

Waverley College

YEAR 11 MATHEMATICS EXTENSION 1 EXAMINATION

Term 1 2011
Weighting: 20%

TIME ALLOWED: 50 mins

STUDENT NUMBER:

INSTRUCTIONS:

Attempt all questions

Calculators may be used

Write in blue or black pen only

Show all working

Marks may be deducted for careless or badly arranged work

Start each question on a new page

Section A	/ 25
Section B	/ 19
Total	/44

SECTION A

25 marks

Question 1: Simplify:

$$(27y^6)^{1/3} y^{-2}$$

(2)

Question 2: Re-write the following expression in the form of $a + b\sqrt{3}$

$$\frac{\sqrt{3}}{2\sqrt{3}+3}$$

(2)

Question 3: Express the following expression as a product of factors

$$\frac{9}{x^2} - \frac{a^2}{b^2}$$

(1)

Question 4: Solve the following equations:

a) $\frac{2}{x} - \frac{1}{x+1} = 1$

(2)

b) $3^{2x-1} = \frac{1}{27}$

(2)

c) $\frac{|x-3|}{x} = 4$ (check your solutions)

(3)

Question 5: Solve the following equation by completing the square:

$$x^2 - 8x - 1 = 0 \text{ (leave your answer in surd form)}$$

(2)

Question 6: Solve the following inequations:

a) $\frac{5y}{3y-2} \leq -1$

(3)

b) $|4x-5| > x+2$

(3)

Question 7: Solve the following simultaneous equation:

$$y - x^2 + 1 = 0$$

$$y - 3x + 3 = 0$$

(3)

Question 8: The solution to the quadratic equation: $ax^2 + bx + c = 0$ is $x = \frac{7 \pm \sqrt{73}}{6}$

Find possible values for $a, b,$ and c

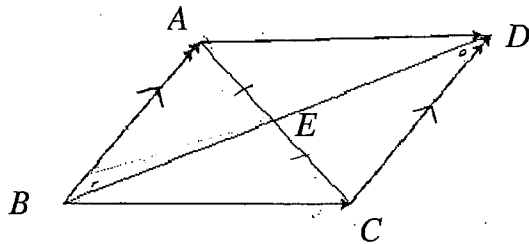
(2)

SECTION B

19 marks

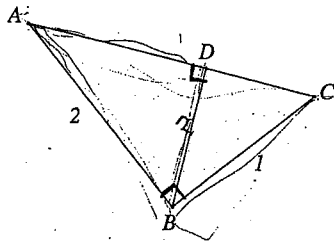
Question 9: By reference to the diagram below, prove:

- a) $\triangle ABE \cong \triangle DCE$ (2)
- b) Hence show that the diagonals of the parallelogram bisect each other (1)

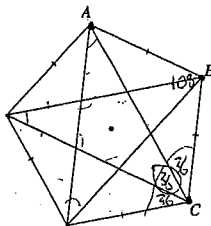


Question 10: By reference to the diagram below

- a) Prove that $\triangle ABC \cong \triangle ADB$ (2)
- b) Hence find the length of BD. Leave your answer in surd form (rationalize any denominator) (2)



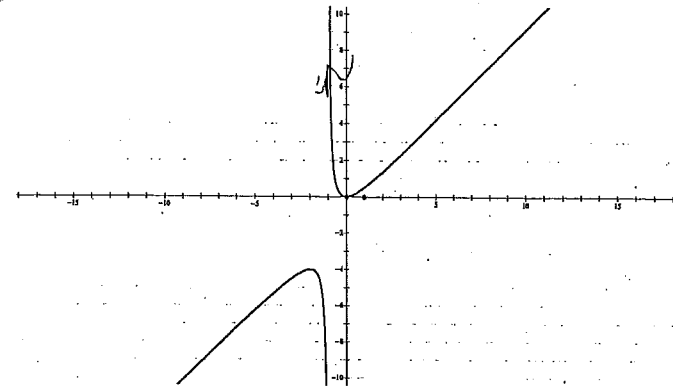
Question 11: By reference to the diagram below, answer the following questions:



- a) Calculate the size of an interior angle of this regular pentagon (1)
- b) Calculate the size of $\angle ACB$ (1)
- c) Hence calculate the size of an acute interior angle of the decagon (star) (1)

Question 12: Consider the equation below and answer the following:

$$h(x) = \frac{x^2}{x+1}$$



- a) What is the domain and range for this function? (2)
- b) Find: (5)

$h(1) =$
 $h(0) =$
 $h(-3) =$
 $h(a) =$
 $h(x^2) =$

- c) Algebraically show if $h(x)$ is even, odd or neither (2)

END OF EXAM
(Check your Working)

1.

<p>1a.</p> $(27y^6)^{1/3} y^{-2}$ $= 3y^2 y^{-2}$ $= 3$	<p>2.</p> $\frac{\sqrt{3}}{2\sqrt{3}+3}$ $= \frac{\sqrt{3}(2\sqrt{3}-3)}{(2\sqrt{3}+3)(2\sqrt{3}-3)}$ $= \frac{6-3\sqrt{3}}{3}$ $= 2-\sqrt{3}$ $a=2, b=1$ <p>✓ ✓</p> <p>1 correct expansion 1 factorisation/ cancelling</p>
<p>3.</p> $\frac{9}{x^2} - \frac{a^2}{b^2}$ $= \frac{9b^2 - a^2 x^2}{b^2 x^2}$ $= \frac{(3b+ax)(3b-ax)}{b^2 x^2}$ <p>or</p> $\frac{9}{x^2} - \frac{a^2}{b^2}$ $= \left(\frac{3}{x} - \frac{a}{b}\right)\left(\frac{3}{x} + \frac{a}{b}\right)$	
<p>4a.</p> $\frac{2}{x} - \frac{1}{x+1} = 1$ $\frac{2(x+1) - x}{x(x+1)} = 1$ $x+2 = x(x+1)$ $x^2 = 2$ $x = \pm\sqrt{2}$	<p>4b</p> $3^{2x-1} = \frac{1}{27}$ $3^{2x-1} = 3^{-3}$ $2x-1 = -3$ $x = -1$
<p>4c.</p> $\frac{ x-3 }{x} = 4$ $x \neq 0$ $(x-3) = 4x \text{ or } -(x-3) = 4x$ $x = -1 \text{ or } x = 3/5$ <p>Check: reject $x = -1$</p> $x = 3/5$ <p>1 2 cases 1 solve correctly 1 reject -1 (checking)</p>	<p>5.</p> $x^2 - 8x - 1 = 0$ $x^2 - 8x + (-4)^2 = 1 + 16$ $(x-4)^2 = 17$ $x = 4 \pm \sqrt{17}$

6a.

$$\frac{5y}{3y-2} \leq -1$$

Find the Critical Point(s)

$$y \neq 2/3$$

$$\frac{5y}{3y-2} = -1$$

$$5y = -3y + 2$$

$$y = 1/4$$

Test points at 0, 5, 1

$$1/4 \leq x < 2/3$$

- $\frac{1}{4} \quad 2/3$
- ① find pts $y = \frac{1}{4}$
 - ① $y \neq \frac{2}{3}$
 - ① correct region.
 - ① x by $(3y-2)^2$
 - ① factorise
 - ① solve correct region.

7.

$$y - x^2 + 1 = 0$$

$$y - 3x + 3 = 0$$

$$y = (x^2 - 1)$$

$$(x^2 - 1) - 3x + 3 = 0$$

$$x^2 - 3x + 2 = 0$$

$$(x-1)(x-2) = 0$$

$$x = 1, x = 2$$

$$y = 0, y = 3$$

6b.

