Waverley College
Year 12
2 Unit Examination
Mid Semester
2007

TIME ALLOWED: 5 MINUTES READING
2 HOURS WRITING

INSTRUCTIONS:

Attempt all questions.
Begin each question in a new booklet as directed.
Approved calculators may be used.
Write in blue or black pen only.
Show all necessary working.
Marks may be deducted for careless or poorly arranged work.

Outcomes:
P1- Demonstrates confidence in using mathematics to obtain realistic solutions to problems
P2 - Provides reasoning to support conclusions which are appropriate to the context
P3-Performs routine arithmetic and algebraic manipulation involving surds, simple rational expressions
P4 - Chooses and applies appropriate arithmetic, algebraic, graphical, trigonometric and geometric techniques
P5- Understands the concept of a function and the relationship between a function and its graph
P6-relates the derivative of a function to the slope of its graph
P7-determines the derivative of a function through routine application of the rules of differentiation
P8-understands and uses the language and notation of calculus
H1 - seeks to apply mathematical techniques to problems in a wide range of practical contexts
H2 - constructs arguments to prove and justify results
H4 – expresses practical problems in mathematical terms based on simple given models
H5 - applies appropriate techniques from the study of calculus, geometry, probability, trigonometry and series to solve problems
H6-uses the derivative to determine the features of the graph of a function
H7-uses the features of a graph to deduce information about the derivative
H8- uses techniques of integration to calculate areas and volumes
H9 - communicates using mathematical language, notation, diagrams and graphs

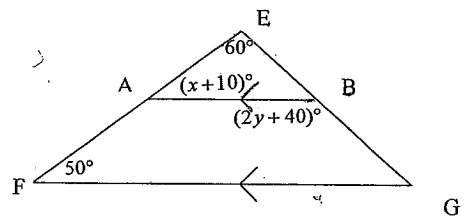
Question 1 Begin a new booklet **(12 marks)**

a) Evaluate: $|-6|-|-7|$ **1**

b) Solve: $5a-7=4(2a+3)$ **2**

c) Factorise: x^4-16 **2**

d) Find the value of all pronumerals, giving clear reasons for your answers. **4**



e) Find the coordinates of the point on the curve $y=3x^2-2x+1$ where the tangent is parallel to the straight line $y=4x-1$. **3**

Question 2 **(12 marks)**

a) If α and β are the roots of the equation $x^2-2x-7=0$, find the values of:

(i) $\alpha+\beta$ **1**

(ii) $\alpha\beta$ **1**

(iii) $\alpha^2+\beta^2$ **2**

b) For what values of k will the equation $x^2+(k-1)x-(2k+1)=0$ have

(i) real roots? **3**

(ii) one root equal to 4? **2**

c) Solve the following equation for all real values of x. **3**

$x^4-3x^2-4=0$

Question 3 Begin a new booklet **(12 marks)**

a) For the parabola $y=x^2-6x+10$

(i) Find the coordinates of the vertex. **2**

(ii) Find the coordinates of the focus. **2**

(iii) Find the equation of the directrix. **1**

b) Let A and B be the fixed points $(-2,0)$ and $(1,0)$ and let the point P be the variable point (x,y).

(i) Find expressions for the distance PA and PB. **2**

(ii) The point P moves so that PA = 2.PB
(i.e. PA = 2 times PB)
Prove that the locus of P is $x^2-4x+y^2=0$. **2**

(iii) Find the centre and radius of this circle. **3**

Question 4 **(8 marks)**

a) A set of 30 cards is labelled from 1 to 30. A card is drawn at random. Find the probability that the card will be

(i) an odd number. **1**

(ii) a multiple of 3. **1**

(iii) a number greater than 10 and a multiple of 5. **1**

b) The probability that a set of traffic lights will show green is $\frac{3}{8}$.

