} = -7$   
 $m_{BC} = \frac{7-6}{8-1} = \frac{1}{7}$  Al. 1

$m_{AC} \cdot m_{BC} = -7 \times \frac{1}{7} = -1$   
 $\therefore AC \perp BC$  Aw. 2

(b)  $d_{AC} = \sqrt{(1-0)^2 + (6-13)^2} = \sqrt{50} = 5\sqrt{2}$  units Al. 1

$d_{BC} = \sqrt{(8-1)^2 + (7-6)^2} = \sqrt{50} = 5\sqrt{2}$  units Aw. 2

(c) Area  $\Delta ABC = \frac{AC \times BC}{2} = \frac{2\sqrt{5} \times 2\sqrt{5}}{2}$  Al. 1  
 $= 25$  units<sup>2</sup> Aw. 2

(d) Centre  $C(1,6)$  and radius  $r = 5\sqrt{2}$  units Al. 1  
 Equation:  $(x-1)^2 + (y-6)^2 = 50$  Aw. 2

(e) For points  $A$  and  $K$ ,  $x = 0$ .  
 $(0-1)^2 + (y-6)^2 = 50$  Al. 1  
 $(y-6)^2 = 49$   
 $y-6 = \pm 7$   
 $y = -1$  or  $13$   
 $\therefore K(0, -1)$  Aw. 2

(f) Area of sector  $ACB$   
 $= \frac{\pi r^2}{4} = \frac{50\pi}{4}$  Al. 1  
 $= \frac{25\pi}{2}$  units<sup>2</sup>  $\approx 39.3$  units<sup>2</sup> Aw. 2

### QUESTION 3.

(a) (i)  $\frac{d}{dx} \left[ (1 + \ln x)^{\frac{1}{2}} \right] = \frac{1}{2} (1 + \ln x)^{-\frac{1}{2}} \times \frac{1}{x}$   
 $= \frac{1}{2x\sqrt{1 + \ln x}}$  Aw. 2

(ii)  $\frac{d}{dx} [xe^x] = 1 \times e^x + x \cdot e^x$   
 $= e^x(1+x)$  Aw. 2

(iii)  $\frac{d}{dx} \left[ \frac{\tan x}{1-x} \right] = \frac{(1-x) \cdot \frac{d}{dx} (\tan x) - \tan x \cdot \frac{d}{dx} (1-x)}{(1-x)^2}$   
 $= \frac{(1-x)\sec^2 x - \tan x(-1)}{(1-x)^2}$   
 $= \frac{\sec^2 x - x\sec^2 x + \tan x}{(1-x)^2}$  Aw. 2

(b) (i)  $\int_1^6 (8x+5)^3 dx = \left[ \frac{(8x+5)^4}{4 \times 8} \right]_1^6$  Al. 1  
 $= \frac{53^4}{32} - \frac{13^4}{32}$   
 $= 245685$  Aw. 2

(ii)  $\int_0^{\frac{\pi}{9}} \sec^2 3x dx = \left[ \frac{\tan 3x}{3} \right]_0^{\frac{\pi}{9}}$  Al. 1  
 $= \frac{\tan \frac{\pi}{3} - \tan 0}{3}$   
 $= \frac{\sqrt{3}}{3}$  Aw. 2

(c)  $V(3, -2)$  and  $a = 3$  Al. 1  
 parabola concave upwards,  $\therefore F(3, 1)$  Aw. 2

#### QUESTION 4.

(a) (i)  $P(R=4, B=6, G=3) = \left(\frac{1}{6}\right)^3 = \frac{1}{216}$  Aw. 1

(ii)  $P(R=4, B=6, G=3) + P(R=4, B=3, G=6)$   
 $+ P(R=3, B=6, G=4) + P(R=3, B=4, G=6)$   
 $+ P(R=6, B=4, G=3) + P(R=6, B=3, G=4)$   
 $= 6 \times \left(\frac{1}{216}\right) = \frac{1}{36}$  Aw. 2

