Waverley College
Year 12 2 Unit Mathematics Examination
Term 1 2005

TIME ALLOWED: 50MINUTES

NAME:

TEACHER:

INSTRUCTIONS:

Attempt all questions
Start each question on a new page
Calculators may be used
Write in blue or black pen only
Show all necessary working
Marks may be deducted for careless or badly arranged work

QUESTION 1	/14
QUESTION 2	/14
QUESTION 3	/14
QUESTION 4	/14
TOTAL	/56
%	

QUESTION 1 (14 Marks)

a) Differentiate the following with respect to x.

i) $y = \dfrac{1}{(2x-3)^2}$ (2)

ii) $f(x) = x\sqrt{x}$ (2)

b) Consider the curve given by the equation $y = x^3 - x^2 - x + 4$

i) Find all stationary points and determine their nature. (4)

ii) Find the co-ordinates of any points of inflexion. (3)

iii) Sketch the curve in the domain $-2 \le x \le 3$ indicating important features. (2)

iv) What is the maximum value of $y = x^3 - x^2 - x + 4$ in the domain $-2 \le x \le 3$? (1)

QUESTION 2 (14 Marks) START A NEW PAGE

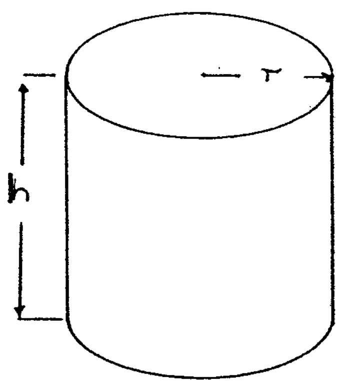
a) A tangent is drawn to the curve $y = x^2 - 2x + 3$ at the point P(2,3).
Find the equation of this tangent. (3)

b) Given the function $y = x^3$;

i) find $\frac{dy}{dx}$ *and* $\frac{d^2y}{dx^2}$ (2)

ii) and hence show that $12\left(\frac{dy}{dx}\right) - \left(\frac{d^2y}{dx^2}\right)^2 + y = x^3$ (2)

a) A closed cylindrical can is to be made out of $600\pi \ cm^2$ of sheet metal. The height of the cylinder is to be $h \ cm$ and the base $r \ cm$.



i) Show that $h = \frac{300}{r} - r$. (3)

ii) Hence find the greatest possible volume for this cylinder. (4)

QUESTION 3 (14 Marks) START A NEW PAGE

a) Find:

$$\int \frac{dx}{(4+4x)^2} \qquad (3)$$

b) Evaluate the following expressions:

i. $\int_0^1 \left(x^2+3x-5\right)dx$ (3)

ii. $\int_1^2 \left(\frac{x^3-2x^2+1}{x^2}\right)dx$ (4)

c) Sketch the curve $y = x^2 - 5x + 6$ and hence find the area bounded by the curve $y = x^2 - 5x + 6$ and the x-axis. (4)

QUESTION 4 (14 Marks) START A NEW PAGE

a) For the following functions:

i. Show that $y = 3x^2$ and $y = 4x - x^2$ intersect at $x = 0$ and $x = 1$. (3)

ii. On the same axes sketch $y = 3x^2$ and $y = 4x - x^2$ indicating any key features. (3)

iii. Find the area of the region bounded by $y = 3x^2$ and $y = 4x - x^2$. (4)

b) What is the volume of the solid formed when the line $y = x - 2$ is rotated about the x-axis from $x = 2$ to $x = 3$. (4)

END OF EXAMINATION

Yr 12 2 unit Mathematics Examination

12/14

Q1.

a. i). $y = \frac{1}{(2x-3)^2}$

$y = (2x-3)^{-2}$

$dy/dx = -2(2x-3)^{-3} \times 2$

$= -4(2x-3)^{-3}$

$= \frac{-4}{(2x-3)^3}$ ✓✓

ii). $f(x) = x\sqrt{x}$

$f(x) = x \cdot x^{1/2}$

$f'(x) = x \cdot \frac{1}{2}x^{-1/2} + x^{1/2}$

$= x \times \frac{1}{2\sqrt{x}} + \sqrt{x}$

$= \frac{x}{2\sqrt{x}} + \sqrt{x}$. ✓✓

b. $y = x^3 - x^2 - x + 4$

i). $dy/dx = 3x^2 - 2x - 1$

$dy/dx = 0$ when $3x^2 - 2x - 1 = 0$

$(3x+1)(x-1) = 0$

$\therefore x = -\frac{1}{3}$ or $x = 1$.

$\begin{matrix} 3x & 1 \\ x & -1 \end{matrix}$

$d^2y/dx^2 = 6x - 2$

at $x = -\frac{1}{3}$ $d^2y/dx^2 = -4$ which is < 0

$\therefore$ max st. pt.

at $x = -\frac{1}{3}$, $y = (-\frac{1}{3})^3 - (-\frac{1}{3})^2 - (-\frac{1}{3})$.

