Waverley College
Year 11 2 unit Accelerated
HSC Task 1
Term 2, 2004

TIME ALLOWED: 50 MINUTES

INSTRUCTIONS:

Attempt all questions
Start each question on a new page
Calculators may be used
Write in blue or black pen only
Show all necessary working
Marks may be deducted for careless or badly arranged work

Question 1	/16
Question 2	/18
Question 3	/16
Total	/50
%	

Question 1 (16 marks)

a) If α and β are the roots of the equation $2x^2 + 3x - 4 = 0$, find the values of:
(i) $\alpha\beta$
(ii) $\alpha + \beta$
(iii) $\frac{1}{\alpha} + \frac{1}{\beta}$ **4 marks**

b) For what values of k will $2x^2 - kx + 12$ be always positive? **2 marks**

c) In the quadratic equation $4x^2 - 3x + k = 0$, one root is double the other. Find the value of k. **3 marks**

d) If $9x^2 + 2x - 5 \equiv ax(x+1) + b(x+1) + c$, evaluate a, b and c. **3 marks**

e) For the quadratic equation $x^2 + (k-1)x = 2k + 1$

(i) Find the discriminant.
(ii) Find the values of k for which the equation has two different, real roots. **4 marks**

Question 2 (18 marks)

a) Solve $3x^4 - 11x^2 + 6 = 0$ **5 marks**

b) Solve $x^2 - 7x - 18 \geq 0$ **3 marks**

c) $y = -x^2 + 2x + 8$ is a parabola. **10 marks**

(i) Find the x intercepts.
(ii) Find the y intercepts.
(iii) Find the axis of symmetry.
(iv) Find the coordinates of the vertex.
(v) Draw a neat sketch of the parabola showing all the necessary features.
(vi) Using this graph, or otherwise, find the maximum value of $y = -x^2 + 2x + 8$.

Question 3 (16 marks)

a) Find the coordinates of the vertex, focus and the equation of the directrix of the parabola $y^2 = -12x$. **3 marks**

b) (i) Find the equation of the normal to the curve $x^2 = 12y$ at the point (6,3). **3 marks**

(ii) This normal meets the parabola again at the point Q. Find the coordinates of Q. **3 marks**

c) Let A and B be the points (0,-1) and (0,2) respectively and let P be a variable point (x, y). **7 marks**

(i) If the point P moves so that $\text{PA} = 2 \times \text{PB}$, show that P moves on the circle $x^2 + y^2 - 6y + 5 = 0$.

(ii) Find the centre and radius of the circle.

Solutions:

Q1 (a) i) $\alpha\beta = -2$ ii) $\alpha+\beta = -3/2$

iii) $\frac{1}{\alpha}+\frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = 3/4$

(b) $f(x) > 0$ if $\Delta < 0$ & $a > 0$

ie $k^2 - 96 < 0 \Rightarrow -4\sqrt{6} < k < 4\sqrt{6}$

(c) $\alpha + 2\alpha = -b/a \rightarrow \alpha = 1/4$

$\alpha \times 2\alpha = c/a \rightarrow k = 1/2$

(d) $9x^2+2x-5 \equiv ax^2+(a+b)x+(b+c)$

$\therefore \left.\begin{array}{l} a=9 \\ a+b=2 \\ b+c=-5 \end{array}\right\} \Rightarrow \begin{array}{l} a=9 \\ b=-7 \\ c=2 \end{array}$

(e) $\Delta = k^2+6k+5 > 0$ for 2 roots

$\therefore \quad k<-5 \;\cup\; k>-1$

Q2 (a) $(3x^2-2)(x^2-3) = 0$

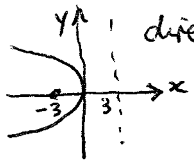
$\therefore x = \pm\sqrt{2/3}$ or $x = \pm\sqrt{3}$

(b) $(x-9)(x+2) \geq 0 \rightarrow x \leq -2 \cup x \geq 9$

(c) $(4,0)$ $(-2,0)$; $(0,8)$; $x=1$; $V=(1,9)$

Max value is 9

Q3 (a) $a=3$, $S=(0,-3)$, $V=(0,0)$



directrix is $x = 3$

(b) At $P(6,3)$ gradient is $m=1 \therefore m_\perp = -1$

(i) $\therefore y-3 = -1(x-6) \rightarrow x+y-9=0$ (Eqn. Normal)

(ii) Solve $\left\{\begin{array}{l} y=-x+9 \\ x^2=12y \end{array}\right\} \rightarrow x^2+12x-108=0$

$\left.\begin{array}{l} x=6 \\ y=3 \end{array}\right\}$ or $\left.\begin{array}{l} x=-18 \\ y=27 \end{array}\right\}$

$\therefore Q = (-18, 27)$

(c) $PA^2 = 4 \times PB^2$ (...using distance formula)

$(x-0)^2+(y+1)^2 = 4\left[(x-0)^2+(y-2)^2\right]$

$\therefore x^2+y^2+2y+1 = 4x^2+4y^2-16y+16$

$\therefore 3x^2+3y^2-18y+15 = 0$

$x^2+y^2-6y+5 = 0$

$x^2+(y-3)^2 = 4$

a Circle, centre $(0,3)$ radius 2