(iii)  $P(R=5, B=5, G=5) = \left(\frac{5}{6}\right)^3 = \frac{125}{216}$  Aw. 1

(iv)  $P(\text{same number}) = 6 \times \left(\frac{1}{6}\right)^3 = \frac{1}{36}$  Aw. 1

(v)  $P(\text{Total} = 4) = P(R=2, B=1, G=1)$   
 $+ P(R=1, B=2, G=1) + P(R=1, B=1, G=2)$   
 $= 3 \times \left(\frac{1}{216}\right) = \frac{1}{72}$  Aw. 2

(b) (i)  $m_{AP} \times m_{BP} = -1$

$\frac{y-4}{x+1} \times \frac{y-12}{x-5} = -1$  Al. 1

$y^2 - 16y + 48 = -x^2 + 4x + 5$   
 $x^2 - 4x + y^2 - 16y + 43 = 0$  Al. 1

$(x-2)^2 + (y-8)^2 = -43 + 4 + 64$   
 $(x-2)^2 + (y-8)^2 = 25$  Aw. 3

(ii) The locus of point  $P$  is a circle with  
 centre in  $(2, 8)$  and radius  $r = 5$  units Aw. 2

#### QUESTION 5.

(a)  $25^x \times 125^{2-x} = 1$   
 $(5^2)^x \times (5^3)^{2-x} = 5^0$   
 $5^{2x} \times 5^{6-3x} = 5^0$   
 $5^{2x+6-3x} = 5^0$   
 $2x+6-3x = 0$   
 $x = 6$  Aw. 2

(b)  $y = 6x^4 - 8x^3 + 3$   
 (i)  $\frac{dy}{dx} = 24x^3 - 24x^2$  Aw. 1

(ii)  $\frac{dy}{dx} = 0$  Al. 1  
 $24x^3 - 24x^2 = 0$   
 $24x(x-1) = 0$   
 Case 1:  $x = 0, y = 3$  Al. 1  
 Case 2:  $x = 1, y = 1$  Aw. 3

(iii)  $\frac{d^2y}{dx^2} = 0$   
 $72x^2 - 48x = 0$   
 $24x(3x-2) = 0$  Al. 1  
 Case 1:  $x = 0$   
 Case 2:  $x = \frac{2}{3}$  Aw. 2

(iv) Nature of stationary point at  $(1, 1)$ :  
 $f''(1) = 72 - 48 = 12 > 0$   
 Stationary point at  $(1, 1)$  is a minimum. Al. 1

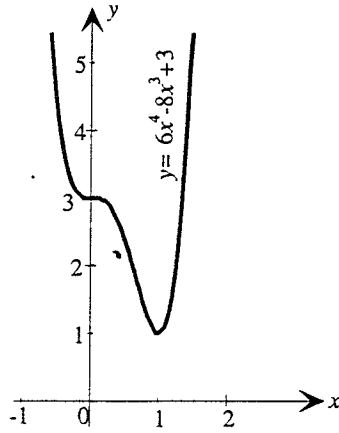
(iv) Nature of stationary point at (0,3) :

$x$	-0.5	0	0.5
$\frac{dy}{dx}$	-9	0	-3

Stationary point at (0,3) is a horizontal inflexion.

Aw. 2

(v)



Aw. 2

### QUESTION 6.

(a) (i)  $P = 70e^{-0.004t}$   
 $P = 70e^{-0.004 \times 365}$   
 $P = 16.3$  watts

Aw. 1

(ii)  $70e^{-0.004t} > 10$   
 $e^{-0.004t} > \frac{1}{7}$   
 $e^{0.004t} < 7$

Al. 1

$0.004t < \ln 7$

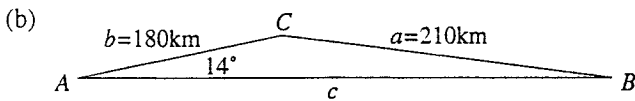
Al. 1

$t < \frac{\ln 7}{0.004}$

$t < 486.4$

$t = 486$  days

Aw. 3



$\frac{a}{\sin A} = \frac{b}{\sin B}$   
 $\frac{210}{\sin 14} = \frac{180}{\sin B}$