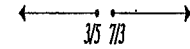
$$|4x-5| > x+2$$

$$4x-5 = x+2 \text{ or } -(4x-5) = x+2$$

$$x \neq 7/3 \text{ or } x \neq 3/5$$

Test: $x=0, x=1, x=3$

$$x < 3/5 \text{ or } x > 7/3$$



- 1 2 cases
- 1 solving eqns
- 1 correct soln. (testing)

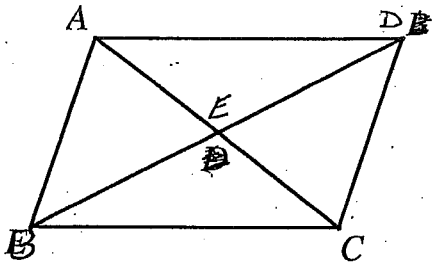
8.

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(3)(-2)}}{2(3)}$$

$$a=3, b=-7, c=-2$$

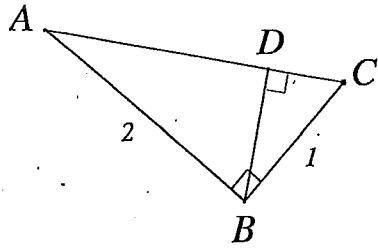
- 1 mark a or b
- 1 mark c.

9.



a) $\triangle ADE$ and $\triangle BDC$ ✓
 $AE=BC$ (opp sides pgram) ✓
 $\angle ADE=\angle BDC$ (vert opp) ✓
 $\angle EAD=\angle ACB$ (alt angles) ✓
 $\triangle ADE \cong \triangle BDC$ (AAS) ✓
 b) $AD=DC$ and $DE=BD$ ✓

10.



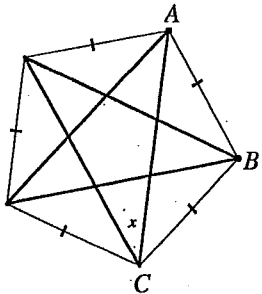
$\angle A$ is common ✓
 $\angle D$ and $\angle B$ are right angles ✓ ✓
 $\therefore \angle ABD=\angle ACB$ ✓
 $\therefore \triangle ABC$ and $\triangle ADB$ similar (AAA) ✓

$$\frac{AB}{AC} = \frac{BD}{BC} = \frac{AD}{AB}$$

$$\frac{2}{\sqrt{5}} = \frac{BD}{1}$$

$$BD = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$$

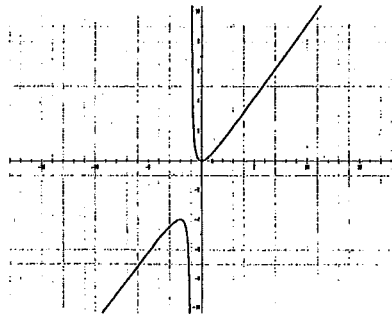
11.



a) $S_n = (n-2)180 = 540$ ✓
 $\angle B = 540/5 = 108^\circ$ ✓
 b) From isosceles $\triangle ABC$ ✓
 $\angle ABC = (180-108)/2 = 36^\circ$ ✓
 c) $2(36) + x = 108^\circ$ ✓
 $x = 36^\circ$ ✓

12.

$$f(x) = \frac{x^2}{x+1}$$



a) D: $x \neq -1$ R: $y \geq 0$, or $y \leq -4$ ✓ ✓

b) $h(1) = 1/2$ ✓
 $h(0) = 0$ ✓
 $h(-3) = -9/2$ ✓
 $f(a) = \frac{a^2}{a+1}$ ✓
 $f(x^2) = \frac{x^4}{x^2+1}$ ✓

(5)

c) $h(x) = \frac{x^2}{x+1}$
 $h(-x) = \frac{(-x)^2}{(-x)+1} = \frac{x^2}{(-x)+1} \neq h(x) \text{ or } -h(x)$