TEST 5 Real Numbers

Marks: /60

Time: 1 hour 30 minutes

Name: ..... Date: .....

INSTRUCTIONS TO CANDIDATES

Section A (30 marks) **Time: 45 minutes**

1. Answer **all** the questions in this section.
2. Calculators may **not** be used in this section.
3. All working must be clearly shown. Omission of essential working will result in loss of marks.
4. The marks for each question is shown in brackets [ ] at the end of each question.

---

1 Write down the exact values of

(a) $-1.2 \times (-0.5)$,

(b) $69.6 \div (-0.04)$.

Answer (a) ..... [1]

(b) ..... [1]

---

2 Evaluate

(a) $-8 - (11 - 21)$,

(b) $\{-12 - [17 + (-8)]\} \div 3$.

Answer (a) ..... [1]

(b) ..... [1]

---

3 Write the correct integer in each box.

Answer

(a) $(-12) \times (-17) + 9 \times (-17) = [(-12) + 9] \times \square$ [1]

(b) $(-3)^3 + (-5)^2 = (-2)^2 \times \square$ [1]

---

4 Fill in the boxes with >, = or <.

Answer

(a) $[3 - (-12) \times (-4)] \div (-15) \square [(-2) \times (-25)] \div (-10)$ [2]

(b) $\dfrac{6 \times (-5) + (-10)^2}{35 \div (-5)} \square \dfrac{(-2)^3 \times (-3)^2}{(-6) \times (-2)}$ [2]

---

5 Express each of the following as a fraction in its lowest terms.

(a) $\frac{1}{\sqrt{25}} - \frac{1}{\sqrt[3]{-8}} - \left(-1\frac{1}{2}\right)^2$

(b) $-\frac{1}{6} + \left[\left(-\frac{1}{4}\right) \times \frac{1}{8} - \left(-\frac{1}{2}\right) - \frac{1}{4}\right] \div \frac{3}{8}$

Answer (a) ..... [2]

(b) ..... [2]

---

6 Evaluate

(a) $(-2)^2 - (-2)^2 + (-2)^3 - 2^3$,

(b) $-1.6 \times 5 - 4.5 \times (-2.3) \div 0.9$,

(c) $\dfrac{-16 + [(-15) \div 3]}{6 - [36 \div (-4)]}$.

Answer (a) ..... [1]

(b) ..... [2]

(c) ..... [2]

---

7 By using as much of the information given below as is necessary, find the values of

(a) $\sqrt{12}$,

(b) $\sqrt[3]{\frac{-11}{64}}$.

$\sqrt{3} = 1.73$, $\sqrt[3]{11} = 2.22$

Answer (a) ..... [1]

(b) ..... [2]

---

8 The boiling points of some noble gases are given below.

(a) Find the average temperature of the four gases.

(b) Find the difference between the highest and lowest temperatures.

Helium	–269°C
Xenon	–108°C
Krypton	–153°C
Radon	–62°C

Answer (a) ..... °C [1]

(b) ..... °C [1]

---

9 Express $\frac{-5 \times (-10.3)}{-2.8 + (-3.2)}$

(a) as a faction in its simplest form,

(b) as a recurring decimal.

Answer (a) ..... [2]

(b) ..... [1]

---

10 The table below shows the temperatures in a number of cities at noon on a particular day.

(a) Find the difference in temperature between

(i) Singapore and Pyongyang,

(ii) Pyongyong and Vancouver.

(b) The temperature in New York is midway between the temperature in Chengdu and Pyongyang.
What is the temperature in New York?

Singapore	32°C
Pyongyang	–11°C
Chengdu	4°C
Vancouver	–3°C

Answer (a) (i) ..... °C [1]

(ii) ..... °C [1]

(b) ..... °C [1]

INSTRUCTIONS TO CANDIDATES

Section B (30 marks) **Time: 45 minutes**

1. Answer **all** the questions in this section.
2. Calculators may be used in this section.
3. All working must be clearly shown. Omission of essential working will result in loss of marks.
4. The marks for each question is shown in brackets [ ] at the end of each question.

---

11 Use your calculator to evaluate each of the following, giving your answer correct to 3 decimal places.

(a) $(-37)^3 \div \left[-11 - \left(\sqrt{\frac{8}{25}} - 8.99^2\right)\right] \times [-2.3 \div (-\pi)]$

(b) $\dfrac{\sqrt{6\frac{7}{18}} - \left(-\frac{8}{9}\right)^2}{\left(-3\frac{1}{3}\right)^3 + \sqrt[3]{-51\frac{5}{8}}}$

(c) $\sqrt[3]{\dfrac{(-3)^3 - (-11)^3 \times (-27)^2}{12^2 - 12 \times (-\frac{\pi}{2})}}$

Answer (a) ..... [2]

(b) ..... [2]

(c) ..... [2]

---

12 Write down the next two terms of each of the following sequences.

(a) 0.1, –0.3, 0.9, –2.7, ...