Al. 1

$\sin B = \frac{180 \times \sin 14}{210}$

Al. 1

$B = 11^\circ 58'$

Al. 1

$\frac{c}{\sin C} = \frac{b}{\sin B}$

$\frac{c}{\sin 154^\circ 02'} = \frac{180}{\sin 11^\circ 58'}$

Al. 1

$c = \frac{180 \times \sin 154^\circ 02'}{\sin 11^\circ 58'}$

Al. 1

$c = \frac{180 \times 0.437848175}{0.20736}$

$c = 380.1$  km

Aw. 6

(ii) The plane flew altogether 390 km, i.e. extra 9.9 km at 300 km/h. needing extra time

$t = \frac{d}{v} = \frac{9.9}{300}$

$= 0.33$  hrs. = 2 min.

Al. 1

$\therefore$  The aircraft arrived at 2.02 PM.

Aw. 2

### QUESTION 7.

(a)  $y = -x^2 + 13x - 12$

$x$	2	4	6	8
$y$	10	24	30	28

Al. 1

$I = \frac{h}{2} [f(2) + 2f(4) + 2f(6) + f(8)]$

$\approx \frac{2}{2} [10 + 2 \times 24 + 2 \times 30 + 28]$

Al. 1

$\approx 10 + 28 + 60 + 28$

$\approx 126$

Aw. 3

(b)  $y = \log_e 2x$

$2x = e^y$

$x = \frac{e^y}{2}$

Al. 1

$V = \pi \int_1^6 \left(\frac{e^y}{2}\right)^2 dy$

Al. 1

$= \frac{\pi}{4} \int_1^6 e^{2y} dy$

$= \frac{\pi}{4} \left[\frac{e^{2y}}{2}\right]_1^6$

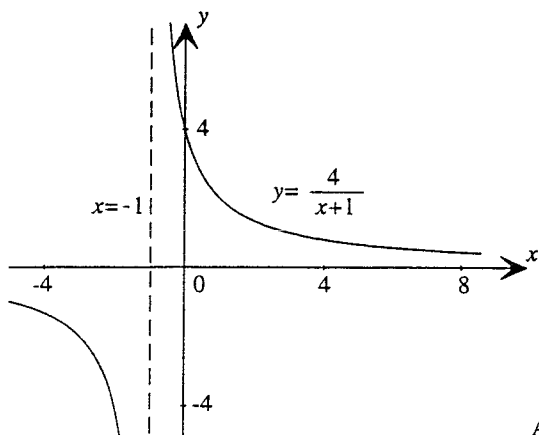
Al. 1

$= \frac{\pi}{8} (e^{12} - e)$  units<sup>3</sup>

Aw. 4

$\approx 63913$  units<sup>3</sup>

(c) (i)



Aw. 1

(ii)

$$\int_0^k \frac{4}{x+1} dx = \int_k^8 \frac{dx}{x+1}$$

Al. 1

$$\int_0^k \frac{dx}{x+1} = \int_k^8 \frac{dx}{x+1}$$

$$[\ln(x+1)]_0^k = [\ln(x+1)]_k^8$$

Al. 1

$$\ln(k+1) - \ln 1 = \ln 9 - \ln(k+1)$$

$$2 \ln(k+1) = \ln 9 + \ln 1$$

$$2 \ln(k+1) = \ln 9$$

Al. 1

$$2 \ln(k+1) = 2 \ln 3$$

$$k+1 = 3$$

$$k = 2$$

Aw. 4

### QUESTION 8.

(a) Geometric Series with  $a = 20$  and  $r = \frac{9}{10}$ .

