

Waverley College

Year 10 Level 3

Mathematics

Half Yearly Examination 2006

Weighting: 80%

Time Allowed: 80 Minutes

Student Name: _____

Teacher's Name: _____

Instructions to Students:

- Use a blue or black pen only.
- Show all working where necessary.
- Approved Scientific calculators may be used.
- Marks may be deducted for careless or poorly arranged work

Section A – Literacy	/10	
Section B – Number & Measurement Review	/10	
Section C – Algebraic Techniques	/15	Executive
Section D – Equations & Inequalities	/15	
Section E – Trigonometry	/25	
TOTAL	/75	%

SECTION A: Literacy (10 Marks)

Use the list of words below to answer the following:

discriminant conjugate surds squares perfect square substitution tan
sine minutes solve scientific notation quadratic ~~binomial~~ cos

- 1 $(\sqrt{2} + \sqrt{5})$ and $(\sqrt{2} - \sqrt{5})$ are _____
- 2 A shorthand way of writing very large or small numbers using powers of ten _____
- 3 The expression $(a + b)^2$ is considered to be a _____
- 4 An equation of the form $ax^2 + bx + c$ is called a _____
- 5 An expression of the form $(a + b)(a - b)$ is called a difference of two _____
- 6 Replacing a variable with a number _____
- 7 Find the solution _____
- 8 Used to determine the number and type of solutions to a quadratic expression _____
- 9 Ratio defined as opposite over hypotenuse _____
- 10 There are 60 of these in 1 degree _____

SECTION B: Number & Measurement Review (10 Marks)

For multiple choice questions 1 to 10, circle the correct response.

1 $\frac{4}{3-\sqrt{3}}$ in simplest form is:

- A $\frac{12+4\sqrt{3}}{3}$
- B $\frac{6+2\sqrt{3}}{6}$
- C $\frac{3+\sqrt{3}}{3}$
- D $\frac{6+2\sqrt{3}}{3}$

2 $\frac{a^{11} \times a^2}{a^{12} \div a^9}$ simplified is:

- A a^{-10}
- B a^{10}
- C a^{11}
- D $\frac{1}{a}$

3 $\frac{4.8 \times 3.6}{\sqrt{2.5}}$ correct to three significant figures is:

- A 10.9
- B 10.92
- C 10.929
- D 11

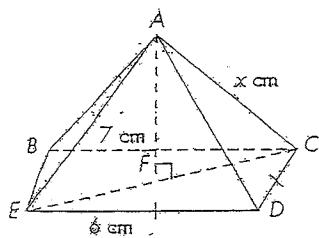
4 $\frac{3.5 \times 10^5 + 6.1 \times 10^8}{4.7 \times 10^3}$ is closest to:

- A 4.80×10^{10}
- B 1.30×10^{11}
- C 4.54×10^{16}
- D 2.04×10^{10}

5. $\frac{\sqrt{60}}{2\sqrt{5}}$ in simplest form is:

- A $\frac{\sqrt{12}}{2}$
- B $\sqrt{6}$
- C $\sqrt{3}$
- D $2\sqrt{3}$

6. The exact length of the slant edge of a square pyramid with base side length 6 cm and height 7 cm would be:



- A $\sqrt{67}$
- B $\frac{\sqrt{67}}{4}$
- C 8
- D $3\sqrt{2}$

7. \$12 000 is invested for 5 years at an interest rate of 3.5% per annum compounded quarterly. The interest earned is:

- A \$5 182
- B \$2 284.08
- C \$2 100
- D \$1 104.97

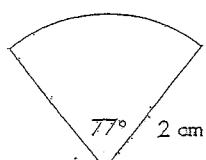
8. At a clearance sale, a television worth \$600 is discounted by 30%, then discounted by a further 20%. The final price of the television is:

- A \$480
- B \$420
- C \$336
- D \$300

9 Which of the following formulae does not give the area of the stated polygon?

- A $A = \frac{1}{2}h(a+b)$ Trapezium
B $A = \frac{1}{2}xy$ Kite
C $A = xy$ Rhombus
D $A = bh$ Parallelogram

10 Find the area of the following figure correct to 1 decimal place:



- A 1.3 cm^2
B 2.7 cm^2
C 12.6 cm^2
D 5.4 cm^2

SECTION C: Algebraic Techniques

1 Expand the following expressions:

3 Marks

(a) $3a(4b + 7)$

.....
.....
.....

