*Waverley College*
*Year 12 Ext 1*
*HSC Task 2*

**TIME ALLOWED: 45 MINUTES**

**NAME:** Valentino Koosasi

**TEACHER:** Ms. Murphy.

**INSTRUCTIONS:**

**Attempt all questions on your own A4 paper**
**Start each question on a new page**
**Calculators may be used**
**Write in blue or black pen only**
**Show all necessary working**
**Marks may be deducted for careless or badly arranged work**

| | |
|---|---|
| **Question 1** | 21 **/23** |
| **Question 2** | 17 **/17** |
| **Total** | **/40** |

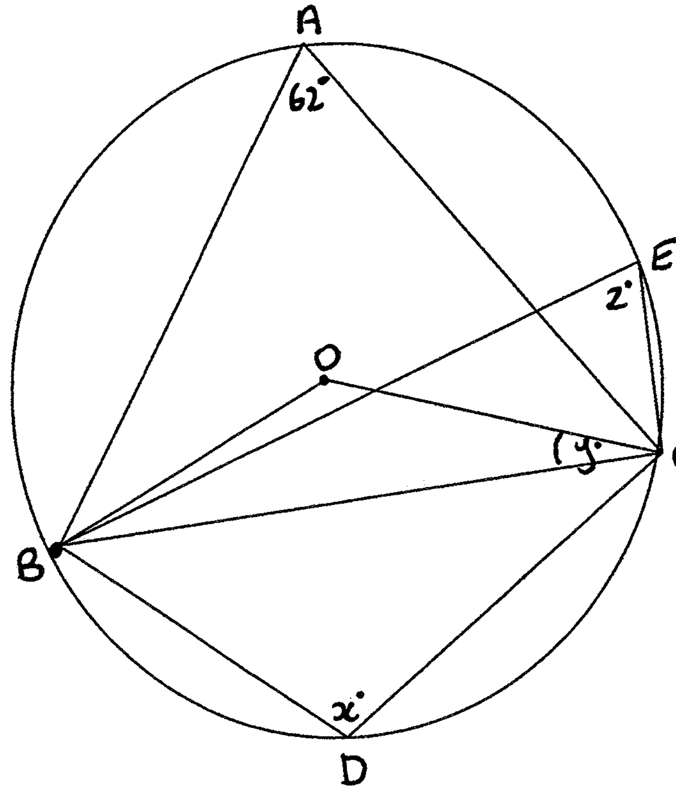
**Outcomes**:

PE 1 – Appreciates the role of mathematics in the solution of practical problems.
PE 2 – Uses multi-step deductive reasoning in a variety of contexts.
PE3 - solves problems involving permutations and combinations, inequalities, polynomials, circle geometry and parametric representations
PE5 - determines derivatives which require the application of more than one rule of differentiation
PE6 - makes comprehensive use of mathematical language, diagrams and notation for communicating in a wide variety of situations
HE1 - appreciates interrelationships between ideas drawn from different areas of mathematics
HE2 - uses inductive reasoning in the construction of proofs

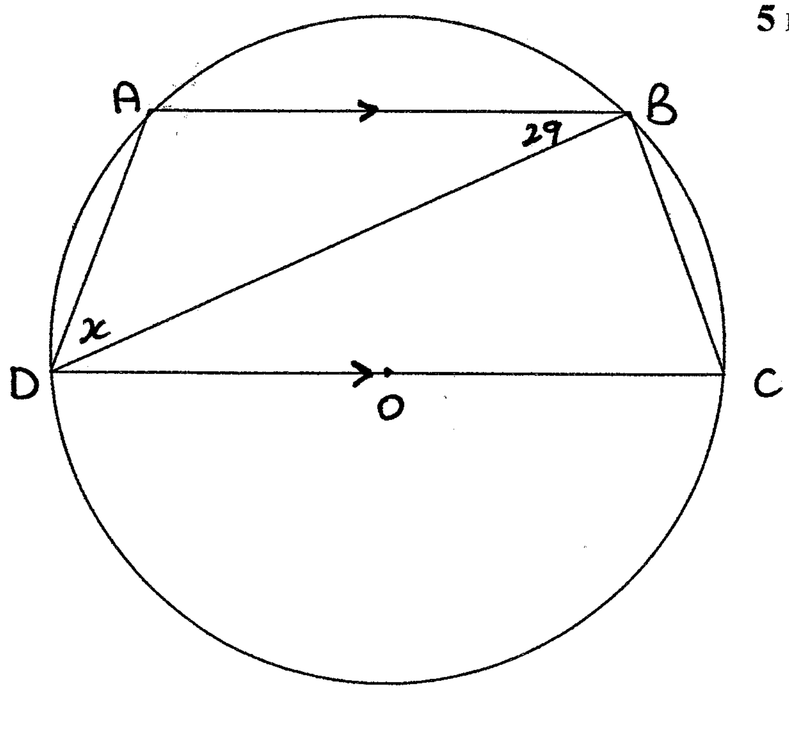
**Question 1 (START ON A NEW PAGE) 23 marks**