$$(i) T_8 = ar^7$$

$$= 20 \times \left(\frac{9}{10}\right)^7$$

Al. 1

$$= 9.566 \text{ min.}$$

$$= 9 \text{ min. } 34 \text{ sec.}$$

Aw. 2

$$(ii) s_8 = \frac{a(1-r)^7}{1-r}$$

$$= \frac{20 \times \left[1 - \left(\frac{9}{10}\right)^7\right]}{1 - \frac{9}{10}}$$

Al. 1

$$= 104.34 \text{ min.}$$

$$= 104 \text{ min. } 30 \text{ sec.}$$

Aw. 2

(b) Arithmetic Series with  $a = 26$  and  $d = 6$ .

$$(i) T_n = a + (n-1)d$$

$$200 = 26 + (n-1) \times 6$$

Al. 1

$$200 = 26 + 6n - 6$$

$$6n = 180$$

$$n = 30$$

On the 30<sup>th</sup> day they completed for the first time 200 laps in one day.

Aw. 2

$$(ii) S_{TOTAL} = S_{30} + S^{**}$$

$$= \frac{n}{2}(a+l) + S^{**}$$

Al. 1

$$= \frac{30}{2}(26 + 200) + [14 \times 200]$$

$$= 3164 + 2800$$

$$= 5964 \text{ laps}$$

Al. 1

The swimmers completed a total of

$$5964 \times 50 \text{ m} = 298.2 \text{ km}$$

Aw. 3

$$(c) A_n = P \times \left(1 + \frac{r}{100}\right)^n$$

Al. 1

$$10000 = P \times \left(1 + \frac{12}{100}\right)^{216}$$

Al. 1

$$10000 = P \times \left(1 + \frac{1}{300}\right)^{216}$$

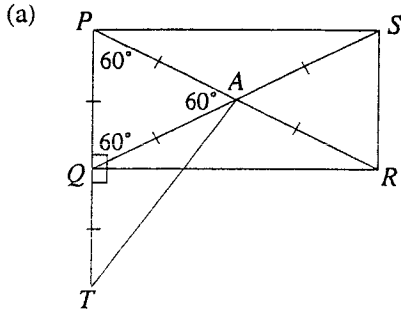
$$10000 = 2.051974826P$$

$$P = \$4873.35$$

Aw. 3



**QUESTION 9.**



$AP = AR = AQ = AS$  (diagonals of a rectangle are equal length and bisect each other)

Al. 1

$PT = PR$  (given)

$$\therefore \frac{1}{2}PT = \frac{1}{2}PR$$

$$\therefore PQ = AP$$

Hence  $AP = AR = AQ = AS = PQ = QT$ .

$\triangle APQ$  is equilateral (all sides are equal length) Al. 1

$\triangle QTA$  is isosceles (2 sides are equal length) Al. 1

$\angle PQA = \angle PAQ = 60^\circ$  (angles in an equilateral  $\triangle$ )

$\angle TQA = 120^\circ$  (exterior angle of a  $\triangle$ )

$$\angle QAT = \frac{180^\circ - 120^\circ}{2} = 30^\circ \text{ (base angle of isosceles } \triangle,$$

given that the sum of interior angles in a  $\triangle = 180^\circ$ ) Al. 1

$$\angle PAT = \angle PAQ + \angle QAT = 60^\circ + 30^\circ = 90^\circ$$

$$\angle RAT = 180^\circ - 90^\circ = 90^\circ \text{ (} PAR \text{ is a straight line)}$$

