2006

Higher School Certificate

Preliminary Examination

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Board approved calculators may be used
- Write using black or blue pen
- A table of standard integrals is provided
- All necessary working should be shown in every question
- Write your student number and/or name at the top of every page

Total marks – 84

- Attempt Questions 1 – 7
- All questions are of equal value

This paper MUST NOT be removed from the examination room

STUDENT NUMBER/NAME ....

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Question 1 *Start a new page* Marks

(a) In a weekend garage sale, $\frac{3}{5}$ of the items on offer were sold on Saturday. $\frac{2}{3}$ of the remaining items, were then sold on Sunday. What percentage of the items remained unsold at the end of the weekend? 2

(b) Simplify by removing parentheses:

(i) $\sqrt{8}\left(\sqrt{2}-\sqrt{3}\right)$ 1

(ii) $\left(2\sqrt{5}-\sqrt{3}\right)^2$. 1

(c) Solve: $x^2+9x=10$. 2

(d) Factorise completely.

(i) m^3+10m^2+25m 1

(ii) $x^3-\frac{1}{8}$ 1

(e) Simplify: $\frac{1}{x+5}-\frac{1}{x-5}$. 2

(f) Evaluate $255(0.0024)^{10}$ giving your answer in scientific notation correct to 3 significant figures. 2

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Question 2 *Start a new page* **Marks**

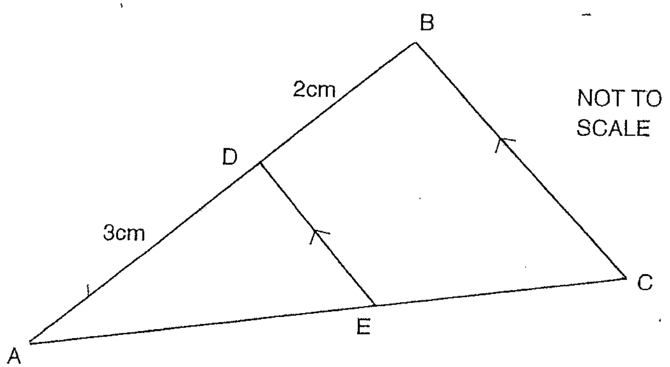
(a) Solve: $|3-2x|>5$ **2**

(b) Solve: $\sin\beta-\frac{1}{2}\sqrt{3}=0$ for $0°\le\beta\le 360°$ **2**

(c) If $\frac{2}{3-\sqrt{7}}=a+b\sqrt{7}$, find the values of a and b. **2**

5.6457513111

(d) **2**



Copy or trace the diagram onto your worksheet.

In ΔABC, points D and E lie on lines AB and AC respectively, such that DE is parallel to BC. If AD = 3centimetres and DB = 2 centimetres, find the ratio $DE : BC$, giving reasons.

(e) Solve the following pair of simultaneous equations: **2**

$$2x-y=-8$$
$$3x+2y=-5$$

(f) ABC is a right angled triangle in which $\angle ACB=90°$. **2**
If $AB=\sqrt{1+x^2}$, $BC=\sqrt{1-x^2}$ and $AC=\sqrt{x}$, find the value(s) of x, giving reasons.

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Question 3 *Start a new page* **Marks**

(a) Differentiate each of the following:

(i) $\frac{3x}{x^2-2x}$. **2**

(ii) $(2-3x)^7$. **1**

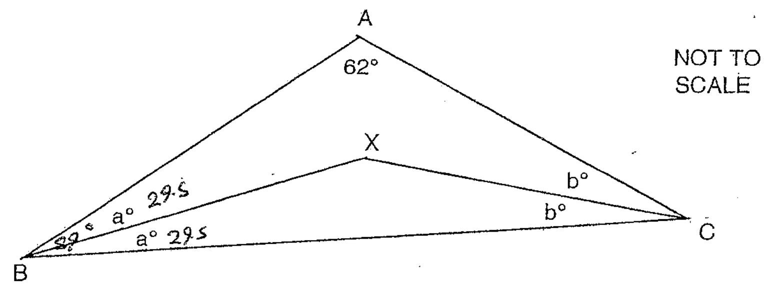
(iii) $x\sqrt{1-x}$. **2**

(iv) $\sqrt[3]{x}-\frac{1}{x}$. **2**

(b) Shade in the region on the number plane where the following inequalities hold simultaneously. **2**

$$y\ge x^2-1$$
$$y\le x+1$$

(c) **3**



In the diagram above, XB and XC bisect $\angle ABC$ and $\angle ACB$ respectively. $\angle BAC=62°$.