(i) What is the probability that the light will not be green? **1**

(ii) A motorist has to drive through two sets of lights.
Draw a tree diagram showing the possible outcomes and the probabilities on each branch. **1**

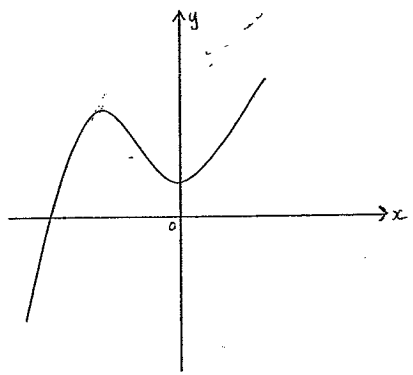
(iii) Find the probability that the lights will both be green. **1**

(iv) Find the probability that the lights will be green at least one time. **2**

Question 5 Begin a new booklet **(12 marks)**

a) For the function $f(x) = 6 - 3x - x^2$, find the values of x for which the function is monotonic increasing. **3**

b) The diagram shows the graph of a function.



(i) Copy this graph.

(ii) On a new set of axes, draw a sketch of the derivative of the function. i.e. y'. **2**

c) The equation for the expense per year (in tens of thousands of dollars) of running a certain business is given by

$$E = x^2 - 8x + 17$$

where x is the number (in hundreds) of items manufactured.

(i) Find the expense of running the business if no items are manufactured. **1**

(ii) Find the number of items needed to minimise the expense of the business. **3**

(iii) Find the minimum expense of the business (consider units). **2**

Question 6 **(12 marks)**

a) Evaluate $\int_1^2 \frac{dx}{x^2}$ **2**

b) Find the area bounded by the x-axis, the curve $y = x^3$ and the lines $x = -1$ and $x = 3$. **4**

c) A surveyor wrote down the following measurements for an irregular block of land. All measurements are in metres.

0	4	8	12	16	20	24
0	7.12	8.92	10.85	9.76	6.80	0

Calculate the area of the block of land using Simpson's Rule, correct to two decimal places. **2**

d) The region which lies between the x-axis and the line $y = x + 1$ from $x = 0$ to $x = 3$ is rotated about the x-axis to form a solid. Find the volume of the solid in terms of π. **4**

Question 7 Begin a new booklet **(12 marks)**

a) The first 3 terms of an arithmetic series are 40 + 34 + 28 +...

(i) Find an expression in simplest form for the nth term. **2**

(ii) Find the 50^{th} term of the series. **1**

(iii) Calculate the sum to 50 terms of the series. **1**

b) The first 4 terms of a geometric series are 3+9+27+81+...

(i) Find the 7^{th} term **2**

(ii) Find the sum of the first 10 terms **2**

(iii) How many terms of the series will give a sum of 21 523 359? **2**

c) Evaluate the limiting sum of the geometric series 32 + 16 + 8 +.... **2**

End of Exam

Question 1

a) $|-6| - |-7| = 6 - 7$

$= -1$ (1)

b) $5a - 7 = 8a + 12$

$3a = -19$ (1)

$a = -\frac{19}{3}$ (1)

c) $x^4 - 16 = (x^2 + 4)(x^2 - 4)$ (1)

$= (x^2 + 4)(x + 2)(x - 2)$ (1)

d) $x + 10 = 50$ (corresponding angles on // lines) (1)

$x = 40$ (1)

$2y + 40 = 60 + 50$ (exterior angle sum Δ) (1)

$2y = 70$

$y = 35$ (1)

e) $y = 3x^2 - 2x + 1$

$\frac{dy}{dx} = 6x - 2$ (1)

$6x - 2 = 4$

$6x = 6$

$x = 1$ (1)

$y = 3(1)^2 - 2(1) + 1$

$y = 3 - 2 + 1$

$y = 2$

$\therefore$ point is $(1, 2)$ (1)

Question 2

a) $x^2 - 2x - 7 = 0$

(i) $\alpha + \beta = 2$ (1)

(ii) $\alpha\beta = -7$ (1)

(iii) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= (2)^2 - 2(-7)$ (1)