a) In each of the following find the value of all pronumerals giving reasons.

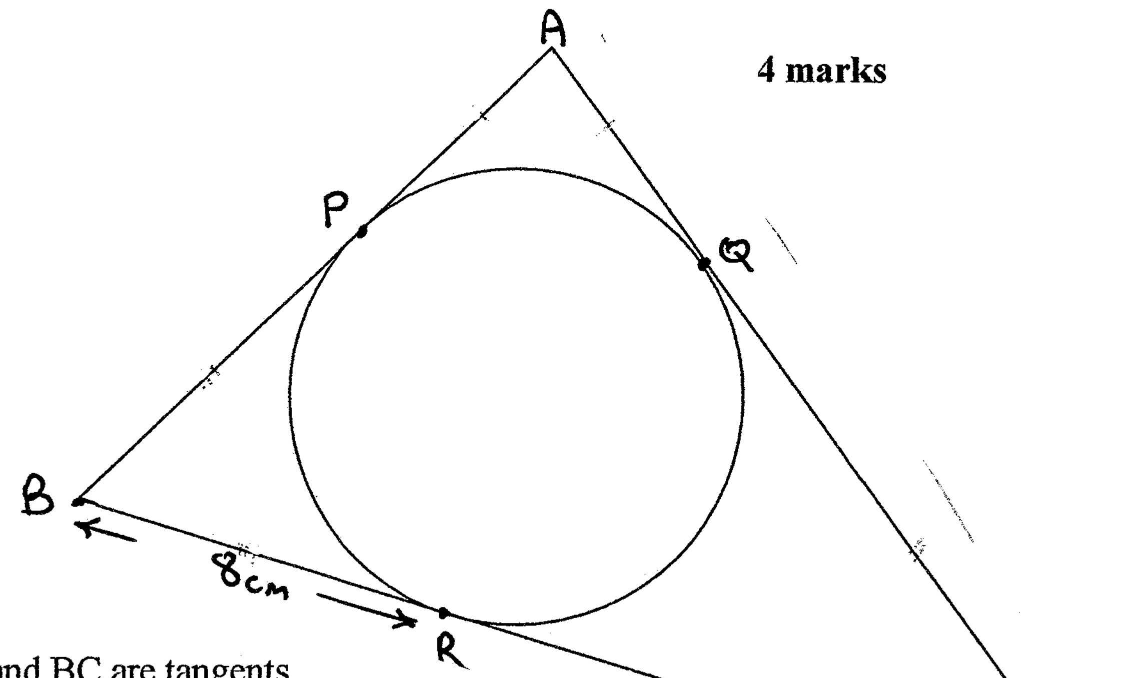
i) **7 marks**




ii) **5 marks**



b) **4 marks**




AB, AC and BC are tangents

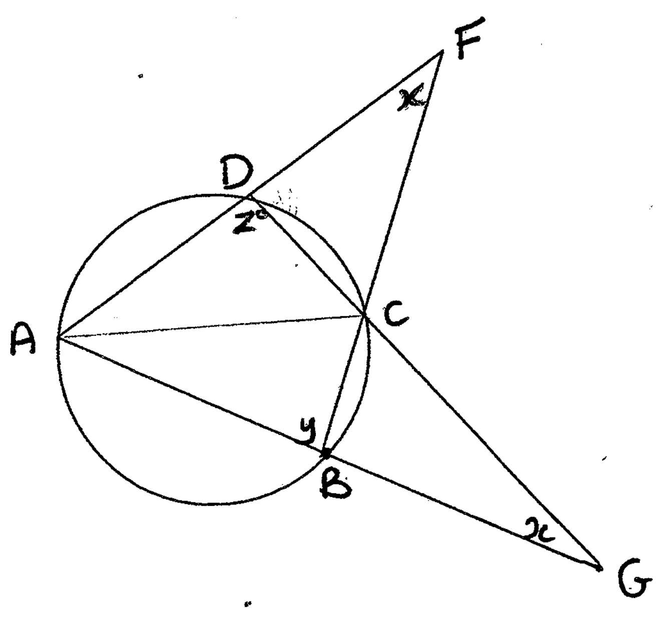
AC=12 cm
BC=15 cm

Find the length of AB – giving all reasons.

C) In the diagram, A, B, C and D are points on a circle.

Given $\angle AFB = \angle AGD$,

i) Show that $\angle ABC = \angle ADC$ (y=z) **4marks**
ii) Prove that AC is a diameter **3marks**



**Question 2 (START ON A NEW PAGE) 17 marks**

**a)** Solve $\cos 2\vartheta = \cos\vartheta$ **for** $0 \le \vartheta \le 2\pi$ **5marks**

**b)** By expressing as the cosine function $\cos(\vartheta - \alpha)$ and its expansion, solve (round $\vartheta$ and $\alpha$ to the nearest minute) **7 marks**

$2\cos\vartheta + 3\sin\vartheta = 1$ **for** $0 \le \vartheta \le 360$

**c)** By using your 't' equations $(\equiv \tan\frac{\vartheta}{2})$, **5 marks**

solve

$5\sin\vartheta + 5\cos\vartheta = 1$ **for** $0 \le \vartheta \le 360$

Valentino Koosasi

Q1 $\frac{21}{23}$

Yr 12 Ext 1.

Q1.

a. i) $x° = 180° - 62°$ (supplementary angles of opp. sides
$= 118°$ ✓ in cyclic quad) ✓