(b) $\frac{2}{3}, -\frac{3}{5}, \frac{4}{7}, -\frac{5}{9}, \ldots$

(c) 2, –1, 3, 1, 6, 5, 11, 11, 18, 19, ...

Answer (a) ..... , ... [1]

(b) ..... , ... [2]

(c) ..... , ... [3]

---

13 (a) Arrange the following numbers in ascending order.

$-5, -\frac{1}{4}, \sqrt{2}, -0.7, -\pi, 2\frac{1}{3}, \sqrt[3]{-10}$

(b) Write down two different irrational numbers, p and q where $\frac{p}{q}$ is a rational number.

(c) Tick [✓] the boxes below the numbers which are rational.

Answer (c)

$\frac{\pi}{5}$	$\sqrt{2}+1$	$(\sqrt{2})^3$	$\sqrt{3}\times\sqrt{12}$	2^3+2^2	$(\sqrt{3})^2$	$\sqrt{3}-1.73205$	$\frac{\sqrt{3}}{\sqrt{2}}$

[2]

Answer (a) ..... [2]

(b) p = ... q = ... [2]

---

14 (a) On January 1, Bettina's bank balance was \$800. During the month, she wrote checks for \$89.70, \$61.20 and \$235.80. She also topped up her cash cards by \$60 using the ATM machine. She also made deposits of \$305.80, \$67.50 and \$250. Find the balance in her account at the end of the month.

(b) The largest bubble chamber in the world is about 4.6 m in diameter and contains about 33 000 litres of liquid hydrogen at a temperature of –247°C. If the temperature is dropped by 5.5°C every $\frac{1}{2}$ hour for 5 consecutive hours, what is the new temperature?

Answer (a) \$ ..... [2]

(b) ..... °C [3]

15 (a) Two slugs were released at the same starting point on a vertical wall. The first slug climbed up $17\frac{1}{7}$ mm, slid down $21\frac{1}{4}$ mm, climbed up a further $12\frac{1}{8}$ mm and slid down $3\frac{1}{2}$ mm. The second slug climbed up $20\frac{1}{3}$ mm, slid down $37\frac{1}{6}$ mm, climbed up 19 mm and slid down $23\frac{2}{3}$ mm. How far apart are both slugs finally?

(b) The original temperature of a chemical was –10.75°C. In an experiment, the chemical was heated until its temperature rose by 11.25°C. After that the chemical was cooled until its temperature fell by 8.9°C. This process was repeated until the temperature of the chemical reaches 1°C. How many times must the process be repeated?

Answer (a) ..... mm [3]

(b) ..... [4]

(c) 1 kg of chocolates
+ 1 kg of candies = \$56.50 (Given)
2 kg of chocolates
+ 2 kg of candies = 2 × \$56.50
= \$113
2 kg of chocolates
+ 5 kg of candies = \$172.10 (Given)
∴ cost of 3 kg of candies
= \$172.10 – \$113
= \$59.10
Cost of 1 kg of candies
$= \frac{\$59.10}{3}$
= \$19.70
∴ cost of 1 kg of chocolates
= \$56.50 – \$19.70
= **\$36.80**

Teacher's Tip
Find the total cost of 2 kg of chocolates and 2 kg of candies first.

Test 5: Real Numbers

Section A

1. (a) $-1.2 \times (-0.5)$
$= 1.2 \times 0.5$
$= \mathbf{0.6}$

(b) $69.6 \div (-0.04)$
$= -(69.6 \div 0.04)$
$= -\left(\frac{6960}{4}\right)$
$= \mathbf{-1740}$

Teacher's Tip
Rules for multiplying and dividing of directed numbers

Multiplication	Division
$(+) \times (+) = (+)$	$(+) \div (+) = (+)$
$(-) \times (-) = (+)$	$(-) \div (-) = (+)$
$(+) \times (-) = (-)$	$(+) \div (-) = (-)$
$(-) \times (+) = (-)$	$(-) \div (+) = (-)$

2. (a) $-8 - (11 - 21)$
$= -8 - (-10)$
$= -8 + 10$
$= \mathbf{2}$

(b) $\{-12 - [17 + (-8)]\} \div 3$
$= \{-12 - 9\} \div 3$
$= -21 \div 3$
$= \mathbf{-7}$

Teacher's Tips
Order of operations
1. Simplify the expressions within the brackets first. (Start with innermost brackets.)
2. Working from left to right, perform multiplication and division before addition and subtraction.