(b) $(x - 6)(x + 7)$

.....
.....
.....

2 The expansion of $(3x - 2)(3x + 2)$ is:

1 Mark

- A $(3x - 2)^2$
B $9x^2 - 4$
C $9x^2$
D $9x^2 + 4$

3 $(5x + 3)^2$ is the same as:

1 Mark

- A $25x^2 + 30x + 9$
B $25x^2 - 15x + 9$
C $25x^2 + 9$
D $25x^2 + 15x + 9$

4 Factorise the following 3 Marks

(a) $x^2 - 100$

(b) $16x^2 - 25y^2$

5 A rectangle with an area $3x^2 + 11x - 20$ has length $x + 5$. Its breadth is: 1 Mark

A $3x - 4$

B $3x^2 + 10x - 15$

C $3x^3 + 26x^2 + 35x - 100$

D $2x - 4$

6 Simplify the fraction: 3 Marks

$$\frac{y^2 - 16}{y^2 + 6y + 8}$$

7 Simplify the following expression: 3 Marks

$$\frac{x-6}{4} + \frac{2x+7}{9}$$

SECTION D: Equations & Inequations

1 Solve to find the value of y :

$$\frac{3y-9}{6} = 3$$

2 Marks

2 Solve the following equations:

(a) $x^2 - 4x + 4 = 0$

1 Mark

3 Use the quadratic formula to solve the following equation

$$-4x^2 - 5x + 6 = 0$$

2 Marks

4 The area of an envelope is 72 cm^2 . If the length is 6 cm longer than the breadth, find the dimensions of the envelope.

3 Marks

5

Solve the following pairs of simultaneous equations.

4 Marks

$$2x^2 + y = 9$$

$$2x + 3y = 7$$

6

Solve:

2 Marks

$$7(x + 3) \leq 14$$

7

Transposing $x = \sqrt{y^2 + z^2}$ to make z the subject gives:

1 Mark

A $z = -\sqrt{x^2 - y^2}$

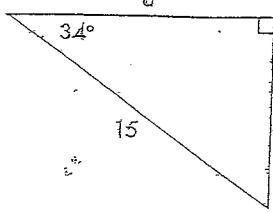
B $z = +\sqrt{x^2 - y^2}$

C $z = x^2 - y^2$

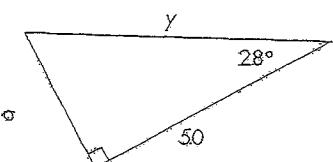
D $z = \pm \sqrt{x^2 - y^2}$

SECTION E: Trigonometry

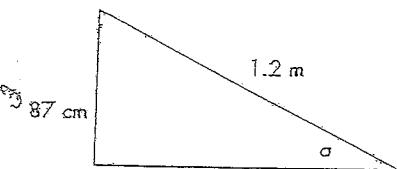
- 1 Find the value of the pronumeral in this diagram correct to the nearest whole number. 2 Marks



- 2 Find the value of the pronumeral in this diagram, correct to 2 decimal places. 3 Marks



- 3 A ramp 1.2 m long reaches a point 87 cm above the ground. The angle the ramp makes with the ground, to the nearest minute, is: 2 Marks