$\therefore AR \perp AT$ . Al. 1

Aw. 6

(b) (i)  $V = \pi r^2 h$

$$250\pi = \pi r^2 h$$

$$h = \frac{250}{r^2}$$

Al. 1

$$S = 2\pi r^2 + 2\pi r h$$

$$S = 2\pi r^2 + 2\pi r \times \frac{250}{r^2}$$

$$S = 2\pi r^2 + \frac{500\pi}{r}$$

Aw. 2

(ii)  $S = 2\pi r^2 + 500\pi r^{-1}$

$$\frac{dS}{dr} = 0$$

$$4\pi r - 500\pi r^{-2} = 0$$

Al. 1

$$4\pi r = \frac{500\pi}{r^2}$$

$$r^3 = 125$$

$$r = 5$$

Al. 1

When  $r = 5$  :

$$\frac{d^2S}{dr^2} = 4\pi + \frac{1000\pi}{r^3}$$

$$= 4\pi + \frac{1000\pi}{125}$$

Minimum value occurs when  $r = 5$  cm. Al. 1

When  $r = 5$ ,  $h = \frac{250}{5^2} = 10$  cm.

$\therefore$  The cylinder will have the least surface area

when  $r = 5$  cm and  $h = 10$  cm.

Aw. 4

**QUESTION 10.**

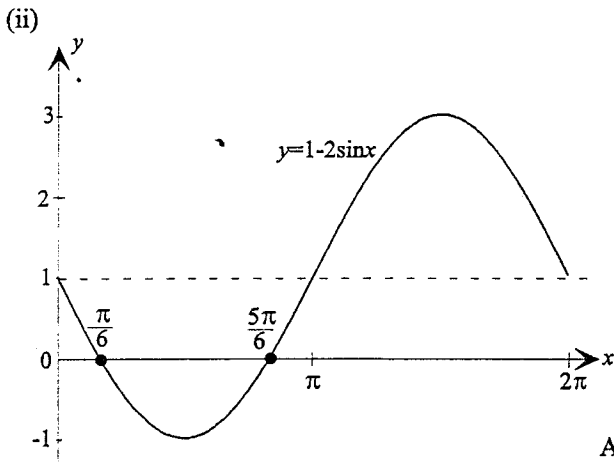
(a) (i)  $1 - 2 \sin x = 0$  for  $0 \leq x \leq 2\pi$

$2 \sin x = 1$

$\sin x = \frac{1}{2}$

$x = \frac{\pi}{6}$  or  $\frac{5\pi}{6}$

Aw. 2



(b) (i)  $x = \int (1 - 2 \sin t) dt$

$x = t + 2 \cos t + C$

Al. 1

When  $t = 0$ ,  $x = 2 \therefore C = 0$

$\therefore x = t + 2 \cos t$

Aw. 2

(ii)  $v(0) = 1 > 0$  particle is moving towards right, away from the origin. Aw. 1

(iii)  $v = 0$  when  $t = \frac{\pi}{6}$  seconds Aw. 1

(iv)  $x = \int_0^{\frac{\pi}{6}} v dt + \left| \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} v dt \right|$  Al. 1

$= \left[ t + 2 \cos t \right]_0^{\frac{\pi}{6}} + \left| \left[ t + 2 \cos t \right]_{\frac{\pi}{6}}^{\frac{\pi}{2}} \right|$

$= \left\{ \left( \frac{\pi}{6} + 2 \cos \frac{\pi}{6} \right) - (0 + 2 \cos 0) \right\}$

$+ \left| \left( \frac{\pi}{2} + 2 \cos \frac{\pi}{2} \right) - \left( \frac{\pi}{6} + 2 \cos \frac{\pi}{6} \right) \right|$  Al. 1

$= \left\{ \left( \frac{\pi}{6} + 2 \times \frac{\sqrt{3}}{2} \right) - (0 + 2) \right\} +$

$+ \left| \left( \frac{\pi}{2} + 0 \right) - \left( \frac{\pi}{6} + 2 \times \frac{\sqrt{3}}{2} \right) \right|$

$\approx 0.255649583 + |-0.684853256|$

$\approx 0.94$  cm Aw. 3

(v) Particle  $Q$  moves in the same direction as particle  $P$  and is at all times 6 cm to the right of point  $P$ . Aw. 1