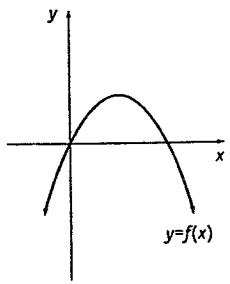
Maths 30 Yr 11 Sept 1999

Marks

Question 1

(a) Sketch the gradient function of



2

(b) Given that $f(x) = x^2 + x$ find the values of b for which $f''(b) = f(b)$ 2

(c) 45% of a population are of blood group O, 40% are of blood group A, and the remainder are neither blood group O nor blood group A. Three people are chosen at random from the population. 4

Find the probability that:

(i) All three people are of blood group A.
(ii) None of the three people are of blood group O.
(iii) Two of the people are of blood group O and the other is neither blood group A nor blood group O.
(iv) At least one is blood group A.

(d) A box contains 7 blue, 9 white and 10 grey envelopes. If one is chosen at random find the probability that a blue or white envelope is selected. 2

Question 2

(a) Differentiate each of the following with respect to x.

(i) $y = 2x^3 + 7x$ 1

(ii) $y = \left(\frac{1}{\sqrt{x}}\right)^3$ 2

(iii) $y = (x^2 - 1)(x + 3)^3$ 3

(b) Find the equation of the normal to $y = \frac{x+2}{x-2}$ at the point where $x = 1$. 4

Marks

Question 3.

(a) The first three terms of an arithmetic series are 50, 43, 36.

(i) Write down a formula for the n^{th} term. 1

(ii) If the last term of the series is –27, how many terms are there in the series? 2

(iii) Find the sum of the series. 2

(b) A geometric progression has an n^{th} term of $T_n = 4(3^{-n})$

(i) Write down the first three terms. 2

(ii) Write down the common ratio. 1

(iii) Find the sum of the first ten terms. 2

Question 4 8

a) Consider the curve given by $f(x) = x^3 - 2x^2 + x + 4$, for $-2 \le x \le 2$

(i) Find the stationary points and determine their nature.
(ii) Find the point of inflexion
(iii) Sketch the curve for $-2 \le x \le 2$
(iv) What is the maximum value for $f(x)$ in the interval $-2 \le x \le 2$?

b) Find the value of a if a , $2a^2$, $3a^3$ are successive terms in an arithmetic series. 2

Question 5

(a) A loan of $15,000 is to be repaid by equal monthly instalments, repayments commencing at the end of the first month of the loan. The interest is calculated at the rate of 0.8% per month and is compounded each month.

If the monthly instalment is M dollars, prove that: 6

(i) The amount owing at the end of the 2^{nd} month is $15,240.96 – 2.008M

(ii) If the loan including interest is exactly repaid at the end of 5 years, find the monthly repayment M.

Marks

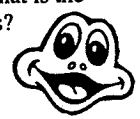
Question 5 (continued)

(b) Angela wishes to draw some cash out of an ATM and requires to enter her 4-digit PIN number. Given there are no restrictions on the digits being entered (e.g. 8444,0099), find the probability that:

(i) She randomly guesses the correct combination given she remembers the first & last digits only. 1

(ii) She randomly guesses the correct combination given she remembers that the middle digits are alike. 1

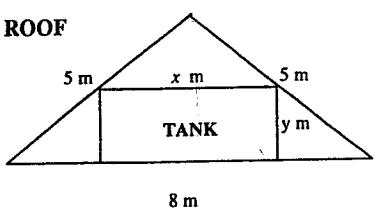
(c) In a group of 20 girls, who all like chocolate, 14 like chocolate freddos and 16 like caramel koalas. If one of the girls is chosen at random what is the probability she will like both chocolate freddos and caramel koalas? (Use a Venn diagram or otherwise). 2



Question 6.

(a) The diagram below shows the cross section of a rectangular hot water tank, x metres wide, y metres high and 1.5 metres long which fits exactly into the roof of a house. The roof is an isosceles triangle with base 8 metres and equal sides 5 metres in length. 8

HOUSE ROOF



(i) Explain why the roof of the house is 3 metres high.

(ii) Show, using similar figures, that $y = \frac{1}{8}(24 - 3x)$

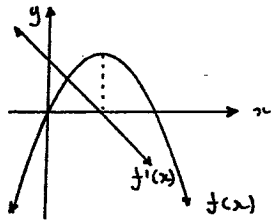
(iii) Show that the volume of the tank is given by : $V = \frac{3}{16}x(24 - 3x)$.

(iv) Calculate the maximum volume of the tank.