- 4 Which of the following statements is untrue? 1 Mark

A $\sin 30^\circ = \frac{1}{2}$

B $\tan 60^\circ = \sqrt{3}$

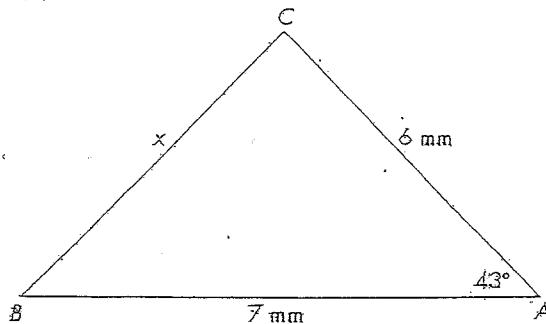
C $\sin 0^\circ = 0$

D $\cos 45^\circ = \frac{1}{\sqrt{3}}$

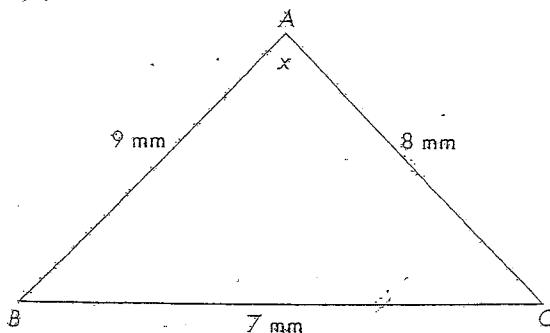
- 5 Find the value of x in each of the following triangles correct to two decimal places or in degrees and minutes.

6 Marks

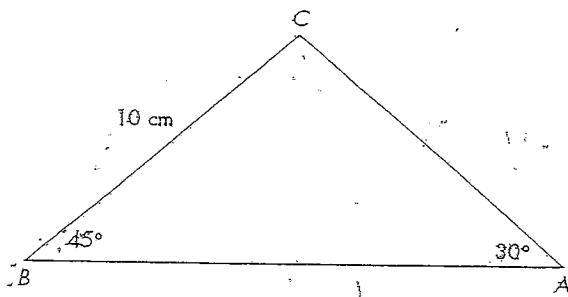
(a)



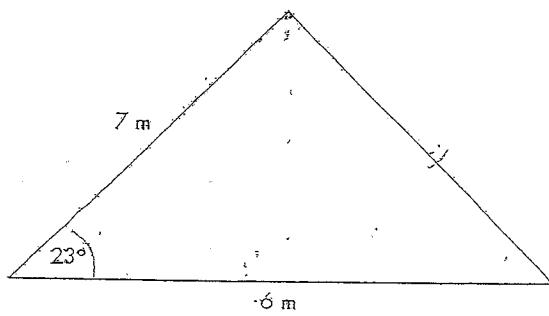
(b)



6 Given the following side and two angles in the diagram below, find the remaining sides and angle correct to the nearest whole number. 3 Marks



7

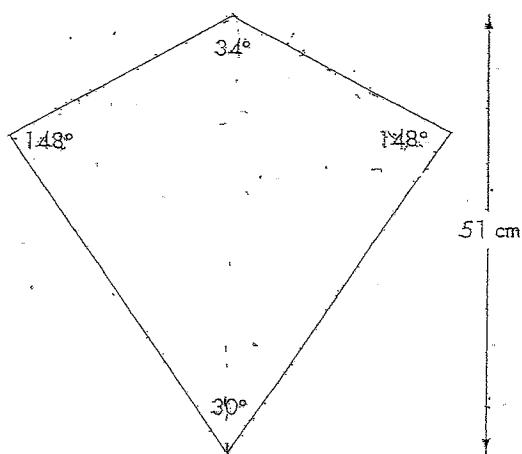


The area of the triangle shown is:

- A 21 m^2
- B 16.42 m^2
- C 42 m^2
- D 8.21 m^2

1 Mark

- 8 Melissa's kite has angles of 148° , 148° , 30° and 34° . If the height from the top to the bottom is 51 cm, find 7 Marks



- (a) the length of all the sides on her kite, correct to two decimal places
(b) the total area of material in cm^2 that covers the kite, correct to two decimal places
(c) the cost to replace the material if it costs 65 cents/ cm^2 .

