Student Number: ....

2000
Year 11, PRELIMINARY EXAMINATION

MATHEMATICS
3 Unit (Additional)
and
3/4 Unit (Common)

Time Allowed - Two (2) hours
(plus 5 minutes reading time)

Directions to Candidates

- Attempt ALL questions
- Show all necessary working, marks may be deducted for careless or untidy work
- Standard integrals are printed on the last page
- Board-approved calculators may be used
- Additional Answer Booklets are available

Directions to School or College

Question One

(a) Solve for x: $\dfrac{6}{|x+1|} \geq 3$ *(3 marks)*

(b) Factorise $ay(3+2x^2)+x(2y^2+3a^2)$ *(2 marks)*

(c) Simplify $\dfrac{y^{-2}-x^{-2}}{y^{-2}-2x^{-1}y^{-1}-3x^{-2}}$ *(3 marks)*

(d) Solve for x: $x+\sqrt{5-x}+1=0$ *(2 marks)*

(e) Differentiate with respect to x: $y=\dfrac{1}{2x^2}-\dfrac{1}{3x}$ *(2 marks)*

Question Two

(a) Solve for x: $3x+2=|1-2x|$ *(2 marks)*

(b) Give the general solution of the trigonometric equation $1+2\cos\theta=0$ *(2 marks)*

(c) (i) Use the factor theorem to find the point of intersection of the curve $y=-x^3$ and the line $x-y+10=0$

(ii) Calculate to the nearest degree the size of the acute angle between the curve and the line at their point of intersection *(5 marks)*

(d) Twelve Prefects are to be chosen from a group of 62 Year 12 students.

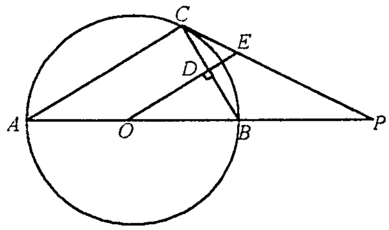
(i) In how many ways can they be chosen?

(ii) In how many ways can they line up to receive their badges if the Head-Prefect is first, followed by the two Vice-Prefects? *(3 marks)*

Question Three

(a) Find the Cartesian equation of the curve whose parametric equations are: $x = \cos 2t$, $y = \sin t$ $(0 \le t \le 360°)$ *(2 marks)*

(b) In the figure, AB is the diameter of the circle centre O. AB is produced to P. PC is a tangent to the circle at C and the perpendicular from O to BC intersects BC at D and PC at E.



(i) Prove that $AC // OE$

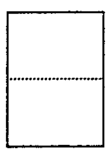
(ii) If $\angle BCP = x$, name, with reasons, two other angles equal to x

(iii) Prove that $OBEC$ is a cyclic quadrilateral

(iv) Prove that $\angle P = 90° - 2x$ *(5 marks)*

(c) Solve for β, $0 \le \beta \le 360°$ if $\cos\beta - \sqrt{3}\sin\beta = \sqrt{3}$ *(3 marks)*

(d) A4 copy-paper has the property that if it is folded in half as indicated, the ratio of length to width is constant

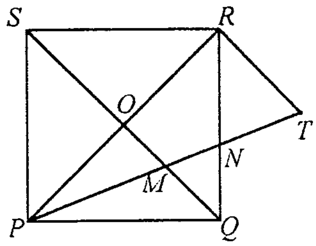


(Figure not to scale)

Calculate the ratio of length to width and express it in terms of a surd *(2 marks)*

Question Four

(a) $PQRS$ is a square with diagonals intersecting at O. PM, with M on QS, bisects $\angle QPO$. PM is produced to T such that $PM = MT$. PT cuts RQ in N.



(i) What is the size of $\angle POM$? Why?

