

**Topic 17: Exercises on Volumes and Shells**  
**Level 2, Part 2**

1. By taking strips parallel to the axis of rotation, use the method of cylindrical shells to find the volume of the solid obtained by rotating the region  $\{(x, y) : 1 \leq x \leq e, 0 \leq y \leq \ln x\}$  about the  $y$ -axis.

$$\frac{\pi}{2}(e^2 + 1) \text{ units}^3$$

2. By taking strips parallel to the axis of rotation, use the method of cylindrical shells to find the volume of the solid obtained by rotating the region  $\{(x, y) : 0 \leq x \leq 1, 0 \leq y \leq \frac{1}{1+x^2}\}$  about the  $y$ - axis.

$$\pi \ln 2 \text{ units}^3$$

3. By taking strips parallel to the axis of rotation, use the method of cylindrical shells to find the volume of the solid obtained by rotating the region  $\{(x, y) : 0 \leq x \leq \sqrt{6}, 0 \leq y \leq 6x^2 - x^4\}$  about the  $y$ -axis.

$36\pi \text{ units}^3$

4. By taking strips parallel to the axis of rotation, use the method of cylindrical shells to find the volume of the solid obtained by rotating the region enclosed by the parabola  $y^2 = 4ax$  between its vertex  $(0, 0)$  and its latus rectum about its latus rectum.

$$\frac{32\pi a^3}{15} \text{ units}^3$$

5. By taking strips parallel to the axis of rotation, use the method of cylindrical shells to find the volume of the solid obtained by rotating the region  $\{(x, y) : x \geq 0, y \geq 0, x^{1/2} + y^{1/2} \leq a^{1/2}\}$  about the  $y$ - axis.

$$\frac{\pi a^3}{15} \text{ units}^3$$

6. By taking strips parallel to the axis of rotation, use the method of cylindrical shells to find the volume of the solid obtained by rotating the region  $\{(x, y) : (x - 1)^2 = y^2 \leq 1\}$  about the  $y$ -axis.

$2\pi^2 \text{ units}^3$
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