(b) Bazza and Blue play a game of darts. The winner is the first person to hit the centre of the target. If Bazza and Blue have a probability of 0.50 and 0.60 respectively of hitting the centre of the target, find the probability of Bazza winning if he goes first. 2

Solutions Yr 11 3U Sept 96

Q1.

a) 

②

b) $f(x) = x^2 + x$
$f(b) = b^2 + b$
$f'(x) = 2x + 1$
$f''(x) = 2 \quad \therefore f''(b) = 2$
then $b^2 + b = 2$
$b^2 + b - 2 = 0$
$(b+2)(b-1) = 0$
$b = -2$ or $b = 1$ ②

c) 45% group O
40% group A ④
15% other
i) $P = (0.4)(0.4)(0.4)$
$= 0.064$
ii) $P = (0.55)(0.55)(0.55)$
$= 0.166375$
iii) $P = 0.45 \times 0.45 \times 0.15 \times 3$ ways
$= 0.091125$
iv) $P = 1 - (\text{None from A})$
$= 1 - (0.6 \times 0.6 \times 0.6)$
$= 0.784$

d) $P(B) + P(W) = \frac{7}{26} + \frac{9}{26}$ ②
$= \frac{8}{13}$

Q2

a) i) $y = 2x^3 + 7x$ ①
$\frac{dy}{dx} = 6x^2 + 7$

ii) $y = \left(\frac{1}{\sqrt{3x}}\right)^2$ ②
$y = (x^{-\frac{1}{2}})^2$
$\frac{dy}{dx} = -\frac{3}{2} x^{-\frac{5}{2}}$
$= \frac{-3}{2x^{\frac{5}{2}}} \quad \left(\frac{-3}{2x\sqrt{x}}\right)$

iii) $y = (x^2-1)(x+3)^3$ ③
$u = x^2 - 1 \qquad v = (x+3)^3$
$\frac{du}{dx} = 2x \qquad \frac{dv}{dx} = 3(x+3)^2(1)$

$\frac{dy}{dx} = (x+3)^3(2x) + (x^2-1)3(x+3)^2$
$= (x+3)^2[(x+3)(2x) + 3(x^2-1)]$
$= (x+3)^2[2x^2 + 6x + 3x^2 - 3]$
$= (x+3)^2[5x^2 + 6x - 3]$

b) $y = \frac{x+2}{x-2}$ ④

$\frac{dy}{dx} = \frac{(x-2)(1) - (x+2)(1)}{(x-2)^2}$

$= \frac{x - 2 - x - 2}{(x-2)^2}$

$= \frac{-4}{(x-2)^2}$

when $x = 1$
$m = \frac{-4}{(1-2)^2}, \quad y = \frac{1+2}{1-2}$

$= -4 \qquad = -3$

Eqn of normal
$y - y_1 = m(x - x_1)$
$y + 3 = \frac{1}{4}(x - 1)$
$4y + 12 = x - 1$
$0 = x - 4y - 13$

Q3

a) Arithmetic 50, 43, 36
i) $a = 50 \quad d = -7$ ①
$T_n = a + (n-1)d$
$= 50 + (n-1)(-7)$
ii) $57 - 7n = -27$ ②
$-7n = -84$
$n = 12$

iii) $S_n = \frac{n}{2}(a + l)$ ②
$= \frac{12}{2}(50 - 27)$
$= 138$

b) $T_n = 4(3^{-n})$ G.P.
i) $4(3^{-1}), 4(3^{-2}), 4(3^{-3})$ ②
$\frac{4}{3}, \quad \frac{4}{9}, \quad \frac{4}{27}$

ii) $\frac{1}{3}$ ①

iii) $S_n = \frac{a(1 - r^n)}{1 - r}$ ②
$= \left(\frac{4}{3}\right)\left(1 - \left(\frac{1}{3}\right)^{10}\right) \div \frac{2}{3}$
$= 1.99997$ (5 d.p.)

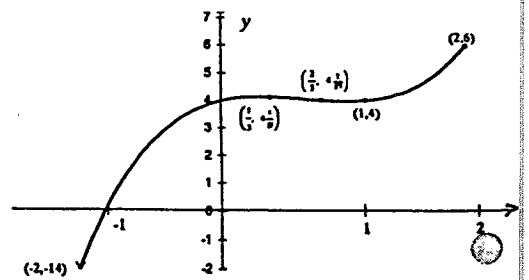
Q4

a) i) $f(x) = x^3 - 2x^2 + x + 4$ ③
$f'(x) = 3x^2 - 4x + 1$
$f''(x) = 6x - 4$
for stny pt. $f'(x) = 0$
$3x^2 - 4x + 1 = 0$
$(3x-1)(x-1) = 0$
$x = \frac{1}{3}$ or $x = 1$
$y = 4\frac{4}{27}$ $\qquad y = 1$
$f''(\frac{1}{3}) < 0 \qquad f''(1) > 0$
$\therefore$ max $\qquad \therefore$ min