(ii) Prove: (α) $\angle NRT = 45°$

(β) $\Delta NRT \,|||\, \Delta NQM$

(γ) $\dfrac{2OM}{QM} = \dfrac{RN}{QN}$ *(6 marks)*

(b) (i) Derive the equation of the tangent to the point $P\left(2ap, ap^2\right)$ on the parabola $x^2 = 4ay$

(ii) If the tangent at P cuts the y-axis at T, show that the midpoint of PT always lies on the x-axis *(3 marks)*

(c) Prove the trigonometric identity:

$$\frac{4\cos\theta - 3\sec\theta}{1 - 2\sin\theta} + \frac{1 - 2\sin\theta}{\cos\theta} = 2\sec\theta$$

(3 marks)

Question Five

(a) Solve for θ $(0 \le \theta \le 360°)$: $5\sin\theta\cos\theta + 5\cos^2\theta = 2$ *(3 marks)*

(b) Solve the cubic equation $3x^3 + 19x^2 + 22x - 24 = 0$ given that two roots are consecutive integers *(4 marks)*

(c) A (1,2) and B (7,9) are two points on a number plane. C divides the interval AB internally in the ratio 2:1 while D divides the interval AB externally in the ratio 4:1

(i) Find the coordinates of both C and D

(ii) Hence show that B is the midpoint of CD *(5 marks)*

Question Six

(a) Given the expansion of $\cos(\alpha - \beta)$, derive the expansion of $\cos(\alpha + \beta)$ *(2 marks)*

(b) Calculate the exact value of cos 79° cos 311° + sin 101° sin 49° without using a calculator *(2 marks)*

(c) (i) Find the equation of the normal to the curve $xy = 6$ at $x = 2$

(ii) Find the coordinates of the point P, at which the normal cuts the curve again *(5 marks)*

(d) (i) Determine the nature of the roots of $x^2 - 2x + 2 = 0$

(ii) Solve for x: $\dfrac{x^2 - 2x + 2}{3 - x} \le 0$ *(3 marks)*

Question Seven

(a) If the polynomial $P(x)$ is divided by $(x+1)(x-2)$, the quotient is $Q(x)$ and the remainder is $ax+b$

(i) Express $P(x)$ in an identity containing all these quantities

(ii) Hence find the remainders if $P(x)$ is divided by $x+1$ and $x-2$ separately

(iii) Now find the values of a and b if these remainders are -2 and 7 respectively *(4 marks)*

(b) From what external point are the tangents to the parabola $y = x^2$ to be drawn so that $6x - 3y - 5 = 0$ is the equation of the chord of contact? *(4 marks)*

(c) Four couples arrive at a holiday resort where there are 4 hotels

(i) How many different accommodation arrangements are possible if there are no restrictions on where they stay?

(ii) If each couple stays at a different hotel, how many arrangements are possible?

(iii) Two couples are close friends and wish to stay at the same hotel. If the other two couples can stay at any other hotel, how many different arrangements are there?

(iv) If the only restriction is that one couple cannot afford to stay at the most expensive hotel, how many arrangements are possible? *(4 marks)*

SUGGESTED ANSWERS, MATHEMATICS 3U - Preliminary 2000

Question 1

(a)

$$\frac{6}{|x+1|} \geq 3$$

$$\frac{|x+1|}{6} \leq \frac{1}{3}$$

$$|x+1| \leq 2$$

$$-2 \leq x+1 \leq 2$$

$$-3 \leq x \leq 1, \text{ but } x \neq -1$$

(b)

$$ay(3+2x^2)+x(2y^2+3a^2)$$

$$= 3ay + 2ax^2y + 2xy^2 + 3a^2x$$

$$= 3ay + 2xy^2 + 3a^2x + 2ax^2y$$

$$= y(3a+2xy) + ax(3a+2xy)$$

$$= (3a+2xy)(y+ax)$$

(c)

$$\frac{y^{-2}-x^{-2}}{y^{-2}-2x^{-1}y^{-1}-3x^{-2}}$$

$$= \frac{\frac{1}{y^2}-\frac{1}{x^2}}{\frac{1}{y^2}-\frac{2}{xy}-\frac{3}{x^2}}$$