$= 18$ (1)

b) (i) $\Delta = (k-1)^2 - 4 \times 1 \times (2k+1)$

$\Delta = k^2 - 2k + 1 + 8k + 4$

$\Delta = k^2 + 6k + 5$ (1)

for real roots $\Delta \geq 0$

$k^2 + 6k + 5 \geq 0$ (1)

$(k+1)(k+5) \geq 0$

$k \leq -5 \quad k \geq -1$ (1)

(ii) Let $x = 4$

$(4)^2 + (k-1) \cdot 4 - (2k+1) = 0$ (1)

$16 + 4k - 4 - 2k - 1 = 0$

$2k = -11$

$k = -\frac{11}{2}$ (1)

c) $x^4 - 3x^2 - 4 = 0$

$(x^2 + 1)(x^2 - 4) = 0$ (1)

$x^2 = -1 \qquad x^2 = 4$

no real $\qquad x = \pm 2$ (1)

soln (1)

Question 3

a) (i) $y = x^2 - 6x + 10$

$x^2 - 6x + 9 = y - 10 + 9$

$(x-3)^2 = y - 1$

$(x-3)^2 = 4 \times \frac{1}{4} \times (y-1)$ (1)

$\therefore$ Vertex is $(3,1)$ (1)

(ii) $a = \frac{1}{4}$ (1)

$\therefore$ Focus is $(3, 1\frac{1}{4})$ (1)

(iii) directrix is $y = \frac{3}{4}$ (1)

b) (i) $PA = \sqrt{(x+2)^2 + y^2}$ (1)

$PB = \sqrt{(x-1)^2 + y^2}$ (1)

(ii) $PA = 2 \cdot PB$

$\sqrt{(x+2)^2 + y^2} = 2\sqrt{(x-1)^2 + y^2}$

$(x+2)^2 + y^2 = 4\left[(x-1)^2 + y^2\right]$ (1)

$x^2 + 4x + 4 + y^2 = 4\left[x^2 - 2x + 1 + y^2\right]$

$x^2 + 4x + 4 + y^2 = 4x^2 - 8x + 4 + 4y^2$

$3x^2 - 12x + 3y^2 = 0$

$x^2 - 4x + y^2 = 0$ (1)

(iii) $x^2 - 4x + 4 + y^2 = 0 + 4$

$(x-2)^2 + y^2 = 4$ (1)

$\therefore$ Centre at $(2,0)$ (1)

and radius = 2 units (1)

Question 4

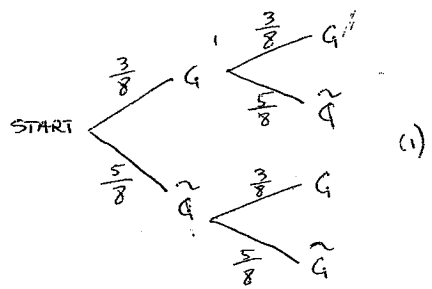
a) (i) $P(\text{odd}) = \frac{1}{2}$ (1)

(ii) $P(\text{multiple of } 3) = \frac{1}{3}$ (1)

(iii) $P(E) = \frac{2}{15}$ (1)

b) (i) $P(\text{not green}) = \frac{5}{8}$ (1)

(ii)



(1)

(iii) $P(GG) = \frac{3}{8} \times \frac{3}{8}$

$= \frac{9}{64}$ (1)

(iv) $P(E) = 1 - P(\tilde{G}\tilde{G})$ (or other suitable method)

$= 1 - \frac{5}{8} \times \frac{5}{8}$ (1)

$= \frac{39}{64}$ (1)