$= 4\frac{5}{27}$

at $x = 1$ $d^2y/dx^2 = 4$ which is > 0

$\therefore$ min st. pt.

at $x = 1$, $y = (1)^3 - (1)^2 - 1 + 4 = 3$

$\therefore (1, 3)$ is min st. pt. ✓✓

and

$(-\frac{1}{3}, 4\frac{5}{27})$ is max st. pt ✓✓

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ii). $d^2y/dx^2 = 0$ when $6x - 2 = 0$

$6x = 2$

$\therefore x = \frac{1}{3}$.

check: at $x < \frac{1}{3}$ $d^2y/dx^2 < 0$

at $x > \frac{1}{3}$ $d^2y/dx^2 > 0$

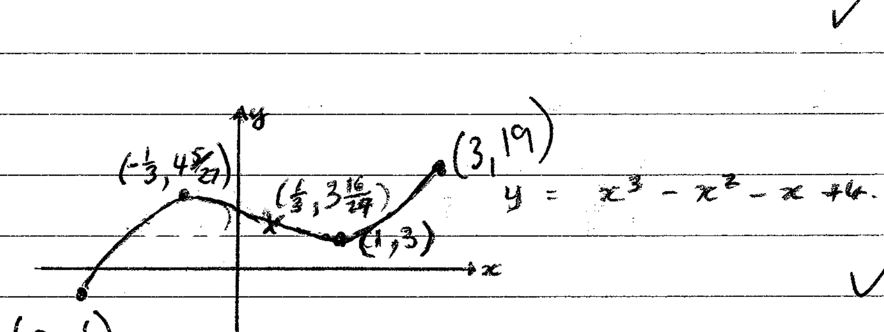
$\therefore x = \frac{1}{3}$ is point of inflation

at $x = \frac{1}{3}$, $y = \left(\frac{1}{3}\right)^3 - \left(\frac{1}{3}\right)^2 - \frac{1}{3} + 4$

$= 3\frac{16}{27}$

$\therefore \left(\frac{1}{3}, 3\frac{16}{27}\right)$ is pt of inflexion

iii).



iv). at $x = -\frac{1}{3}$ $d^2y/dx^2 < 0$

$\therefore$ it's a local maximum at $x = -\frac{1}{3}$

here $y = 4\frac{5}{27}$

at $x = 3$ $y = 19$ and at $x = -2$ $y = -6$

$\therefore y = 19$ is the max value

in the domain $-2 \leq x \leq 3$

Q2.

14/14

a. $y = x^2 - 2x + 3$ at $P(2,3)$

$dy/dx = 2x - 2$

at $x = 2$

$dy/dx = 2(2) - 2$

$= 2$

$\therefore$ m of tangent $= 2$

$\therefore$ eqn of tangent

$y - y_1 = m(x - x_1)$

$y - 3 = 2(x - 2)$

$y - 3 = 2x - 4$

$y = 2x - 1$.

$\therefore$ eqn of tangent $y = 2x - 1$ ✓✓✓

b. $y = x^3$

i). $dy/dx = 3x^2$ ✓

$d^2y/dx^2 = 6x$. ✓

ii). show $12\,(dy/dx) - (d^2y/dx^2)^2 + y = x^3$

but $y = x^3$; $dy/dx = 3x^2$ and $d^2y/dx^2 = 6x$

$\therefore$ LHS $= 12(3x^2) - (6x)^2 + x^3$

$= \cancel{36x^2} - \cancel{36}x^2 + x^3$

$= x^3$

$=$ RHS ✓✓.

$\therefore 12\,(dy/dx) - (d^2y/dx^2)^2 + y = x^3$

c. i). $A = 2\pi r^2 + 2\pi r h = 600\pi$

$2\pi r h = 600\pi - 2\pi r^2$

$\therefore h = \dfrac{600\pi - 2\pi r^2}{2\pi r}$

$= \dfrac{\cancel{2\pi}(300 - r^2)}{\cancel{2\pi} r}$ ✓✓✓

$\therefore h = \dfrac{300 - r^2}{r} = \dfrac{300}{r} - r$.

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ii) $V = \pi r^2 h$

but $h = \frac{300}{r} - r$

$V = \pi r^2 \left(\frac{300}{r} - r\right)$

~~$dV/dr = 2\pi r\left(\frac{300}{r} - r\right)$~~

~~$dV/dr = 0$ when $4\pi r\left(\frac{300}{r} - r\right) = 0$.~~

$V = \pi r^2 \left(\frac{300}{r} - r\right)$

$= \frac{300\pi r^2}{r} - \pi r^3$

$= 300\pi r - \pi r^3$

$dV/dr = 300\pi - 3\pi r^2$

$dV/dr = 0$ when $300\pi - 3\pi r^2 = 0$

$3\pi r^2 = 300\pi$

$r^2 = \frac{300\pi}{3\pi}$

$r^2 = 100$

$r = \pm 10$

but $r \neq 0$

$\therefore r = 10$.