Copy or trace the diagram onto your worksheet.

Find the size of $\angle BXC$.

STUDENT NUMBER/NAME..

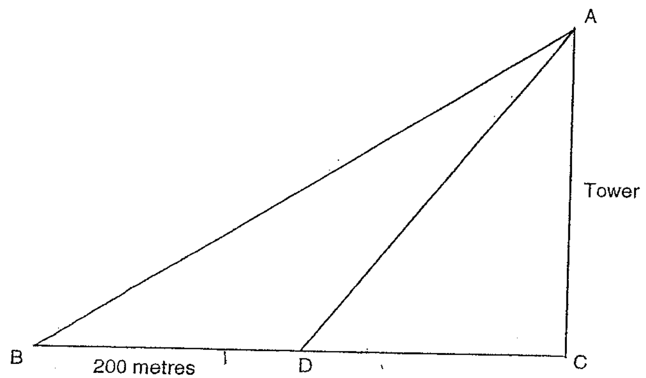
Question 4 *Start a new page* **Marks**

(a) Solve, leaving your answers in exact form: **3**

$$x^6 + 6x^3 - 16 = 0.$$

(b) Find the equation of the normal to the curve $y = x^3 - \overset{5}{\cancel{7}}x$ at the point (1, -2). **3**

(c)



NOT TO SCALE

The diagram above shows a vertical tower AC. Points B, C and D are in a straight line on level ground. The distance from B to D is 200 metres.
A surveyor found that the angle of elevation to the top of the tower from point B was 38°. She then moved to point D and measured the angle of elevation as 54°.

(i) Copy or trace the diagram and fill in all the information given. **1**

(ii) Show that the length of AD $= \dfrac{200\sin 38^\circ}{\sin 16^\circ}$ **2**

(iii) Calculate the height of the tower. **1**

(d) A circle has equation $x^2 - 2x + y^2 + 6y = 6$. By writing the equation in the form $(x-a)^2 + (y-b)^2 = c$, or otherwise, find the radius of the circle and the coordinates of the centre. **2**

STUDENT NUMBER/NAME..

Question 5 *Start a new page* **Marks**

(a) Prove that $\dfrac{\sin\theta}{1-\cos\theta} - \dfrac{\sin\theta}{1+\cos\theta} = 2\cot\theta$ **4**

(b) A ship sails from point A on a bearing of 237°T for a distance of 423 kilometres. to point B
The ship then turns and sails due South to point C. The bearing of A from C is found to be 41°T.

(i) Draw a diagram showing the above information. **1**

(ii) Find the size of $\angle BAC$. **1**

(iii) Calculate the total distance sailed by the ship. **2**

(c) Differentiate $y = x^2 + x$ from first principles

using $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h) - f(x)}{h}$ **4**

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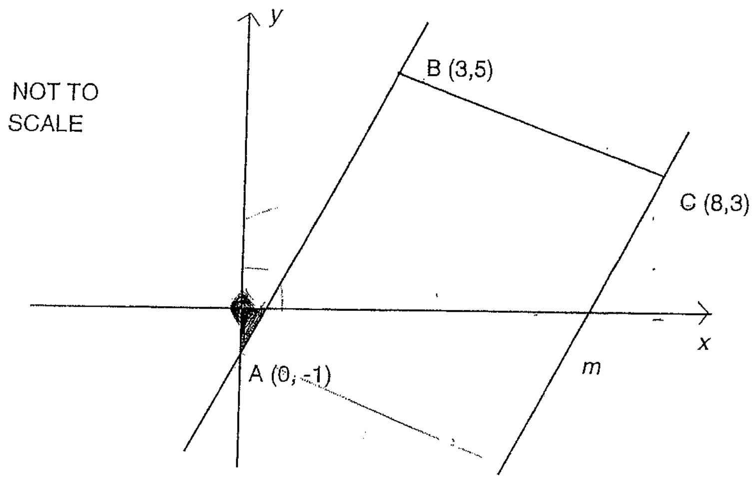
Question 6 *Start a new page* **Marks**

(a)



In the diagram, A (0,-1), B (3,5) and C (8,3) are points on the number plane.

