THE UNIVERSITY OF
NEW SOUTH WALES

UNSW Foundation Year

Mathematics C
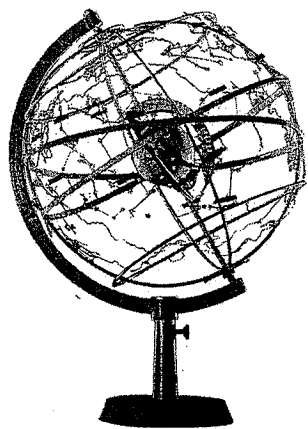
FINAL
Examination Paper

Sample L

Time Allowed: 3 hours

Reading Time: 5 minutes

Question 1   Use a SEPARATE book clearly marked Question 1

(i)   Given $T = 2\pi\sqrt{\dfrac{L}{g}}$ find the value of T correct to 3 significant figures, if $L = 60.7$ and $g = 9.8$.

(ii)   Solve $\dfrac{x}{2} - 3 = x$.

(iii) Factorize $15x^2 y - 20xy^3$.

(iv)   List in ascending order: $3^{-1}, \dfrac{1}{\pi}, \dfrac{3}{10}$.

(v)   Write y as the subject of $xy = 2y + 5$.

(vi)   Find the point where the curve $y = 2^{x+4}$ crosses the y-axis.

(vii) Find the centre and the radius of the circle with equation $(x-4)^2 + y^2 = 9$.

(viii) Find the gradient of the tangent to the curve $y = x^2 + 4$ at the point $(-1, 5)$.

(ix)   Find the derivative of e^{5x+6}.

(x)   If a card is chosen at random from a regular pack of 52 cards, find the probability that it is a black Queen.

(xi)   Sketch the function $y = 2|x|$.

(xii) A committee of 5 is to be chosen from 12 people, of whom X and Y are two. Find the number of possible committees if X and Y are both included.

Question 2   Use a SEPARATE book clearly marked Question 2

(i)   Find the indefinite integral of e^{-2x}.

(ii)   A person purchasing a new car has several options: 7 exterior colour choices, 5 interior colour choices and 2 choices of body style. Find the number of different cars possible if one choice is made for each option.

(iii)   If $\int_{2}^{5} f(x)\,dx = 12$, find the value of $\int_{2}^{5} \frac{f(x)}{4}\,dx$.

(iv)   If y is a function in x, find $\frac{dy}{dx}$ for $3x + y^2 = 4$.

(v)   Two groups of people, adults and children were asked their preferences in television programs from among three new programs. The results are shown in the table below.

Group of People	Program X	Program Y	Program Z
Adults	35	25	40
Children	15	10	25

Find the probability that a person selected at random will be an Adult or prefer program X.

(vi)   Evaluate $\int_{1}^{4} \frac{dt}{\sqrt{t}}$.

(vii)   Find $\lim_{x \to 3} \frac{2x^2 - x - 15}{3 - x}$.

(viii) For $2\log_a m - \log_a n = 5$, find a relationship between a, m and n which does not involve logarithms.

(ix)   Find the values of x for which the curve $y = x^3 - 6x^2 + 11x + 9$ is concave up.

Question 3   Use a SEPARATE book clearly marked Question 3

(i)   Diffferentiate with respect to x:

(a) $(7 - x)^8$.

(b) $\log_e \left[\frac{2x+1}{x-7} \right]$.

(ii)   Find the equation of the normal to $y = \frac{5}{x-1}$ at the point where $x = 2$.

(iii)   A box contains 5 white cards and 4 black cards.
        Three cards are drawn in succession at random without replacement.
        Find the probability of selecting :

(a)   no white cards.

(b)   at least 2 black cards.

(iv)   (a)   Find the amount to which $\$7\,000$ grows when invested at 8% compounded quarterly for 6 years.

        (b)   Find the effective rate which gives the same annual return as 8% compounded quarterly.

Question 4 Use a SEPARATE book clearly marked Question 4

(i) Find the primitive function (i.e. indefinite integral) of each of the following :

   (a) $(9x-7)^3$.

   (b) $\dfrac{2}{3-2x}$.

   (c) $7xe^{x^2}$.

(ii) According to one theory of learning, the number of items $N(t)$ that a person will have learnt after t hours of continuous instruction is given by:

$$N(t) = 15\sqrt[3]{t^2}, \quad 0 \le t \le 9.$$

   Find the rate of learning at the end of 8 hours of continous instruction.

(iii) The first three terms of a geometric sequence are $x-4, x, 5x-12$.
   If all three terms are positive find the value of x.

(iv) Use the substitution of $m = e^x$, or otherwise, to form a quadratic equation and solve $8e^{-x} - e^x = 2$.

Question 5 Use a SEPARATE book clearly marked Question 5

(i) (a) Sketch the parabola $y = x^2 - 4x$ showing the vertex and the x-intercepts.

   (b) Find the area enclosed by the curve $y = x^2 - 4x$ and the x-axis.

(ii) (a) Sketch the curve $y = e^x + 1$ showing the essential features.

   (b) Find the volume of the solid generated when the curve $y = e^x + 1$ for $0 \le x \le 1$ is rotated about the x-axis. [Leave the answer in exact form]

(iii) Find the median for the data tabulated below.

Score Interval	Frequency
0.5-3.5	1
3.5-6.5	5
6.5-9.5	7
9.5-12.5	2

(iv) The probability of a machine component being defective is 1%.
   A sample of five machine components are selected at random.