$z° = 62°$ (Angles in the same segment are equal) ie $\angle BAC = \angle BEC$.

$\angle BOC = 2 \times 62°$ (angle on the centre is double the
$= 124°$ ✓ angle on the circumference subtended by same arc.)

$BO = OC$ ✓ (radius of a circle ✓) ∴ $\triangle BOC$ is isosceles $\triangle$

∴ $y = \frac{180° - 124}{2}$ ✓ (base angles of isosceles $\triangle$). ✓

$= 28°$. ✓

5

ii). $\angle DBC = 90°$ ✓ (angle on semi circle is a right ✓ angle)

$\angle BDC = 29°$ ✓ (alternate angles on parallel lines). ✓

$\angle ABC = 90° + 29°$

$= 119°$ ✓

$\angle ADC = 180° - 119°$ (supplementary angles of opp.
$= 61°$. ✓ side in cyclic quad) ✓

but $\angle ADC = x + \angle BDC$ ✓

$61° = x + 29°$

∴ $x = 32°$. ✓

5 Good.

b. $AP = AQ$ ✓ (tangents drawn from the external point of a circle are equal) ✓

∴ $CQ = CR$ ✓ ( ——✓—— )

$BP = BR$ ✓ ( ——✓—— ).

$BC = 15$ cm (given). & $BR = 8$ cm ✓ (given)

∴ $CR = 15 - 8$ ✓

$= 7$ cm ✓

∴ $CQ = 7$ cm ✓ ($CQ = CR$ proven above ✓).

$AC = 12$ cm (given ✓)

$CQ = 7$ cm ✓

∴ $AQ = 12 - 7$

$= 5$ cm ✓

$AQ = AP$ (proven above) ✓

$= 5$ cm. ✓

and $BP = BR$ (proven above) ✓

$= 8$ cm ✓

∴ $AB = 5 + 8$ ✓

$= 13$ cm. ✓

4 Good.

c. $\angle AFB = \angle AGD$ (given)

$= x$.