3. (a) $(-12) \times (-17) + 9 \times (-17)$
$= [(-12) + 9] \times \boxed{-17}$

Teacher's Tip
Use the distributive law over addition.
$a \times b + a \times c = a \times (b + c)$

(b) $(-3)^3 + (-5)^2$
$= -27 + 25$
$= -2$
$(-2)^2 \times \left(-\frac{1}{2}\right)$
$= 4 \times \left(-\frac{1}{2}\right)$
$= -2$
$\therefore (-3)^3 + (-5)^2 = (-2)^2 \times \boxed{-\frac{1}{2}}$

Within brackets, perform multiplication before subtraction.

4. (a) $[3 - (-12) \times (-4)] \div (-15)$
$= [3 - 48] \div (-15)$
$= (-45) \div (-15)$
$= 45 \div 15$
$= 3$
$[(-2) \times (-25)] \div (-10)$
$= 50 \div (-10)$
$= -(50 \div 10)$
$= -5$
$\therefore [3 - (-12) \times (-4)] \div (-15) \boxed{>} [(-2) \times (-25)] \div (-10)$

(b) $\frac{6 \times (-5) + (-10)^2}{35 \div (-5)}$
$= \frac{-30 + 100}{-7}$
$= \frac{70}{-7}$
$= -10$
$\frac{(-2)^3 \times (-3)^2}{(-6) \times (-2)}$
$= \frac{-8 \times 9}{12}$
$= \frac{-72}{12}$
$= -6$
$\therefore \frac{6 \times (-5) + (-10)^2}{35 \div (-5)} \boxed{<} \frac{(-2)^3 \times (-3)^2}{(-6) \times (-2)}$

5. (a) $\frac{1}{\sqrt{25}} - \frac{1}{\sqrt[3]{-8}} - \left(-1\frac{1}{2}\right)^2$

$= \frac{1}{5} - \left(\frac{1}{-2}\right) - \left(-\frac{3}{2}\right)^2$

$= \frac{1}{5} + \frac{1}{2} - \frac{9}{4}$

$= \frac{4 + 10 - 45}{20}$ ← The LCM of 5, 2 and 4 is 20.

$= \frac{-31}{20}$

$= -1\frac{11}{20}$

Within brackets, perform multiplication first before subtraction.

(b) $-\frac{1}{6} + \left[\left(-\frac{1}{4}\right) \times \frac{1}{8} - \left(-\frac{1}{2}\right) - \frac{1}{4}\right] \div \frac{3}{8}$

$= -\frac{1}{6} + \left[-\frac{1}{32} + \frac{1}{2} - \frac{1}{4}\right] \div \frac{3}{8}$

$= -\frac{1}{6} + \left[\frac{-1 + 16 - 8}{32}\right] \div \frac{3}{8}$

$= -\frac{1}{6} + \frac{7}{32} \div \frac{3}{8}$ ← Change '÷' to '×' and invert the divisor.

$= -\frac{1}{6} + \frac{7}{\cancel{32}_4} \times \frac{\cancel{8}^1}{3}$ ← Perform multiplication before addition.

$= -\frac{1}{6} + \frac{7}{12}$

$= \frac{-2 + 7}{12}$

$= \frac{5}{12}$

6. (a) $(-2)^2 - (-2)^2 + (-2)^3 - 2^3$

$= 4 - 4 + (-8) - 8$

$= 0 - 8 - 8$

$= -16$

(b) $-1.6 \times 5 - 4.5 \times (-2.3) \div 0.9$

$= -8 - (-10.35) \div 0.9$

$= -8 - (-11.5)$

$= -8 + 11.5$

$= 3.5$

Teacher's Tip

Do multiplication and division working from left to right before addition and subtraction.

(c) $\frac{-16 + [(-15) \div 3]}{6 - [36 \div (-4)]}$ ← Simplify expressions within brackets first.