END OF EXAM



SOLUTIONS



Waverley College

Year 10 Level 3

Mathematics

Half Yearly Examination

Weighting: 80%

Time Allowed: 80 Minutes

Student Name: Jason Lam-Henig Teacher's Name: Mrs Farnell

Instructions to Students:

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- Show all working where necessary.
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- Marks may be deducted for careless or poorly arranged work

Section A – Literacy	/10	
Section B – Number & Measurement Review	/10	
Section C – Algebraic Techniques	/15	
Section D – Equations & Inequalities	/15	
Section E – Trigonometry	/25	
TOTAL	175	%

SECTION A – Literacy (10 Marks)

Use the list of words below to answer the following:

discriminant conjugate surds squares perfect square substitution
 sine minutes solve scientific notation quadratic trinomial tan
 cos

1. $(\sqrt{2} + \sqrt{5})$ and $(\sqrt{2} - \sqrt{5})$ are conjugate surds ✓
2. A shorthand way of writing very large or small numbers using powers of ten Scientific notation ✓
3. The expression $(a + b)^2$ is considered to be a perfect square ✓
4. An equation of the form $ax^2 + bx + c$ is called a quadratic trinomial ✓
5. An expression of the form $(a + b)(a - b)$ is called a difference of two squares ✓
6. Replacing a variable with a number substitution ✓
7. Find the solution value ✓
8. Used to determine the number and type of solutions to a quadratic expression discriminant ✓
9. Ratio defined as opposite over hypotenuse sine ✓
10. There are 60 of these in 1 degree minutes ✓

SECTION B: Number & Measurement Review (10 Marks)

For multiple choice questions 1 to 10, circle the correct response.

1 $\frac{4}{3-\sqrt{3}}$ in simplest form is:

$$\frac{4}{3-\sqrt{3}} \times \frac{3+\sqrt{3}}{3+\sqrt{3}} = \frac{12+4\sqrt{3}}{9-3} = \frac{12+4\sqrt{3}}{6}$$

$$= \frac{12+4\sqrt{3}}{6} = \frac{6+2\sqrt{3}}{3}$$

- A $\frac{12+4\sqrt{3}}{3}$
- B $\frac{6+2\sqrt{3}}{6}$
- C $\frac{3+\sqrt{3}}{3}$
- D $\frac{6+2\sqrt{3}}{3}$



2 $\frac{a^{11} \times a^2}{a^{12} + a^9}$ simplified is:

$$\frac{a^{13}}{a^{12}}$$

- A a^{-10}
- B a^{10}
- C a^{11}
- D $\frac{1}{a}$



3 $\frac{4.8 \times 3.6}{\sqrt{2.5}}$ correct to three significant figures is:

- A 10.9
- B 10.92
- C 10.929
- D 11



4 $\frac{3.5 \times 10^5 + 6.1 \times 10^8}{4.7 \times 10^3}$ is closest to:

- A 4.80×10^{10}
- B 1.30×10^{11}
- C 4.54×10^{16}
- D 2.04×10^{10}



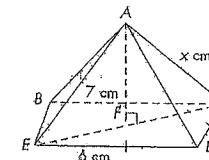
5 $\frac{\sqrt{60}}{2\sqrt{5}}$ in simplest form is:

$$\frac{\sqrt{60}}{2\sqrt{5}} = \frac{\sqrt{12}}{2} = \sqrt{3}$$

- A $\frac{\sqrt{12}}{2}$
- B $\sqrt{6}$
- C $\sqrt{3}$
- D $2\sqrt{3}$



6 The exact length of the slant edge of a square pyramid with base side length 6 cm and height 7 cm would be:



- A $\sqrt{67}$
- B $\frac{\sqrt{67}}{4}$
- C 8
- D $3\sqrt{2}$

$$\begin{aligned} EC^2 &= 6^2 + 6^2 = 72 \\ EC &= \sqrt{72} = 6\sqrt{2} \\ FC &= 3\sqrt{2} \\ x^2 &= (3\sqrt{2})^2 + 72 \\ &= 18 + 49 = 67 \\ x &= \sqrt{67} \end{aligned}$$



7 \$12 000 is invested for 5 years at an interest rate of 3.5% per annum compounded quarterly. The interest earned is:

- A \$5 182
- B \$2 284.08
- C \$2 100
- D \$1 104.97

$$\begin{aligned} A &= 12000 \left(1 + \frac{0.035}{4}\right)^{20} \\ &= 14284.08 - \\ &\quad \underline{12000} \\ &= 2284.08 \end{aligned}$$



8 At a clearance sale, a television worth \$600 is discounted by 30%, then discounted by a further 20%. The final price of the television is:

- A \$480
- B \$420
- C \$336
- D \$300

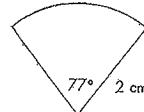
$$\begin{aligned} &\frac{30}{100} \times 600 = \$180 \\ 600 - 180 &= \$420 \\ \frac{20}{100} \times 420 &= \$84 \\ 420 - 84 &= \$336 \end{aligned}$$

4

9 Which of the following formulae does not give the area of the stated polygon?

- A $A = \frac{1}{2}h(a+b)$ Trapezium
- B $A = \frac{1}{2}xy$ Kite
- C $A = xy$ Rhombus
- D $A = bh$ Parallelogram

10 Find the area of the following figure correct to 1 decimal place:



- A 1.3 cm^2
- B 2.7 cm^2
- C 12.6 cm^2
- D 5.4 cm^2

$$\begin{aligned} & \frac{\pi r^2 \theta}{360} \\ &= \frac{\pi \times 2^2 \times 77}{360} \\ &= 2.7 \text{ cm}^2 \end{aligned}$$

SECTION C: Algebraic Techniques

1 Expand the following expressions:

(a) $3a(4b+7)$

$12ab + 21a$

(b) $(x-6)(x+7)$

$x^2 + 7x - 6x - 42$

$x^2 + x - 42$

2 The expansion of $(3x-2)(3x+2)$ is:

A $(3x-2)^2$

B $9x^2 - 4$

C $9x^2$

D $9x^2 + 4$

$(5x+3)(5x+3)$

$$\begin{aligned} &= 25^2 + 15x + 15x + 9 \\ &= 25^2 + 30x + 9 \end{aligned}$$

3 $(5x+3)^2$ is the same as:

A $25x^2 + 30x + 9$

B $25x^2 - 15x + 9$

C $25x^2 + 9$

D $25x^2 + 15x + 9$

4 Factorise the following:

(a) $x^2 - 100$

$(x+10)(x-10)$

(b) $16x^2 - 25y^2$

$(4x-5y)(4x+5y)$

5 A rectangle with an area $3x^2 + 11x - 20$ has length $x+5$. Its breadth is:

A $3x-4$

B $3x^2 + 10x - 15$

C $3x^3 + 26x^2 + 35x - 100$

D $2x-4$

$(x+5) = 3x^2 + 11x - 20$

$(3x-4)(x+5)$

$\checkmark 3x^2 + 15x - 4x - 20$

$= 3x^2 + 11x - 20$

6 Simplify the fraction:

$$\frac{y^2 - 16}{y^2 + 6y + 8} = \frac{(y+4)(y-4)}{(y+4)(y+2)} = \frac{(y-4)}{(y+2)}$$

$y-4$

$y+2$

7 Simplify the following expression:

$$\frac{x-6}{4} + \frac{2x+7}{9}$$

$9(x-6) + 4(2x+7)$

36

$9x - 54 + 8x + 28$

36

$17x - 26$

36

SECTION D: Equations & Inequations

1 Solve to find the value of y :

$$\frac{3y-9}{6} = 3$$

$$3y - 9 = 18$$

$$3y = 27$$

$$y = 9$$

2 Marks

2 Solve the following equations:

(a) $x^2 - 4x + 4 = 0$

$$(x-2)(x-2) = 0$$

$$x = 2$$

1 Mark

3 Use the quadratic formula to solve the following equation

2 Marks

$$-4x^2 - 5x + 6 = 0$$

$$ax = -4 \quad b = 5 \quad c = 6$$

$$(-5) \pm \sqrt{(-5)^2 - 4(-4)(6)} =$$

$$-4 \times 2$$

$$2. \quad \frac{5 \pm \sqrt{25+48}}{-8}$$

$$= \frac{5 \pm \sqrt{73}}{-8}$$

$$x = -5/2 \text{ or } x = \frac{3}{4}$$

4 The area of an envelope is 72 cm^2 . If the length is 6 cm longer than the breadth, find the dimensions of the envelope.