$z + y = 180°$ ✓ (supplementary angles of opp. sides in cyclic quad). ✓

i). Since $\angle AFB = \angle AGD = x$ (given)

and $\angle FAB = \angle GAD$ (common) ✓

∴ $\triangle BAF \parallel\!| \triangle DAG$. ✓ (all angles are equal) ✓

∴ $\angle ABC = \angle ADC$ ✓

∴ $y° = z°$ ✓

4 Good.

ii). Since $y° = z°$

and $y + z = 180$ ✓ (opposites angles on ✓ cyclic quad are supplementary)

∴ $y = z = 90°$ ✓

Since $y = z = 90°$

∴ AC is the diameter (angle on semi circle is a right angle ✓).

3 Good.

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Q2 17/17

Q2.

a. solve $\cos 2\theta = \cos\theta$ for $0 \le \theta \le 2\pi$

$2\cos^2\theta - 1 = \cos\theta$

$2\cos^2\theta - \cos\theta - 1 = 0$

let $x$ be $\cos\theta$

$\therefore 2x^2 - x - 1 = 0$ $\quad$ $2x \times 1$, $x \times -1$

$(2x+1)(x-1) = 0$

but $x = \cos\theta$

$\therefore (2\cos\theta + 1)(\cos\theta - 1) = 0$

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$\therefore 2\cos\theta + 1 = 0$ or $\cos\theta - 1 = 0$

S | A
T | C

$\frac{\pi}{3}$

$\cos\theta = -\frac{1}{2}$ $\qquad$ $\cos\theta = 1$

$\therefore \theta = \frac{2\pi}{3}, \frac{4\pi}{3}$ $\qquad$ $\theta = 0$

$\frac{5\pi}{3} \cdot \frac{\pi}{3}$

$\therefore \theta = 0, \frac{2\pi}{3}, \frac{4\pi}{3}, 2\pi$

b. solve $2\cos\theta + 3\sin\theta = 1$ for $0 \le \theta \le 360$.

$A = \sqrt{2^2 + 3^2}$

$= \sqrt{4+9}$

$= \sqrt{13}$

$\sqrt{13}\left(\frac{2}{\sqrt{13}}\cos\theta + \frac{3}{\sqrt{13}}\sin\theta\right) = 1$

$\sqrt{13}\left(\cos\theta\,\frac{2}{\sqrt{13}} + \sin\theta\,\frac{3}{\sqrt{13}}\right) = 1$

$\cos(\theta - \alpha) = \cos\theta\cos\alpha + \sin\theta\sin\alpha$

$\therefore \cos\alpha = \frac{2}{\sqrt{13}}$ or $\sin\alpha = \frac{3}{\sqrt{13}}$

$\therefore \alpha = 56°19'$

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$\therefore \sqrt{13}\cos(\theta - 56°19') = 1$.

$\cos(\theta - 56°19') = \frac{1}{\sqrt{13}}$

$\therefore \theta - 56°19' = 73°54', 286°6'$

$\therefore \theta = 130°13', 342°25'$

c. solve $5\sin\theta + 5\cos\theta = 1$ for $0 \le \theta \le 360$

$5\left(\frac{2t}{1+t^2}\right) + 5\left(\frac{1-t^2}{1+t^2}\right) = 1$

$\frac{10t}{1+t^2} + \frac{5-5t^2}{1+t^2} = 1$

$10t + 5 - 5t^2 = 1 + t^2$

$6t^2 - 10t - 4 = 0$.

5

$3t^2 - 5t - 2 = 0$ $\quad$ $3t \times 1$, $t \times -2$

$(3t+1)(t-2) = 0$

but $t = \tan\frac{\theta}{2}$

$\therefore 3\tan\frac{\theta}{2} + 1 = 0$ or $\tan\frac{\theta}{2} - 2 = 0$

$\tan\frac{\theta}{2} = -\frac{1}{3}$ or $\tan\frac{\theta}{2} = 2$

$\therefore \frac{\theta}{2} = 161°34'$ $\qquad$ $\frac{\theta}{2} = 63°26'$

$\therefore \theta = 323°8'$ $\qquad$ $\theta = 126°52'$

$\therefore \theta = 126°52', 323°8'$.