Waverley College

Year 12 2 Unit Mathematics Examination

Term 2 2012

TIME ALLOWED: 50 MINUTES

STUDENT NUMBER/NAME: ____________ ____________

General Instructions

- Write using black or blue pen.
- Board-approved calculators and templates may be used.
- A list of standard integrals is provided at the end of the examination paper.
- All necessary working should be shown in every question.
- Record your solutions to the multiple choice on the tear-off sheet provided.
- Start Questions 7 & 8 on a new piece of paper.

SECTION 1	**Multiple Choice**	/6
SECTION 2	**Q 7**	/15
	Q 8	/ 13
TOTAL		/ 34
%		

SECTION I - Multiple Choice.

Answers for this section should be recorded on the separate answer sheet handed out with this examination paper.

QUESTION ONE

When simplified, $\ln e + \ln e^e$ is equal to:

(A) 2e

(B) 1 + e

(C) 2

(D) e

QUESTION TWO

For the function $y = \ln(x^2 + 1)$, the derivative $\frac{dy}{dx}$ is equal to:

(A) $2x$

(B) $2x\ln(x^2 + 1)$

(C) $\frac{2x}{x^2+1}$

(D) $2xe^{x^2+1}$

QUESTION THREE

The indefinite integral $\int \frac{15}{1-3x}$ is equal to:

(A) $5\log_e(1 - 3x) + C$

(B) $-45\log_e(1 - 3x) + C$

(C) $-5\log_e(1 - 3x) + C$

(D) ~~$45\log_e(1 - 3x)$~~ $+ C$

QUESTION FOUR

What is the value of $\sum_{r=-1}^{3} (4r - 1)$?

(A) 16

(B) 15

(C) 21

(D) 27

QUESTION FIVE

The amount of interest earned when $10 000 is invested for a period of three years at 6% p.a and compounding monthly is given by

(A) $10000(1\cdot06)^{36}$

(B) $10000(1\cdot005)^{3}$

(C) $10000-10000(1\cdot06)^{36}$

(D) $10000(1\cdot005)^{36}-10000$

QUESTION SIX

For what values of x does the Geometric Series $(2x - 3) + (2x - 3)^2 + (2x - 3)^3 + \ldots\ldots$ have a limiting sum?

(A) $-1 < x < 1$
(B) $-1 < x < 2$
(C) $0 < x < 1$
(D) $1 < x < 2$

SECTION 2
Total Marks - 28

Start each question on a new piece of paper.

QUESTION SEVEN

a) Evaluate $\log_{15} 21$ to four significant figures. **1**

b) Differentiate $y = \frac{e^{3x}}{3x+1}$ **2**

c) Find the gradient of the tangent to the curve $y = e^{5x}$ at the point $(2, e^{10})$ and hence find the equation of the tangent to the curve $y = e^{5x}$ at the point $(2, e^{10})$. **3**

d) Find the exact area enclosed between the curve $y = e^{-x}$, the x-axis and the lines $x = -\ln 3$ and $x = 1$. **2**

e) Differentiate

i) $y = \log_e(2x - 5)$ **1**

ii) $y = x \ln x$ **2**

f) Find $\int \frac{3x^2+2}{x^3+2x+5}\, dx$. **1**

g) Evaluate $\int_1^2 \frac{3}{7-2x} dx$ correct to four significant figures. **3**

QUESTION EIGHT

(a) The first three terms of a series are $12\cdot 5+12\cdot 7+12\cdot 9+\ldots$

(i) Find the fiftieth term. **2**

(ii) Find the sum of the first fifty terms. **2**

(b) The first three terms of a geometric series are $2+6+18+\ldots$

Find the first term of the series which is greater than $100\,000$. **3**

(c) When Julia started working she began paying \$100 at the beginning of each month into a superannuation fund. The contributions are compounded monthly at an interest rate of 6% per annum. She intends to retire after having worked 35 years.

(i) Let \$$P$ be the final value of Julia's superannuation when she retires after 35 years(420 months). Show that \$$P$ = \$143 183 to the nearest dollar. **2**

(ii) 15 years after she started working Julia read a magazine article about retirement and realised that she would need \$800 000 in her fund when she retires. At the time of reading the magazine article she had \$29 227 in her fund. For the remaining 20 years she intends to work, she decides to pay a total of \$$M$ into her fund at the beginning of each month. The contributions continue to attract the same interest rate of 6% per annum, compounded monthly. At the end of n months after starting the new contributions, the amount in the fund is \$$A_n$.