3 Marks

$$x+6$$

$$x$$

$$A = 72 \text{ cm}^2$$

$$x(x+6) = 72$$

$$x^2 + 6x = 72$$

$$x^2 + 6x - 72 = 0$$

$$(x+12)(x-6) = 0$$

$$\therefore x = 6 \text{ but } x \neq -12$$

Sides are 6 and 12

5 Solve the following pairs of simultaneous equations.

$$2x^2 + y = 9$$

$$2x + 3y = 7$$

$$6x^2 + 3y = 27 \quad (1)$$

$$2x + 3y = 7 \quad (2)$$

(1) - (2):

$$6x^2 - 2x = 20 \quad (2)$$

$$3x^2 - x - 10 = 0$$

$$(3x+5)(x-2) = 0$$

$$\therefore x = 2 \text{ or } x = -\frac{5}{3} \quad \leftarrow \text{Subs. into (1). } y = 9 - 2x^2$$

$$\therefore y = 9 - 2(2)^2 \quad \& \quad y = 9 - 2\left(-\frac{5}{3}\right)^2$$

$$= 9 - 8 \quad = 9 - 2\left(\frac{25}{9}\right)$$

$$= 1 \quad = 9 - \frac{50}{9}$$

$$= \frac{31}{9}$$

$x = 2$	$\&$	$x = -\frac{5}{3}$
$y = 1$	$\&$	$y = \frac{31}{9}$

6 Solve:

2 Marks

$$7(x+3) \leq 14$$

$$7x + 21 \leq 14$$

$$7x \leq -7 \quad (\div 7)$$

$$x \leq -1$$

7 Transposing $x = \sqrt{y^2 + z^2}$ to make x the subject gives:

1 Mark

A $z = -\sqrt{x^2 - y^2}$

B $z = +\sqrt{x^2 - y^2}$

C $z = x^2 - y^2$

D $z = \pm \sqrt{x^2 - y^2}$

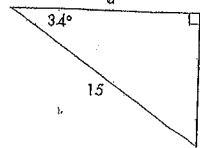
$$x^2 = y^2 + z^2$$

$$x^2 - y^2 = z^2$$

$$\pm \sqrt{x^2 - y^2} = z$$

SECTION E: Trigonometry

- 1 Find the value of the pronumeral in this diagram correct to the nearest whole number. 2 Marks

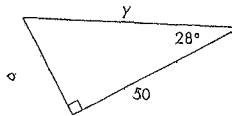


$$\cos 34^\circ = \frac{a}{15}$$

$$15 \cos 34^\circ = a$$

$\cos 34^\circ = \frac{a}{15}$ $a = 12$ ✓ ✓

- 2 Find the value of the pronumeral in this diagram, correct to 2 decimal places. 3 Marks



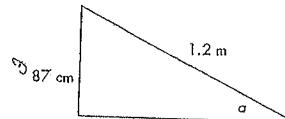
$$\cos 28^\circ = \frac{y}{50}$$

$$y \cos 28^\circ = 50$$

$$y = \frac{50}{\cos 28^\circ}$$

$y = 56.63$ ✓ ✓ ✓

- 3 A ramp 1.2 m long reaches a point 87 cm above the ground. The angle the ramp makes with the ground, to the nearest minute, is: 2 Marks



$$\sin \alpha = \frac{87}{120}$$

$$\sin \alpha = 0.725$$

$$\alpha = 46^\circ 28'$$

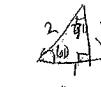
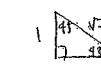
- 4 Which of the following statements is untrue?