$$= \frac{x^2-y^2}{x^2-2xy-3y^2}$$

$$= \frac{(x-y)(x+y)}{(x-y)(x+3y)}$$

$$= \frac{x+y}{x+3y}$$

(d)

$$x+\sqrt{5-x}+1=0$$

$$\sqrt{5-x} = -x-1$$

$$5-x = (-x-1)^2$$

$$5-x = x^2+2x+1$$

$$x^2+3x-4=0$$

$$(x+4)(x-1)=0$$

$$x=-4, \ x=1$$

(e)

$$y = \frac{1}{2x^2} - \frac{1}{3x}$$
$$= \frac{x^{-2}}{2} - \frac{x^{-1}}{3}$$
$$\frac{dy}{dx} = \frac{-2x^{-3}}{2} - \frac{(-x^{-2})}{3}$$
$$= \frac{-1}{x^3} + \frac{1}{3x^2}$$

Question Two

(a)

$$3x + 2 = 1 - 2x$$
$$5x = -1$$
$$x = -\frac{1}{5}$$

Test:

$$LHS = \frac{-3}{5} + 2$$
$$= \frac{+7}{5}$$
$$RHS = \left|1 + \frac{2}{5}\right|$$
$$= \frac{7}{5}$$

$\therefore$ Valid

or

$$3x + 2 = -(1 - 2x)$$
$$3x + 2 = -1 + 2x$$
$$x = -3$$

Test:

$$LHS = -7$$
$$RHS = |1 + 6|$$
$$= 7$$

$\therefore$ Not valid

$\therefore$ Only solution is $x = \frac{-1}{5}$

(b) $1 + 2\cos\theta = 0$

$\therefore \cos\theta = -\frac{1}{2} = \cos\left(\frac{2\pi}{3}\right)$ or $\cos 120°$

$\therefore \quad \theta = \pm\frac{2\pi}{3} + 2n\pi$ or $\pm 120° + n(360°)$

(c) (i) $\left.\begin{array}{l} y=-x^3 \\ x-y+10=0 \end{array}\right\}$ $\therefore$ Let $f(x)=+x^3+x+10$

$f(1)=+1+1+10\neq 0$

$f(-1)=-1-1+10\neq 0$

$\therefore$ $x\pm 1$ are not factors

$f(2)=+8+2+10\neq 0$

$f(-2)=-8-2+10=0$

$\therefore$ $x+2$ is a factor

$$\begin{array}{r|l} & x^2-2x+5 \\ \hline x+2 & x^3 \qquad\quad +x+10 \\ & \underline{x^3+2x^2} \\ & \quad -2x^2+x \\ & \quad \underline{-2x^2-4x} \\ & \qquad\qquad 5x+10 \end{array}$$

$\therefore$ $x^3+x+10=0$

$(x+2)(x^2-2x+5)=0$

$\therefore$ $x=-2$

$x^2-2x+5=0$ has no real roots

$\therefore$ Point of intersection is at (-2,8)

(ii) Let $g(x)=-x^3$

$g'(x)=-3x^2$

$g'(-2)=-12$ which is the gradient of the curve at $x=-2$

gradient of the straight line is 1

$\therefore$ $\tan\alpha=\left|\dfrac{m_1-m_2}{1+m_1m_2}\right|$

$=\left|\dfrac{1-(-12)}{1+(1)(-12)}\right|$

$=\dfrac{13}{11}$

$\therefore$ $\alpha=50°$ to the nearest degree

(d) (i) The number of ways in which they may be chosen

$={}^{62}C_{12}=2.16\times 10^{12}$

(ii) They can line up in $2\times 10!$ ways, ie. 7 257 600 ways

Question Three

(a) $x = \cos 2t$

$\therefore$ $x = 1 - 2\sin^2 t$

$y = \sin t$

$\therefore$ Cartesian equation is $x = 1 - 2y^2$

(b) (i) $\angle ACB = 90°$ *(angle in a semi-circle)*

$\therefore AC \,//\, OE$ *(corresponding angles are equal)*

(ii) $\angle A = \angle BCP$ *(angle between tangent and chord)*

$\angle BOE = \angle A$ *(corresponding angles; AC // OE)*

$\therefore \angle A = \angle BOE = x$

(iii) $\angle BCP = x$ *(given)*

$\angle BOE = x$ *(proved above)*

$\therefore \angle BPC = \angle BOE$

$\therefore$ *OBEC* is a cyclic quadrilateral

(angles in the same segment of a circle)