(α) Show that: $A_2 = 29\,227\times 1.005^2 + M(1.005+1.005^2)$ **1**

(β) Find the value of M so that Julia will have \$800 000 in her fund after the remaining 20 years(240 months). **3**

END OF EXAMINATION

Yr 12 2U, T2 2012

Q1	B	Q4	B
Q2	C	Q5	D
Q3	C	Q6	D

Q7 a) $\log_{15} 21 = \frac{\log 21}{\log 15}$

$= 1.124$ ✓

b) $y = \frac{e^{3x}}{3x+1}$ $\quad u = e^{3x}$, $u' = 3e^{3x}$, $v = 3x+1$, $v' = 3$

$y' = \frac{3e^{3x}(3x+1) - 3e^{3x}}{(3x+1)^2}$ ✓

$= \frac{9xe^{3x} + 3e^{3x} - 3e^{3x}}{(3x+1)^2}$

$= \frac{9xe^{3x}}{(3x+1)^2}$ ✓

c) $y = e^{5x}$

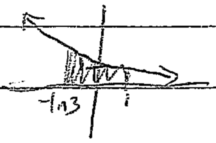
$y' = 5e^{5x}$ ✓

At $x = 2$, $y' = 5e^{10}$ ✓

$y - e^{10} = 5e^{10}(x-2)$

$y - e^{10} = 5e^{10}x - 10e^{10}$

$0 = 5e^{10}x - y - 9e^{10}$ ✓

d) 

$A = \int_{-\ln 3}^{1} e^{-x}\,dx$

$= \left[-e^{-x}\right]_{-\ln 3}^{1}$ ✓

$= -\frac{1}{e} + e^{\ln 3}$

$= 3 - \frac{1}{e}$ units^2 ✓

e) i) $y = \log_e(2x-5)$

$y' = \frac{2}{2x-5}$ ✓

ii) $y = x\ln x$ $\quad u = x$, $u' = 1$, $v = \ln x$, $v' = \frac{1}{x}$

$y' = \ln x + 1$ ✓ ✓

f) $\int \frac{3x^2+2}{x^3+2x+5}\,dx = \ln(x^3+2x+5) + C$ ✓

g) $\int_1^2 \frac{3}{7-2x}\,dx = \frac{3}{-2}\int_1^2 \frac{-2}{7-2x}$ ✓

$= \frac{3}{-2}\left[\ln(7-2x)\right]_1^2$

$= \frac{3}{-2}\left[\ln 3 - \ln 5\right]$

$= 0.7662$ ✓

Q8

a) i) $T_{50} = 12.5 + 49 \times 0.2$ ✓

$= 22.3$ ✓

ii) $S_{50} = \frac{50}{2}[2 \times 12.5 + 49 \times 0.2]$ ✓

$= 870$ ✓

b) $a = 2$ $T_n = 2(3)^{n-1}$

$r = 3$

$2(3)^{(n-1)} > 100\,000$ ✓

$3^{n-1} > 50\,000$

$(n-1)\log 3 > \log 50\,000$

$n-1 > \frac{\log 50\,000}{\log 3}$

$n > \frac{\log 50000}{\log 3} + 1$

> 10.8

∴ The 11th term is greater than 100000 ✓

$T_{11} = 2(3)^{10}$

$= 118098$ ✓

d) $r = \frac{0.06}{12} = 0.005$

i) $A_1 = 100(1.005)^{420}$

$A_2 = 100(1.005)^{419}$

⋮

$A_{420} = 100(1.005)$ ✓

∴ $A_{TOTAL} = 100[1.005 + 1.005^2 + \ldots + 1.005^{420}]$

$= 100\left[\frac{1.005(1.005^{420}-1)}{0.005}\right]$ ✓

$= \$143\,183$

ii) $A_1 = 29\,227(1.005) + M(1.005)$

$A_2 = A_1(1.005) + M(1.005)$

$= 29227(1.005)^2 + M(1.005)^2 + M(1.005)$

$= 29\,227(1.005)^2 + M(1.005 + 1.005^2)$ ✓

⋮

$A_{240} = 29\,227(1.005)^{240} + M(1.005 + 1.005^2 + \ldots + 1.005^{240})$ ✓

$800\,000 = 29\,227(1.005)^{240} + M\left[\frac{1.005(1.005^{240}-1)}{0.005}\right]$ ✓

$M = \frac{(800\,000 - 29\,227(1.005)^{240}) \times 0.005}{1.005(1.005^{240}-1)}$

$= \$1514.48$ ✓