



Waverley College
Year 12 2 unit Mathematics Examination
Term 4 2006

TIME ALLOWED: 45 MINUTES

INSTRUCTIONS:

- Attempt all questions**
- Approved calculators may be used**
- Write in blue or black pen only**
- Show all necessary working**
- Marks may be deducted for careless or badly arranged work**

Section 1	/35	Section 2	/ 10
Total	/45	Percentage	%

Outcomes: A student

- P4-chooses and applies appropriate arithmetic, algebraic, graphical, trigonometric and geometric techniques.
- P6-relates the derivative of a function to the slope of its graph.
- P7-determines the derivative of a function through routine application of the rules of differentiation.
- P8-understands and uses the language and notation of calculus.
- H2-constructs arguments to prove and justify results.
- H9-communicates using mathematical language, notation, diagrams and graphs.

SECTION 1 The Quadratic Polynomial

Question 1

- a) Write down the discriminant of $3x^2 - kx + 3 = 0$. 1
- b) Find the value(s) of k for which the equation $3x^2 - kx + 3 = 0$ has
- (i) exactly one real root. 2
 - (ii) no real roots. 2

Question 2

Prove the function $y = x^2 - x + 6$ is positive definite. ✓ 3

Question 3

Find the axis of symmetry of $y = x^2 - 6x + 10$ and hence find the minimum value of $y = x^2 - 6x + 10$. 2 ✓

Question 4

If α and β are the roots of $x^2 - 2x - 7 = 0$, find the values of:

- a) $\alpha + \beta$ 1 ✓
- b) $\alpha\beta$ 1 ✓
- c) $\frac{1}{\alpha} + \frac{1}{\beta}$ 2 ✓
- d) $\alpha^2 + \beta^2$ 2 ✓
- e) $\alpha^2\beta^3 + \alpha^3\beta^2$ 2 ✓

Question 5

For what values of k will the equation $x^2 + 4x - 2k = 0$ have roots which are the reciprocal of each other? 2 ✓

Question 6

Solve the following equations for all real values of x :

- a) $3^{2x} - 4(3^x) + 3 = 0$ 5 ✓
- b) $4x^4 = 4x^2 + 8$ 5 ✓

Question 7

Find the values of A , B and C for which $2x^2 - 7x - 4 \equiv A(x+2)^2 + B(x+2) + C$ 5 ✓

SECTION 2 Locus

Question 8

Find the equation of the circle with radius 6 units and centre $(2,-5)$.

(Do not expand)

2

Question 9

Let A and B be the fixed points $(-2,0)$ and $(1,0)$ and let the variable point be $P(x,y)$.

- a) The point P moves so that PA is twice the distance of PB. Prove that the locus of P is $x^2 - 4x + y^2 = 0$.
- b) This locus is a circle. Find the centre and radius of this circle.

5
3

Section 1

(a) $\Delta = (-k)^2 - 4 \times 3 \times 3$

$\Delta = k^2 - 36$ (1)

b) (i) For exactly one real root, $\Delta = 0$

$k^2 - 36 = 0$ (1)

$k = \pm 6$ (1)

(ii) For no real roots, $\Delta < 0$

$k^2 - 36 < 0$ (1)

$-6 < k < 6$ (1)

2) $\Delta = (-1)^2 - 4 \times 1 \times 6$

$\Delta = -23$ (1)

$\Delta < 0$

$a = 1$ (1)

$a > 0$

For $\Delta < 0$ and $a > 0$ $y = x^2 - x + 6$ is positive definite (1)

3) $x = \frac{6}{2 \times 1}$

$x = 3$ (1)

$y = (3)^2 - 6(3) + 10$

$y = 1$

\therefore min value = 1 (1) $\frac{1}{2}$ off if vertex given rather than value.

$$4) a) \alpha + \beta = 2 \quad (1)$$

$$b) \alpha\beta = -7 \quad (1)$$

$$c) \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} \quad (1)$$

$$= -\frac{2}{7} \quad (1)$$

$$d) \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta \quad (1)$$

$$= 2^2 - 2(-7)$$

$$= 18 \quad (1)$$

$$e) \alpha^2\beta^3 + \alpha^3\beta^2 = \alpha^2\beta^2(\beta + \alpha)$$

$$= (\alpha\beta)^2(\alpha + \beta) \quad (1)$$

$$= (-7)^2 \times 2$$

$$= 98 \quad (1)$$

5) Let the roots be α and $\frac{1}{\alpha}$

$$\alpha \times \frac{1}{\alpha} = -2k \quad (1)$$

$$1 = -2k$$

$$k = -\frac{1}{2} \quad (1)$$

$$6) a) 3^{2x} - 4(3^x) + 3 = 0$$

$$(3^x)^2 - 4(3^x) + 3 = 0$$

$$\text{Let } m = 3^x$$

$$m^2 - 4m + 3 = 0 \quad (1)$$

$$(m-1)(m-3) = 0 \quad (1)$$

$$m=1 \quad \text{OR} \quad m=3 \quad (1)$$

$$\therefore 3^x = 1 \quad \therefore 3^x = 3$$

$$x=0 \quad x=1 \quad (2)$$

$$6b) \quad 4x^4 - 4x^2 - 8 = 0 \quad (1)$$

$$\text{Let } m = x^2$$

$$4m^2 - 4m - 8 = 0 \quad (1)$$

$$4(m^2 - m - 2) = 0$$

$$4(m+1)(m-2) = 0 \quad (1)$$

$$\therefore m = -1 \quad \text{or} \quad m = 2$$

$$x^2 = -1 \quad x^2 = 2$$

no real soln.

$$x = \pm\sqrt{2} \quad (2)$$

$$7) \quad 2x^2 - 7x - 4 \equiv A(x^2 + 4x + 4) + B(x+2) + C$$

$$2x^2 - 7x - 4 \equiv Ax^2 + 4Ax + 4A + Bx + 2B + C$$

$$2x^2 - 7x - 4 \equiv Ax^2 + (4A+B)x + 4A + 2B + C \quad (2)$$

$$\therefore A = 2 \quad (1)$$

$$\text{and } 4A + B = -7$$

$$4(2) + B = -7$$

$$B = -15 \quad (1)$$

$$\text{and } 4A + 2B + C = -4$$

$$4(2) + 2(-15) + C = -4$$

$$8 - 30 + C = -4$$

$$C = 18 \quad (1)$$

Section 2

$$8) (x-2)^2 + (y+5)^2 = 36 \quad (2)$$

$$9) a) PA = 2PB \quad (1)$$

$$PA^2 = 4PB^2$$

$$(x+2)^2 + y^2 = 4[(x-1)^2 + y^2] \quad (2)$$

$$x^2 + 4x + 4 + y^2 = 4x^2 - 8x + 4 + 4y^2$$

$$0 = 3x^2 - 12x + 3y^2 \quad (1)$$

$$x^2 - 4x + y^2 = 0 \quad (1)$$

$$b) x^2 - 4x + 4 + y^2 = 0 + 4$$

$$(x-2)^2 + y^2 = 4 \quad (1)$$

$$\therefore \text{Centre } (2,0) \text{ and radius} = 2 \text{ units} \quad (2)$$