(iv) In Δ ACP: $\angle P + \angle BCP + \angle ACB + \angle A = 180°$

(angles of a triangle)

But $\angle BCP = \angle A = x$

And $\angle ACB = 90°$

$\therefore \angle P + x + x + 90° = 180°$

$\therefore \angle P = 90° - 2x$

(c) $\cos\beta - \sqrt{3}\sin\beta = \sqrt{3}$

$\sqrt{3}\sin\beta - \cos\beta = r\sin(\beta - x)$

$r = \sqrt{(\sqrt{3})^2 + (-1)^2}$ and $\tan\alpha = \dfrac{1}{\sqrt{3}}$

$r = 2$ $\therefore \alpha = 30°$

$\therefore$ $2\sin(\beta - 30°) = -\sqrt{3}$

$\sin(\beta - 30°) = -\dfrac{\sqrt{3}}{2} = \sin(-60°)$

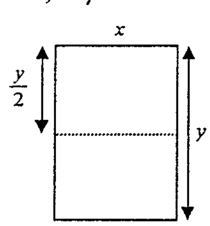
$\therefore$ $\beta - 30° = (-1)^n(-60°) + 180°n$

$\beta = 30° + (-1)^n(-60°) + 180°n$

When $n = 1$, $\beta = 30° + 60° + 180° = 270°$

When $n = 2$, $\beta = -30° + 360° = 330°$

(d)



$\dfrac{y}{x} = \dfrac{x}{\frac{y}{2}}$

$\therefore \dfrac{y^2}{2} = x^2$

$\dfrac{y^2}{x^2} = 2$

$\dfrac{y}{x} = \sqrt{2}$

Question Four

(a) (i) $\angle POM = 90°$ since diagonals of a square are perpendicular to each other

(ii) (α) O is the midpoint of PR

(diagonals of a square bisect each other)

M is the midpoint PT

(given $PM = MT$)

$\therefore OM \,//\, RT$ *(midpoint theorem)*

$\angle POM = 90°$ *(shown above)*

$\therefore \angle ORT = 90°$

(corresponding angles; $OM \,//\, RT$)

But $\angle ORQ = 45°$

(diagonals of a square bisect angles at vertices)

$\therefore \angle NRT = 45°$

($\angle ORQ + \angle NRT = 90°$)

(β) $\angle NQM = 45°$

(diagonals of a square bisect angles at vertices)

$\therefore \angle NQM = \angle NRT$

$\angle RNT = \angle MNQ$

(vertically opposite angles)

$\therefore \Delta NRT$ and ΔNQM are equiangular

$\therefore \Delta NRT \,|||\, \Delta NQM$

(equiangular triangles are similar)

(γ) $\dfrac{RN}{RT} = \dfrac{QN}{QM}$ ($\Delta NRT \,|||\, \Delta NQM$)

But $RT = 2OM$

(midpoint theorem in ΔPRT)

$\therefore \dfrac{RN}{2OM} = \dfrac{QN}{QM}$

$\therefore \dfrac{2OM}{QM} = \dfrac{RN}{QN}$

(b) (i)