$d^2V/dr^2 = -6\pi r$

at $r = 10$

$d^2V/dr^2 < 0$

$\therefore$ at $r = 10$, the V is max

$\therefore$ max V at $r = 10$

$= \pi r^2 h$

$= \pi \times 10^2 \times \frac{300}{10} - 10$

$= 100\pi \times 20$

$= 2000\pi\ u^3$

$\therefore$ max V is $2000\pi\ u^3$.

Q3.

a. $\int \frac{dx}{(4+4x)^2} = \int \frac{1}{(4+4x)^2}\,dx$

$= \int (4+4x)^{-2}\,dx$

$= -\frac{1}{4}(4+4x)^{-1} + c.$ 3

$\frac{13\frac{1}{2}}{14}$

b. i). $\int_0^1 (x^2+3x-5)\,dx$

$= \left[\frac{1}{3}x^3 + \frac{3}{2}x^2 - 5x\right]_0^1$

$= \left(\frac{1}{3}(1)^3 + \frac{3}{2}(1)^2 - 5(1)\right) - \left(\frac{1}{3}(0)^3 + \frac{3}{2}(0)^2 - 5(0)\right)$

$= \left(\frac{1}{3} + \frac{3}{2} - 5\right) - 0$

$= -3\frac{1}{6}.$ 3

ii). $\int_1^2 \left(\frac{x^3 - 2x^2 + 1}{x^2}\right) dx$

$= \int_1^2 \left(\frac{x^3}{x^2} - \frac{2x^2}{x^2} + \frac{1}{x^2}\right) dx$

$= \int_1^2 \left(x - 2 + x^{-2}\right) dx$

$= \left[\frac{1}{2}x^2 - 2x - x^{-1}\right]_1^2$

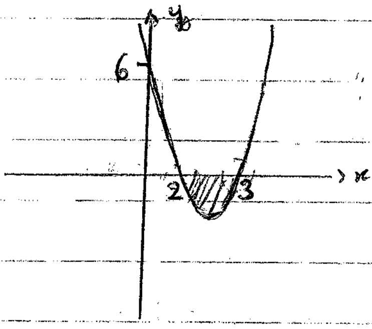
$= \left(\frac{1}{2}(2)^2 - 2(2) - 2^{-1}\right) - \left(\frac{1}{2}(1)^2 - 2(1) - 1^{-1}\right)$

$= \left(2 - 4 - \frac{1}{2}\right) - \left(\frac{1}{2} - 2 - 1\right)$

$= -2\frac{1}{2} + 2\frac{1}{2}$

$= 0$

c. $y = x^2 - 5x + 6$
$(x-3)(x-2)$



$A = \left| \int_2^3 x^2 - 5x + 6 \; dx \right|$

$= \left| \left[\frac{1}{3}x^3 - \frac{5}{2}x^2 + 6x\right]_2^3 \right|$

$= \left| \left(\frac{1}{3}(3)^3 - \frac{5}{2}(3)^2 + 6(3)\right) - \left(\frac{1}{3}(2)^3 - \frac{5}{2}(2)^2 + 6(2)\right) \right|$

$= \left| \left(9 - \frac{45}{2} + 18\right) - \left(\frac{8}{3} - 10 + 12\right) \right|$

$= \left| 4\frac{1}{2} - 4\frac{2}{3} \right|$

$= \left| -\frac{1}{6} \right|$

$= \frac{1}{6}\,u^2.$

Q4.

a. i). $y = 3x^2$

$y = 4x - x^2$

$3x^2 = 4x - x^2$

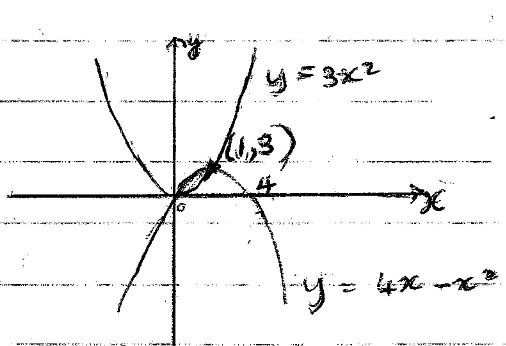
$3x^2 - 4x + x^2 = 0$

$4x^2 - 4x = 0$

$4x(x-1) = 0$

$\therefore x = 0, 1 \quad \therefore$ intersect at $x = 0$ and $x = 1$.

ii).



iii). $\int_0^1 (4x - x^2) - 3x^2 \, dx$

$= \left[2x^2 - \frac{1}{3}x^3 - x^3 \right]_0^1$

$= \left[2(1)^2 - \frac{1}{3}(1)^3 - (1)^3 \right] - 0$

$= 2 - \frac{1}{3} - 1$

$= \frac{2}{3} \, u^2$.

b. $V = \pi \int_2^3 (x-2)^2 \, dx$

$= \pi \left[\frac{1}{3}(x-2)^3 \right]_2^3$

$= \pi \left[\frac{1}{3}(3-2)^3 \right] - \left[\frac{1}{3}(0)^3 \right]$

$= \pi \left[\frac{1}{3} \right]$

$V = \frac{1}{3}\pi \, u^3$.