Mathematics C

Indefinite Integration involving Exponential Functions

Syllabus Reference: 12-1(b), 12-2(b)

Suggested Exercises: Textbook 12-1/13,27,53,69
12-2/3,15,29,35

THE UNIVERSITY OF NEW SOUTH WALES

UNIVERSITY FOUNDATION YEAR
NewSouth Global

$\int e^x\,dx = e^x + c$	$\int e^{ax+b}\,dx = \dfrac{e^{ax+b}}{a} + c$
$\int a^x\,dx = \dfrac{a^x}{\ln a} + c$	$\int f'(x)\,e^{f(x)}\,dx = e^{f(x)} + c$

1. Find the following indefinite integrals:

(a) $\int e^{5x}\,dx$ (b) $\int 3^x\,dx$ (c) $\int e^{\frac{x}{3}}\,dx$

(d) $\int e^{3x+1}\,dx$ (e) $\int e^{-x}\,dx$ (f) $\int e^{2-5x}\,dx$

(g) $\int -2e^{4x}\,dx$ (h) $\int (e^{5u} + 5u + 2)\,du$ (i) $\int \sqrt{e^x}\,dx$

(j) $\int \dfrac{1}{e^{4x}}\,dx$ (k) $\int e^{-x}(3 + 2e^x)\,dx$ (l) $\int (e^x + 3)(e^x - 1)\,dx$

(m) $\int (e^x + e^{-x})^2\,dx$ (n) $\int \dfrac{e^{6x} - e^x}{e^{3x}}\,dx$ (o) $\int 2xe^{x^2}\,dx$

(p) $\int x^2 e^{x^3}\,dx$ EXT.1 (q) $\int \dfrac{e^{2\sqrt{x}}}{\sqrt{x}}\,dx$ Use $u = \sqrt{x}$ EXT.1 (r) $\int \dfrac{e^x}{(e^x + 1)^2}\,dx$ Use $u = e^x + 1$

2. Differentiate $x e^x$ and hence find $\int x e^x\,dx$.

3. If $\dfrac{dy}{dx} = e^x + 2$ find y when $x = 1$, given that $y = 4$ when $x = 0$.

4. For a certain function $y = f(x)$, $f''(x) = 2e^x - 3e^{-x}$.
If $f'(0) = 5$ and $f(0) = 6$ find $f(1)$.

5. The rate of healing for a wound to the skin is approximately $A'(t) = -0.9e^{-0.1t}$ square centimetres each day. If the original wound has an area of $9\,\text{cm}^2$, find the area of the wound after 5 days.

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1, a) $\int e^{5x}\,dx$

$= \frac{1}{5}e^{5x} + C$ ✓

b) $\int 3^x\,dx$

$= \int e^{\ln 3^x}\,dx$

$= \int e^{x\ln 3}\,dx$

$= \frac{1}{\ln 3}e^{x\ln 3} + C$ ✓

c) $\int e^{\frac{x}{3}}\,dx$

$= 3e^{x/3} + C$ ✓

d) $\int e^{3x+1}\,dx$

$= \frac{1}{3}e^{3x+1} + C$ ✓

e) $\int e^{-x}\,dx$

$= -e^{-x} + C$ ✓

f) $\int e^{2-5x}\,dx$

$= -\frac{1}{5}e^{2-5x} + C$ ✓

g) $\int -2e^{4x}\,dx$

$= -\frac{1}{2}e^{4x} + C$ ✓

h) $\int (e^{5u} + 5u + 2)\,du$

$= \frac{1}{5}e^{5u} + \frac{5u^2}{2} + 2u + C$ ✓

i) $\int \sqrt{e^x}\,dx$

$= \int e^{x/2}\,dx$

$= 2e^{x/2} + C$ ✓

j) $\int \frac{1}{e^{4x}}\,dx$

$= \int e^{-4x}\,dx$

$= -\frac{1}{4}e^{-4x} + C$ ✓

k) $\int e^{-x}(3 + 2e^x)\,dx$

$= \int 3e^{-x} + 2\,dx$

$= -3e^{-x} + 2x + C$ ✓

l) $\int (e^x + 3)(e^x - 1)\,dx$

$= \int e^{2x} + 2e^x - 3\,dx$

$= \frac{1}{2}e^{2x} + 2e^x - 3x + C$ ✓

m) $\int (e^x + e^{-x})^2\,dx$

$= \int e^{2x} + 2 + e^{-2x}\,dx$

$= \frac{1}{2}e^{2x} - \frac{1}{2}e^{-2x} + 2x + C$ ✓

n) $\int \frac{e^{6x} - e^x}{e^{3x}}\,dx$

$= \int \frac{e^{6x}}{e^{3x}} - \frac{e^x}{e^{3x}}\,dx$

$= \int e^{3x} - e^{-2x}\,dx$

$= \frac{1}{3}e^{3x} + \frac{1}{2}e^{-2x} + C$ ✓

o) $\int 2xe^{x^2}\,dx$

$= e^{x^2} + C$ ✓

p) $\int x^2 e^{x^3}$

$= \frac{1}{3}e^{x^3} + C$ ✓

q) $\int \frac{e^{2\sqrt{x}}}{\sqrt{x}}\,dx$

$= e^{2\sqrt{x}} + C$ ✓

r) $\int \frac{e^x}{(e^x+1)^2}\,dx$

$= \int e^x (e^x+1)^{-2}\,dx$

$= -(e^x+1)^{-1} + C$

$= \frac{-1}{e^x+1} + C$ ✓

* 2. $\frac{d}{dx}(xe^x)$

$= xe^x + e^x$

$= e^x(1+x)$ ✓

$xe^x = \int xe^x + e^x\,dx$

$xe^x = \int xe^x dx + \int e^x dx$

$\therefore \int xe^x dx = xe^x - \int e^x dx$

$= xe^x - e^x + C$ ✓

3. $y = e^x + 2x + C$

Put $x=0; y=4$. $\quad 4 = e^0 + 2(0) + C$

$C = 3$. ✓

$\therefore y = e^x + 2x + 3$

when $x=1$, $y = e^1 + 2 + 3$

$= e^1 + 5$ ✓

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4. $f'(x) = 2e^x + 3e^{-x} + C$

Put $x=0$, $f'(0)=5$. $\quad \therefore 5 = 2e^0 + 3e^{-0} + C$

$5 = 2 + 3 + C$

$C = 0$

$\therefore f'(x) = 2e^x + 3e^{-x}$ ✓

$f(x) = 2e^x - 3e^{-x} + C$

Put $x=0$, $f(0)=6$ $\quad \therefore 6 = 2e^0 - 3e^{-0} + C$

$6 = 2 - 3 + C$

$C = 7$

$\therefore f(x) = 2e^x - 3e^{-x} + 7$ ✓

When $x=1$, $f(1) = 2e - 3e^{-1} + 7$

$= 2e - \frac{3}{e} + 7$ ✓

5. $\int A'(t)\,dt$

$= \int -0.9e^{-0.1t}\,dt$

$= 9e^{-0.1t} + C$ ✓

Put $t=0$, $A=9$

$\therefore \quad 9 = 9e^{-0.1(0)} + C$

$9 = 9 + C$

$C = 0$. ✓

$\therefore \quad A(t) = 9e^{-0.1t}$

when $t=5$, $A = 9e^{-0.5}$

$\approx 5.459\ cm^2$ ✓ (3 d.p.)