Topic 17: Exercises on Volumes and Shells
Level 3, Part 1

1. By taking strips parallel to the axis of rotation, use the method of cylindrical shells to find the volume of the solid obtained by rotating the region $\{(x, y) : 0 \le x \le 1, x^{1/3} \le y \le 1\}$ about the line $x = 1$.

$\frac{5\pi}{14}$ units^3

2. By taking strips parallel to the axis of rotation, use the method of cylindrical shells to find the volume of the solid obtained by rotating the region $\{(x, y): 0 \leq x \leq \frac{\pi}{2}, 0 \leq y \leq \sin x\}$ about the y-axis.

2π units^3

3. By taking strips parallel to the axis of rotation, use the method of cylindrical shells to find the volume of the solid obtained by rotating the region $\{(x, y): 0 \le x \le \frac{\pi}{2}, 0 \le y \le \cos x\}$ about the line $x = \frac{\pi}{2}$.

2π units^3

4. By taking strips parallel to the axis of rotation, use the method of cylindrical shells to find the volume of the solid obtained by rotating the region $\{(x, y) : 0 \le x \le 1, e^x \le y \le e\}$ about the line $y = e$.

$\frac{\pi}{2}\left(-e^2 + 4e - 1\right)$ units^3

5. By taking strips parallel to the axis of rotation, use the method of cylindrical shells to find the volume of the solid obtained by rotating the region $\{(x, y) : 0 \leq x \leq 1, e^x \leq y \leq e\}$ about the line $x = 1$.

$\pi(4 - e)$ units^3

6. By taking strips parallel to the axis of rotation, use the method of cylindrical shells to find the volume of the solid obtained by rotating the region $\{(x, y): 0 \leq x \leq 1, 0 \leq y \leq \tan^{-1} x\}$ about the y-axis.

$\frac{\pi}{2}(\pi - 2)$ units^3

7. By taking strips parallel to the axis of rotation, use the method of cylindrical shells to find the volume of the solid obtained by rotating the region $\{(x, y) : 0 \leq x \leq 1, 0 \leq y \leq e^{-x^2}\}$ about the y-axis.

$\pi\left(1 - e^{-1}\right)$ units^3