$= \frac{-16 + (-5)}{6 - (-9)}$

$= \frac{-16 - 5}{6 + 9}$

$= \frac{-21}{15}$

$= -1\frac{2}{5}$

7. $\sqrt{3} = 1.73$, $\sqrt[3]{11} = 2.22$ (Given)

(a) $\sqrt{12} = \sqrt{3 \times 4}$

$= \sqrt{3} \times \sqrt{4}$

$= 1.73 \times 2$

$= 3.46$

(b) $\sqrt[3]{\frac{-11}{64}} = \frac{\sqrt[3]{-11}}{\sqrt[3]{64}}$

$= \frac{-2.22}{4}$

$= -0.555$

8. (a) $[-269 + (-108) + (-153) + (-62)] \div 4$

$= \frac{-(269 + 108 + 153 + 62)}{4}$

$= \frac{-592}{4}$

$= -148°C$

(b) Difference $= -62°C - (-269°C)$

$= -62°C + 269°C$

$= 207°C$

9. (a) $\frac{-5 \times (-10.3)}{-2.8 + (-3.2)}$

$= \frac{5 \times 10.3}{-2.8 - 3.2}$

$= \frac{51.5}{-6}$ ← Multiply the numerator and denominator by 10.

$= -\frac{\cancel{515}^{103}}{\cancel{60}_{12}}$

$= -8\frac{7}{12}$

(b) $-8\frac{7}{12} = -8.58\dot{3}$

$$\begin{array}{r} 0.5833 \\ 12\overline{)\,7.0000} \\ \underline{60} \\ 100 \\ \underline{96} \\ 40 \\ \underline{36} \\ 40 \\ \underline{36} \\ 4 \end{array}$$

10. (a) (i) Difference = 32°C – (–11°C)
= 32°C + 11°C
= **43°C**

(ii) Difference = –3°C – (–11°C)
= –3°C + 11°C
= **8°C**

(b) Temperature in New York
$= \frac{-11°C + 4°C}{2}$
$= \frac{-7°C}{2}$
= **–3.5°C**

Section B

11. (a) $(-37)^3 \div \left[-11 - \left(\sqrt{\frac{8}{25}} - 8.99^2\right)\right] \times [-2.3 \div (-\pi)]$
= –535.4706...
≈ **–535.471** (correct to 3 d.p.)

(b) $\frac{\sqrt{6\frac{7}{18}} - \left(-\frac{8}{9}\right)^2}{\left(-3\frac{1}{3}\right)^3 + \sqrt[3]{-51\frac{5}{8}}}$
= –0.0426...
≈ **–0.043** (correct to 3 d.p.)

(c) $\sqrt[3]{\frac{(-3)^3 - (-11)^3 \times (-27)^2}{12^2 - 12 \times (\frac{-\pi}{2})}}$
= 18.1287....
≈ **18.129** (correct to 3 d.p.)

Teacher's Tips
To round off a decimal to a specified place:
1. Include one extra digit (on the right) for consideration.
2. Drop the extra digit if it is less than 5.
3. If it is 5 or more, add 1 to the previous digit before dropping the extra digit.

12. (a) 0.1, –0.3, 0.9, –2.7,...

Rule: Multiply (–3) to the term before to get the next term.

∴ the next 2 terms are
$-2.7 \times (-3) = 8.1$ and
$8.1 \times (-3) = -24.3$.

(b) $\frac{2}{3}, -\frac{3}{5}, \frac{4}{7}, -\frac{5}{9}, \ldots$

Alternate numbers are negative.
Numerator: Add 1 to get to the next term.
Denominator: Add 2 to get to the next term.

∴ the next 2 terms are $\frac{6}{11}$ and $-\frac{7}{13}$.

(c) 2, –1, 3, 1, 6, 5, 11, 11, 18, 19, 27, 29
(–3, –2, –1, –0, +1, +2; +4, +5, +6, +7, +8)

∴ the next two terms are
19 + 8 = **27** and
27 + 2 = **29**.

13. (a) $-5, -\frac{1}{4}, \sqrt{2}, -0.7, -\pi, 2\frac{1}{3}, \sqrt[3]{-10}$
$-\frac{1}{4} = -0.25$, $\sqrt{2} = 1.41\ldots$, $-\pi = -3.14\ldots$, $2\frac{1}{3} = 2.\dot{3}$, $\sqrt[3]{-10} = -2.15\ldots$

∴ the numbers arranged in ascending order are
$-5, -\pi, \sqrt[3]{-10}, -0.7, -\frac{1}{4}, \sqrt{2}, 2\frac{1}{3}$.