$x^2 = 4ay$

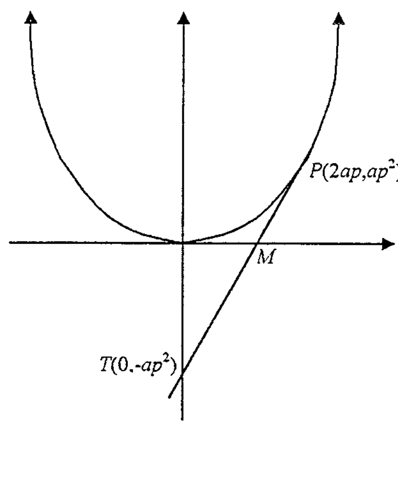
$y = \dfrac{x^2}{4a}$

$\dfrac{dy}{dx} = \dfrac{2x}{4a}$

$= \dfrac{2(2ap)}{4a}$ when $x = 2ap$

$= p$

$\therefore$ Equation of tangent at P is:



$$y - y_P = m(x - x_P)$$
$$y - ap^2 = p(x - 2ap)$$
$$y = px - 2ap^2 + ap^2$$
$$y = px - ap^2$$

(ii) The tangent at P cuts the y-axis when $x = 0$,

ie. $y = -ap^2$

$\therefore \ T(0, -ap^2)$

Let the mid-point of PT be M.
Then:

$$x_M = \frac{x_P + x_T}{2} = \frac{2ap + 0}{2} = ap \qquad y_M = \frac{y_P + y_T}{2} = \frac{ap^2 - ap^2}{2} = 0$$

$\therefore \ M(ap, 0)$ so that M lies on the x-axis.

(c)
$$LHS = \frac{4\cos\theta - 3\sec\theta}{1 - 2\sin\theta} + \frac{1 - 2\sin\theta}{\cos\theta} = \frac{\cos\theta(4\cos\theta - 3\sec\theta) + (1 - 2\sin\theta)^2}{(1 - 2\sin\theta)\cos\theta}$$
$$= \frac{4\cos^2\theta - 3 + 1 - 4\sin\theta + 4\sin^2\theta}{(1 - 2\sin\theta)\cos\theta}$$
$$= \frac{-3 + 1 - 4\sin\theta + 4(\sin^2\theta + \cos^2\theta)}{(1 - 2\sin\theta)\cos\theta}$$
$$= \frac{-3 + 1 - 4\sin\theta + 4}{(1 - 2\sin\theta)\cos\theta}$$
$$= \frac{2 - 4\sin\theta}{(1 - 2\sin\theta)\cos\theta}$$
$$= \frac{2(1 - 2\sin\theta)}{(1 - 2\sin\theta)\cos\theta}$$
$$= \frac{2}{\cos\theta}$$
$$= 2\sec\theta$$
$$= RHS$$

Question Five

(a)
$$5\sin\theta\cos\theta + 5\cos^2\theta = 2$$
$$5\sin\theta\cos\theta + 5\cos^2\theta = 2(\sin^2\theta + \cos^2\theta)$$
$$\therefore \quad 2\sin^2\theta - 5\sin\theta\cos\theta - 3\cos^2\theta = 0$$
$$(2\sin\theta + \cos\theta)(\sin\theta - 3\cos\theta) = 0$$

$2\sin\theta + \cos\theta = 0$ or $\sin\theta - 3\cos\theta = 0$

$2\sin\theta = -\cos\theta$ $\qquad$ $\sin\theta = 3\cos\theta$

$\tan\theta = -\dfrac{1}{2}$ $\qquad$ $\tan\theta = 3$

$\theta = 180° - 26°34', \ 360° - 26°34', \ 71°34, \ 180° + 71°34'$

$\theta = 153°26', \ 333°26', \ 71°34, \ 251°34'$

Alternative Method: Divide throughout by $\cos^2\theta(\cos\theta \neq 0)$:

$\therefore$

$$5\tan\theta + 5 = 2\sec^2\theta$$

$$5\tan\theta + 5 = 2(1+\tan^2\theta)$$

$$2\tan^2\theta - 5\tan\theta - 3 = 0$$

$$(2\tan\theta + 1)(\tan\theta - 3) = 0$$

$$\tan\theta = -\tfrac{1}{2}, \tan\theta = 3 \text{ (as above)}$$

(b) Suppose that the roots are α, $\alpha + 1$, β:

Then $2\alpha + \beta + 1 = -\frac{b}{a} = -\frac{19}{3}$ ....(1)

$\alpha(\alpha+1) + \alpha\beta + (\alpha+1)\beta = \frac{c}{a} = \frac{22}{3}$

$\alpha^2 + \alpha + \alpha\beta + \alpha\beta + \beta = \frac{22}{3}$

$\alpha^2 + 2\alpha\beta + \beta = \frac{22}{3}$ ....(2)

From (1): $\beta = -\frac{19}{3} - 1 - 2\alpha$

Sub in (2): $\alpha^2 + \alpha + 2\alpha(-\frac{19}{3} - 1 - 2\alpha) + (-\frac{19}{3} - 1 - 2\alpha) = \frac{22}{3}$

$\alpha^2 + \alpha + \frac{38\alpha}{3} - 2\alpha - 4\alpha^2 - \frac{19}{3} - 1 - 2\alpha = \frac{22}{3}$

$\therefore$ $-9\alpha^2 - 47\alpha - 44 = 0$

$9\alpha^2 + 47\alpha + 44 = 0$

$(9\alpha + 11)(\alpha + 4) = 0$

$9\alpha + 11 = 0, \ \alpha + 4 = 0$

$\alpha = -\frac{11}{9}$ $\qquad$ $\alpha = -4$

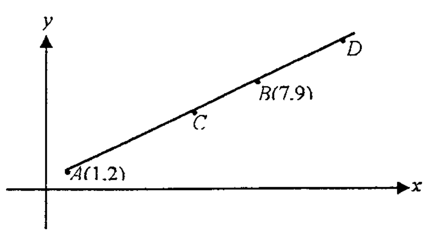
rejected since α is an integer

$\therefore$ Roots are -4, -3 and $\beta = -\frac{19}{3} - 1 - 2(-4)$

$= \frac{2}{3}$

ie. -4, -3, $\frac{2}{3}$

(c)



(i) For internal division:

$$x_c = \frac{mx_2 + nx_1}{m+n} \quad , \quad y_c = \frac{my_2 + ny_1}{m+n}$$

$$x_c = \frac{2(7)+1(1)}{2+1} \qquad y_c = \frac{2(9)+1(2)}{2+1}$$

$$= \tfrac{15}{3} \qquad = \tfrac{20}{3}$$

$$= 5$$

$$\therefore \quad C\,(5, \tfrac{20}{3})$$

For external division:

$$x_D = \frac{mx_2 - nx_1}{m-n} \quad , \quad y_D = \frac{my_2 - ny_1}{m-n}$$

$$= \frac{4(7)-1(1)}{4-1} \qquad = \frac{4(9)-1(2)}{4-1}$$

$$= \tfrac{27}{3} \qquad = \tfrac{34}{3}$$

$$= 9$$

$$\therefore \quad D\,(9, \tfrac{34}{3})$$

(ii) Midpoint of CD:

$$x = \frac{x_1 + x_2}{2} \quad , \quad y = \frac{y_1 + y_2}{2}$$

$$= \frac{5+9}{2} \qquad = \frac{\tfrac{20}{3} + \tfrac{34}{3}}{2}$$

$$= \tfrac{14}{2} \qquad = \frac{\tfrac{54}{3}}{2}$$

$$= 7 \qquad = 9$$

(7,9) which are the coordinates of B.