Question 5

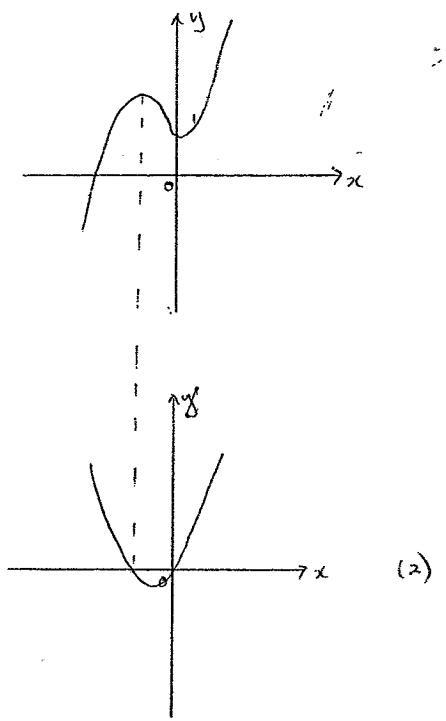
a) $f'(x) = -3 - 2x$ (1)

$-3 - 2x > 0$ (1)

$-2x > 3$

$x < -\frac{3}{2}$ (1)

b) (i)



(2)

c) (i) $E = x^2 - 8x + 17$

$E = (0)^2 - 8(0) + 17$

$E = 17$

∴ Expense is \$170 000 (1)

(ii) $\frac{dE}{dx} = 2x - 8$ (1)

$2x - 8 = 0$

$x = 4$ (1)

$\frac{d^2E}{dx^2} = 2$

which is > 0

∴ min st. pt. at $x = 4$

∴ min number of items is 400 (1)

(iii) $E = (4)^2 - 8(4) + 17$ (1)

$E = 1$

∴ Min expense is \$10 000 (1)

Question 6

a) $\int_1^2 (x^{-2})\,dx = \left[-x^{-1}\right]_1^2$ (1)

$= -\left[\frac{1}{x}\right]_1^2$

$= -\left[\frac{1}{2} - 1\right]$

$= \frac{1}{2}$ (1)

b) $A = \left|\int_{-1}^{0} x^3\,dx\right| + \int_0^3 x^3\,dx$ (1)

$A = \left|\left[\frac{1}{4}x^4\right]_{-1}^{0}\right| + \left[\frac{1}{4}x^4\right]_0^3$ (1)

$A = \left|0 - \frac{1}{4}(-1)^4\right| + \frac{1}{4}(3^4 - 0^4)$ (1)

$A = \frac{1}{4} + \frac{81}{4}$

$A = \frac{82}{4}$

$A = \frac{41}{2}$ units^2 or $20\frac{1}{2}$ units^2 (1)

c) $A \doteq \frac{4}{3}\left[0 + 0 + 4(7.12 + 10.85 + 6.8) + 2(8.92 + 9.76)\right]$ (1)

$A \doteq 181.92$ m^2 (1)

d) $V = \pi \int_0^3 (x+1)^2 \, dx$ (1)

$V = \pi \int_0^3 (x^2 + 2x + 1) \, dx$

$V = \pi \left[\frac{1}{3}x^3 + x^2 + x \right]_0^3$ (1)

$V = \pi \left\{ \left[\frac{1}{3}(3)^3 + (3)^2 + 3 \right] - 0 \right\}$ (1)

$V = \pi \{ 9 + 9 + 3 \}$

$V = 21\pi$ units^3 (1)

Question 7

a) (i) $T_n = 40 + (n-1)x - 6$ (1)

$T_n = 46 - 6n$ (1)

(ii) $T_{50} = 46 - 6 \times 50$

$= -254$ (1)

(iii) $S_{50} = \frac{50}{2}[40 - 254]$

$= -5350$ (1)

b) (i) $T_7 = 3 \times 3^6$ (1)

$= 2187$ (1)

(ii) $S_{10} = \frac{3(3^{10}-1)}{3-1}$ (1)

$= 88\,572$ (1)

(iii) $\frac{3(3^n-1)}{3-1} = 21\,523\,359$ (1)

$3^n = 14\,348\,907$

$n = 15$ (1)

c) $S_\infty = \frac{32}{1 - \frac{1}{2}}$ (1)

$S_\infty = 64$ (1)

12, 14 @ 1.5 $\quad 7\frac{1}{8}$ / 15 $\quad 14\frac{1}{16}$ / 25 $\quad 33,7$ / 55