   (a) Find the probability that the sample contains exactly 4 defective machine components.

   (b) Find the probability that there is at least one defective machine component in the sample.

(i)   Find $\displaystyle\int_1^3 (x-1)(x^2 - 2x + 5)^3$.

(ii)   A publishing company has published a new magazine for young adults. The monthly sales S (in thousands) is given by $S(t) = \dfrac{800t}{t+2}$, where t is the number of months since the first issue was published. Find $S(3)$ and $S'(3)$ and interprete these results.

(iii)   A $400 bicycle is insured against theft for an annual premium of $30. If the probability that the bicycle will be stolen during the year is 0.02 find the expected value of the policy to the policy holder.

(iv)   (a)   Show that $\dfrac{x^3}{x^2 + 2} = x - \dfrac{2x}{x^2 + 2}$.

(b)   Hence find the indefinite integral $\displaystyle\int \dfrac{x^3}{x^2 + 2}\,dx.$

(i)   Show that $f(x) = x^3\sqrt{1 - x^2}$ is an odd function.

(ii)   Find the number of different arrangements in a line of the letters of the place name HONG KONG.

In parts (iii) and (iv) below, the following formulas may be of assistance:

$$FV = PMT\left[\frac{(1+i)^n - 1}{i}\right] \qquad PV = PMT\left[\frac{1 - (1+i)^{-n}}{i}\right]$$

(iii)   The body coorporate of a block of 12 townhouses wishes to set up a sinking fund to cover the anticipated expenses involved in painting and maintenance work in 5 years time. Find the quarterly levy required from each owner if the funds are invested at 10% compounded quarterly and are to amount to $40000 in 5 years time.

(iv)   A home was purchased for $65000. The purchaser financed this by paying 20% deposit, borrowing the balance from a bank and signing a mortgage agreement to repay the loan plus interest over a 25 year period by making equal monthly repayments. Interest is charged at the rate of 9% compounded monthly.

Find:

(a)   the amount borrowed.

(b)   the monthly repayment.

(c)   the amount still owing 14 years after the home was purchased.

(d)   the purchaser's equity in the home 14 years after purchase if the market value of the home had by then increased to $100000 [Answer to the nearest dollar].

Question 8   Use a SEPARATE book clearly marked Question 8

(i)   Glenn scored 80% on his Term 1 Maths exam. The mean and standard deviation of the class on this exam were 60% and 10% respectively. Glenn was absent for the Term II Maths exam. The mean and standard deviation of the class on this exam were 54% and 15% respectively. The teacher wants to give Glenn an estimated mark on the Term II exam based on his performance in Term I. What is a suitable estimated mark for Glenn on the second test? Justify your answer.

(ii)   The rate of decay of a radioactive substance is given by $\dfrac{dM}{dt} = kM$ where M is the mass in grams of the substance present at time t years and k is a constant.
A mass of 10 grams of this substance will reduce to 8 grams in 5 years.

    (a)   Show that $k = \dfrac{\ln 0.8}{5}$

    (b)   Hence find the time taken for the mass of substance present to be reduced by 25% of its original value.

(iii)   An arithmetic sequence has a first term of a_1 and a common difference of d.
The eleventh term is five times the sixth term and the sum of the eighth and ninth terms is 40.

    (a)   Show that $a_1 = -20$ and $d = \dfrac{16}{3}$.

    (b)   Hence find the sum of the first 6 terms of this sequence.

Question 9   Use a SEPARATE book clearly marked Question 9

(i)   Find the coefficient of x^3 in the expansion of $(5 - 2x)^6$.

(ii)   A manufacturing company which makes breakfast cereals finds that if they set the price of a box of Wheaties at $\$C$ then they will sell $100x$ boxes per week where $C = 5 - \ln x$. Each box of Wheaties costs $\$1 \cdot 35$ to produce.

    (a)   Write an expression for the weekly profit $P(x)$ dollars.

    (b)   How should the cereal be priced in order to maximize weekly profit?

(iii)   A particular jet engine is using fuel at a rate given by $\dfrac{dM}{dt} = \dfrac{t}{\sqrt{t-2}}$ for $t \geq 2$ where M kg is the amount of fuel used in t seconds.
By using the substitution $u = \sqrt{t-2}$ find the total amount of fuel used during the time period from $t = 6$ to $t = 11$.

(i)   A ball is dropped vertically from a height of $1 \cdot 8$ metres. After falling to the ground it bounces vertically to a height of $1 \cdot 2$ metres. It then falls to the ground again after which it bounces vertically to a height of $0 \cdot 8$ metres. The ball continues to fall and bounce in this way with the height of each successive bounce being $\dfrac{2}{3}$ the height of the previous bounce. Find the total distance travelled by the ball before it finally comes to rest on the ground.

(ii)   Mr and Mrs Smith are a married couple. The probability that Mr Smith watches a certain TV show is $0 \cdot 4$ and the probability that Mrs Smith watches the show is $0 \cdot 5$. The conditional probability that Mr Smith watches the show, given that Mrs Smith does, is $0 \cdot 7$. Find:

    (a)   the probability that both Mr and Mrs Smith watch the show.

    (b)   the probability that Mrs Smith watches the show given that Mr Smith does.

    (c)   the probability that at least one person of the married Smith couple will watch the show.

(iii)   Let X be a variable which has a normal distribution with $\mu = 2$ and $\sigma = 1.2$.

    (a)   Find the probability that X is negative.

    (b)   Find the constant a such that $P(X > a) = 0.65$