Question Six

(a) $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

Put $\beta = -\gamma$

then $\cos(\alpha + \gamma) = \cos\alpha\cos\gamma + \sin\alpha\sin(-\gamma)$

$= \cos\alpha\cos\gamma + \sin\alpha(-\sin\gamma)$

$= \cos\alpha\cos\gamma - \sin\alpha\sin\gamma$

Thus $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

(b) $\cos 79° \cos 311° + \sin 101° \sin 49°$

$= \cos 79°\cos(360° - 311°) + \sin(180° - 101°)\sin 49°$

$= \cos 79°\cos 49° + \sin 79°\sin 49°$

$= \cos(79° - 49°)$

$= \cos 30°$

$= \dfrac{\sqrt{3}}{2}$

(c) (i) $xy = 6$

$\therefore \quad y = \frac{6}{x} = 6x^{-1}$

$\frac{dy}{dx} = -6x^{-2}$

$= -\frac{6}{x^2}$

At $x = 2$, $\frac{dy}{dx} = -\frac{6}{2^2} = -\frac{6}{4} = -\frac{3}{2}$

Which is the gradient of the tangent at $x = 2$

$\therefore$ Gradient of normal at $x = 2$ is $\frac{2}{3}$

When $x = 2$, $y = \frac{6}{2} = 3$

$\therefore$ Equation of normal at $x = 2$:

$y - y_1 = m(x - x_1)$

$y - 3 = \frac{2}{3}(x - 2)$

$3y - 9 = 2x - 4$

$2x - 3y + 5 = 0$ ....(1)

(ii) Solving simultaneously $2x - 3y + 5 = 0$ ....(1)

$y = \frac{6}{x}$ ....(2)

$2x - 3(\frac{6}{x}) + 5 = 0$

$2x^2 - 18 + 5x = 0$

$2x^2 + 5x - 18 = 0$

$(2x + 9)(x - 2) = 0$

$2x + 9 = 0 \quad , \quad x - 2 = 0$

$x = -\frac{9}{2} \quad , \quad 2$

$\therefore$ P has x-coordinate $-\frac{9}{2}$

$y = \frac{6}{x} = \frac{6}{-\frac{9}{2}} = 6 \times (-\frac{2}{9}) = -\frac{4}{3}$

$\therefore \quad P\left(-\frac{9}{2}, -\frac{4}{3}\right)$

(d) (i) For $x^2 - 2x + 2 = 0$,

$\Delta = b^2 - 4ac$

$= 4 - 4(1)(2)$

$= -4 < 0$

$\therefore$ Roots are unreal

(ii) $\dfrac{x^2-2x+2}{3-x} \le 0$

Multiplying throughout by $(3-x)^2$

$(x^2-2x+2)(3-x) \le 0$

Since x^2-2x+2 is positive definite

$3-x \le 0$

ie. $x \ge 3$ but since $x \ne 3$

$x > 3$ is the solution

Question Seven

(a) (i) $P(x) = (x+1)(x-2)Q(x) + ax + b$

(ii) $P(-1) = \quad 0 - a + b \quad = -a + b$

$P(2) \;\; = \quad 0 + 2a + b \quad = 2a + b$

(iii) $\left.\begin{array}{l} -a+b=-2 \\ 2a+b=7 \end{array}\right\}$

Subtracting : $-3a = -9$

$a = 3$

$\therefore b = 1$

(b) Equation of chord of contact: $xx_0 = 2a(y + y_0)$

$y = x^2 \quad \Rightarrow \quad x^2 = y \qquad \therefore\ a = \frac{1}{4}$

$\therefore \quad xx_0 = 2(\frac{1}{4})(y + y_0)$

$= \frac{1}{2}(y + y_0)$

$6x - 3y - 5 = 0$

$x - \frac{y}{2} - \frac{5}{6} = 0$

$x = \frac{y}{2} + \frac{5}{6}$

$x.1 = \frac{1}{2}(y + \frac{5}{3})$

$\therefore x.1 = 2(\frac{1}{4})(y + \frac{5}{3})$

$\therefore$ Coordinates of external point : $(1, \frac{5}{3})$

(c) (i) Number of arrangements $= 4^4 = 256$

(ii) Number of arrangements $= 4! = 24$

(iii) $4 \times 3^2 = 36$

(iv) $3 \times 4^3 = 192$