A $\sin 30^\circ = \frac{1}{2}$

B $\tan 60^\circ = \sqrt{3}$

C $\sin 0^\circ = 0$

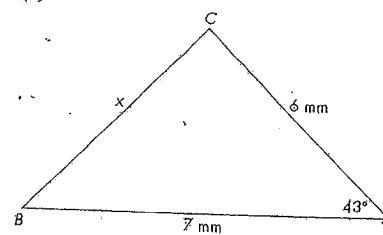
D $\cos 45^\circ = \frac{1}{\sqrt{3}}$



1 Mark

- 5 Find the value of x in each of the following triangles correct to two decimal places or in degrees and minutes. 6 Marks

(a)



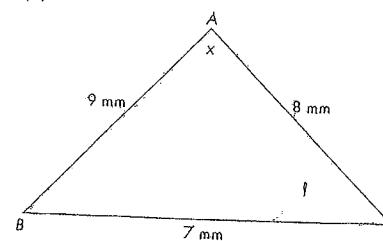
$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$a^2 = 6^2 + 7^2 - 2 \times 7 \times 6 \cos 43^\circ$$

$$a^2 = 23.57$$

$$a = 4.85 \text{ mm}$$

(b)



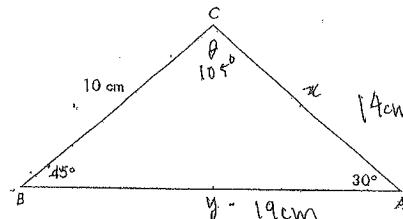
$$\cos A = \frac{8^2 + 9^2 - 7^2}{2 \times 8 \times 9}$$

$$\cos A = \frac{96}{144}$$

$$\cos A = 0.666$$

$$\cos A = 48^\circ 11'$$

- 5 Given the following side and two angles in the diagram below, find the remaining sides and angle correct to the nearest whole number. 13 Marks



$$45 + 30 = 75$$

$$180 - 75 = 105^\circ$$

$$\theta = 105^\circ$$

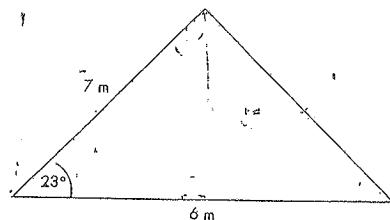
$$\frac{10}{\sin 30} = \frac{x}{\sin 45}$$

$$x = 14 \text{ cm}$$

$$\frac{10}{\sin 30} = \frac{y}{\sin 105}$$

$$y = 19.86 \text{ cm}$$

7



$$\text{Area} = \frac{1}{2} ab \sin C$$

$$= \frac{1}{2} \times 7 \times 6 \times \sin 23^\circ$$

$$= 8.205$$

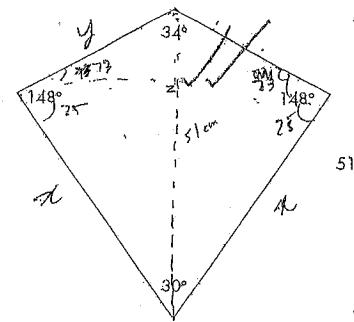
1 Mark

The area of the triangle shown is:

- A 21 m^2
- B 16.42 m^2
- C 42 m^2
- D 8.21 m^2

()

- 8 Melissa's kite has angles of 148° , 148° , 30° and 34° . If the height from the top to the bottom is 51 cm, find 7 Marks



(a) the length of all the sides on her kite, correct to two decimal places

(b) the total area of material in cm^2 that covers the kite, correct to two decimal places

(c) the cost to replace the material if it costs 65 cents/ cm^2 .

$$\text{a) } \frac{51}{\sin 148} = \frac{x}{\sin 17} \quad \frac{51}{\sin 148} = \frac{y}{\sin 115}$$

$$x = 28.14 \text{ cm} \quad y = 24.91 \text{ cm}$$

$$\text{b) } A = \frac{1}{2} ab \sin C$$

$$= \frac{1}{2} \times 28.14 \times 24.91 \times \sin 148^\circ$$

$$= 371.456 \text{ cm}^2$$

$$\text{c) } \$241.456 \text{ per } \text{m}^2$$

$$\$241.456$$

END OF EXAM