ii) for a pt of inflexion ②
$\frac{d^2y}{dx^2} = 0$
$6x - 4 = 0$
$x = \frac{2}{3} \qquad y = 4\frac{2}{27}$
for x a little less than $\frac{2}{3}$
$\frac{d^2y}{dx^2} < 0$

of inflexion at $(\frac{2}{3}, 4\frac{2}{27})$

iii) when $x = -2$, $y = -14$ ②
" $x = 2$, $y = 6$
on y axis $x = 0$, $y = 4$



iii) Max Value 6 when ①
$x = 2$

b) Arithmetic $\therefore$ ②
$2a^2 - a = 3a^3 - 2a^2$
$3a^3 - 4a^2 + a = 0$
$a(3a^2 - 2a + 1) = 0$
$3(3a-1)(a-1) = 0$
$\therefore a = 0, \frac{1}{3}, 1$
but $a \neq 0 \quad \therefore a = \frac{1}{3}$ or $a = 1$

Q5

a) i) Amount owing at end of Month 1 $= 15000 \times 1.008 - M$ ⑥
$= 15120 - M$
Amount owing at end of Month 2 $= (15120 - M)1.008 - M$
$= 15240.96 - 1.08M - M$
$= 15240.96 - 2.08M$

ii) $n = 60$
After n months the amount owing is $=$
$15000(1.08)^n - M(1 + 1.08 + \ldots 1.08^{n-1})$

$$m = \frac{15000(1.008)^{60}}{1 + 1.008 + \ldots + 1.008^{59}}$$

Now denominator geometric $a = 1$, $r = 1.008$, $n = 60$ and its sum is given by

$$S_n = \frac{1(1.008^{60} - 1)}{1.008 - 1}$$

$$\therefore M = \frac{15000(1.08)^{60} \times 0.008}{1.008^{60} - 1}$$

$$= \$315.76$$

b) i) P(of guessing second digit correctly) $= \frac{1}{10}$ ①

P(of guessing third digit correctly) $= \frac{1}{10}$

∴ required probability $= \frac{1}{10} \times \frac{1}{10}$

$= \frac{1}{100}$

ii) P(of guessing first digit correctly) $= \frac{1}{10}$

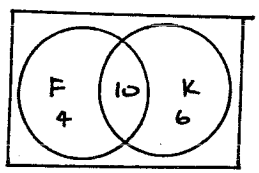
P(of guessing middle digit correctly) $= \frac{1}{10}$

P(of guessing last digit correctly) $= \frac{1}{10}$

∴ required probability $= \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10}$

$= \frac{1}{1000}$

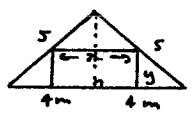
c)



$P = \frac{10}{20}$

$= \frac{1}{2}$

Q6 ⑧

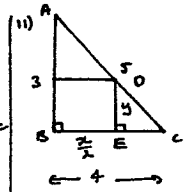


i) $5^2 = 4^2 + h^2$

$h = 3$ m

[Pyth and perp bisector isosc △]

ii)



△ABC ||| △DCE (equiangular)

$\frac{y}{3} = \frac{4 - \frac{x}{2}}{4}$

$\frac{y}{3} = \frac{8 - x}{8}$

$y = \frac{1}{8}(24 - 3x)$

iii) $V = LBH$

$= (x)\frac{1}{8}(24 - 3x)(1.5)$

$= \frac{3}{16}x(24 - 3x)$

$= \frac{3}{16}(24x - 3x^2)$

iv) $\frac{dV}{dx} = \frac{3}{16}(24 - 6x)$

$\frac{d^2V}{dx^2} = \frac{3}{16}(-6)$

$= \frac{-18}{16}$

< 0 ∴ max

for max volume $\frac{dV}{dx} = 0$

$0 = \frac{3}{16}(24 - 6x)$

$0 = 24 - 6x$

$x = 4$

$V = \frac{3}{16}(4)(24 - 12) = 9\,m^3$

b) P(Bazza winning on first go) $= 0.5$

for Bazz to win on second attempt Blue must miss)

∴ $0.5 \times 0.4 \times 0.5$

P(Bazza winning third go)

$= 0.5 \times 0.4 \times 0.5 \times 0.4 \times 0.5)$ etc

ie) $P = 0.5 + (0.5^2)(0.4) + (0.5)^3(0.4)^2$

Infinite sum of G.P.

$a = 0.5$, $r = (0.5)(0.4)$

$S_\infty = \frac{a}{1 - r}$

$= \frac{0.5}{1 - (0.5)(0.4)}$

$= 0.5$