(b) Accept other possible answers.
$p = 3\pi$, $q = 2\pi$
$\frac{3\pi}{2\pi} = 1.5$
Alternative answer
$p = \sqrt{12}$, $q = \sqrt{3}$
$\frac{\sqrt{12}}{\sqrt{3}} = \sqrt{\frac{12}{3}} = \sqrt{4} = 2$

(c)

$\frac{\pi}{5}$	$\sqrt{2}+1$	$(\sqrt{2})^3$	$\sqrt{3} \times \sqrt{12}$	$2^3 + 2^2$	$(\sqrt{3})^2$	$\sqrt{3} - 1.73205$	$\frac{\sqrt{3}}{\sqrt{2}}$
			✓	✓	✓		

$\sqrt{3} \times \sqrt{12} = \sqrt{3 \times 12} = \sqrt{36} = 6$
$2^3 + 2^2 = 8 + 4 = 12$
$(\sqrt{3})^2 = \sqrt{3} \times \sqrt{3} = \sqrt{3 \times 3} = \sqrt{9} = 3$

14. (a) Balance in account
= \$800 – \$89.70 – \$61.20 – \$235.80 – \$60 + \$305.80 + \$67.50 + \$250
= **\$976.60**

(b) New temperature
= –247 – 5 × 2 × 5.5
= –247 – 55
= **–302°C**

15. (a) Height of 1st slug from starting point

$= 17\frac{1}{7} - 21\frac{1}{4} + 12\frac{1}{8} - 3\frac{1}{2} = 4\frac{29}{56}$ mm

Height of 2nd slug from starting point

$= 20\frac{1}{3} - 37\frac{1}{6} + 19 - 23\frac{2}{3} = -21\frac{1}{2}$ mm

Distance between both slugs

$= 4\frac{29}{56} - \left(-21\frac{1}{2}\right)$

$= 4\frac{29}{56} + 21\frac{1}{2}$

$= \mathbf{26\frac{1}{56}}$ **mm**

(b) Difference in temperature in each process
$= 11.25°C - 8.9°C$
$= 2.35°C$
Temperature after
1st process: $-10.75 + 2.35 = -8.4°C$
2nd process: $-8.4 + 2.35 = -6.05°C$
3rd process: $-6.05 + 2.35 = -3.7°C$
4th process: $-3.7 + 2.35 = -1.35°C$
5th process: $-1.35 + 2.35 = 1°C$
$\therefore$ the process must be repeated **5 times** for the temperature of the chemical to reach 1°C.

Test 6: Estimation and Approximation

Section A

1. (a) $23.086 \approx$ **20** (correct to 1 sig. fig.)
(b) $23.086 \approx$ **23.09** (correct to 2 d.p.)
(c) $23.086 \approx$ **23** (correct to the nearest integer)

2. (a) (i) $0.008186 \approx$ **0.00819** (correct to 3 sig. fig.)
(not significant)

Teacher's Tip
Zeros preceding the first non-zero digit are not significant.

(ii) 62 6(5)9 $\approx$ **62 700** (correct to 3 sig. fig.)

If the extra digit is 5 or more, add 1 to the previous digit and replace the extra digits with zeros to keep the place value.

(b) 496 mm $= \frac{496}{10}$ cm ← 1 cm = 10 mm
$= 49.6$ cm
$\approx$ **50 cm** (correct to the nearest cm)

3. (a) $54.96703 - 18.24075$
$\approx 55 - 18$
$= 37$
$\approx$ **40** (correct to 1 sig. fig.)

(b) $\frac{40}{0.00496}$
$\approx \frac{40}{0.0050}$
$= \frac{40\,000}{5}$
$=$ **8000** (correct to 1 sig. fig.)

Teacher's Tip
To estimate to 1 sig. fig., estimate to 2 sig. fig. in the working and then round off to 1 sig. fig. in the final answer.

4. (a) \$615.90 $\approx$ **\$620** (correct to the nearest ten dollars)
(b) 8.03454 $\approx$ **8.03** (correct to 3 sig. fig.)

Zero between non-zero digits are significant.

8.03 has **2 decimal places.**

(c) $10\pi - 18.93 = 10 \times 3.14159 - 18.93$
$\approx 10 \times 3.14 - 18.9$
$= 31.4 - 18.9$
$= 12.5$
$\approx$ **13** (correct to 2 sig. fig.)

Teacher's Tip
To estimate to 2 sig. fig., estimate to 3 sig. fig. in the working and then round off to 2 sig. fig. in the final answer.

5. (a) $367.0457 \approx$ **367.05** (correct to 2 d.p.)
(b) $367.0457 \approx$ **367.0** (correct to 4 sig. fig.)
(c) $367.0457 \approx$ **400** (correct to the nearest hundred)

6. (a) $19.98 \times 30.03 - 59.84$
$\approx 20 \times 30 - 60$
$= 600 - 60$
$= 540$
$\approx$ **500** (correct to 1 sig. fig.)

(b) $\frac{48.12}{1.61} \approx \frac{48}{1.6}$
$= \frac{480}{16}$
$=$ **30** (correct to 1 sig. fig.)