Copy or trace the diagram onto your worksheet.

(i) Find the length of AB. **1**

(ii) Find the equation of AB. **1**

(iii) Find the equation of the line m passing through C parallel to AB. **2**

(iv) The point D lies on m such that BC is parallel to AD. Find the coordinates of D. **2**

(v) What type of quadrilateral is $ABCD$? Give reasons. **1**

(vi) Find the perpendicular distance from C to AB. **1**

(vii) Find the area of $ABCD$. **1**

(b) Sketch $y=|x|$ and state the domain and range. **3**

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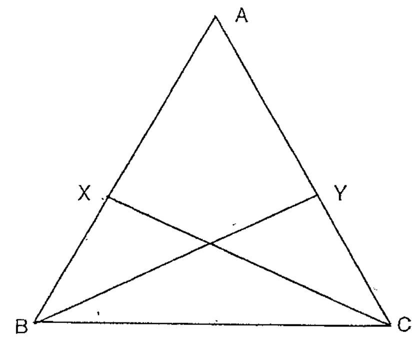
Question 7 *Start a new page* **Marks**

(a)



In the diagram above, ΔABC is isosceles with $AB = AC$. Points X and Y lie on AB and AC respectively such that $AX = AY$.

Copy or trace the diagram onto your worksheet.

(i) Prove that $\Delta BXC \equiv \Delta CYB$. **3**

(ii) Hence or otherwise prove that $\angle XBY = \angle YCX$. **1**

(b) A function $f(x)$ is defined as follows:

$$f(x)=\begin{cases} \frac{1}{x} & \text{for } x>0 \\ 1 & \text{for } x=0 \\ 2^x & \text{for } x<0 \end{cases}$$

(i) Draw a neat sketch of the graph $y=f(x)$ **2**

(ii) Evaluate: $f(2)+f(0)+f(-2)$. **2**

(c) (i) Simplify: $2\cos^2 B+3\sin^2 B-2$. **2**

(ii) Hence or otherwise, solve: $2\cos^2 A+3\sin^2 A-3=0$. **2**

End of Paper

YEAR 11 2UNIT PRELIMINARY EXAM 2006 SOLUTIONS

Question 1

a) Amt unsold $= \frac{1}{3} \times \frac{2}{5}$

$= \frac{2}{15}$ (1)   or similar working out for "sold items"

% unsold $= \frac{2}{15} \times 100$

$= 13\frac{1}{3}\%$. (1) (decimal approx accept)

b) (i) $\sqrt{8}(\sqrt{2}-\sqrt{3}) = \sqrt{16} - \sqrt{24}$

$= 4 - 2\sqrt{6}$ (1)

(ii) $(2\sqrt{5} - \sqrt{3})^2 = 20 - 4\sqrt{15} + 3$

$= 23 - 4\sqrt{15}$ (1)

c) $x^2 + 9x - 10 = 0$

$(x+10)(x-1) = 0$ (1)

$x = -10, 1$ (1)

d) (i) $m^3 + 10m^2 + 25m = m(m^2 + 10m + 25)$ ($\frac{1}{2}$)

$= m(m+5)(m+5)$

$= m(m+5)^2$ ($\frac{1}{2}$)

(ii) $x^3 - \frac{1}{8} = (x - \frac{1}{2})(x^2 + \frac{1}{2}x + \frac{1}{4})$ (1)

e) $\frac{1}{x+5} - \frac{1}{x-5} = \frac{(x-5)-(x+5)}{(x+5)(x-5)}$ (1)

$= \frac{x-5-x-5}{x^2-25}$

$= -\frac{10}{x^2-25}$ (1)

f) 1.62×10^{-24} (1 scientific notation, 1 3 sig figs)

(10)

Question 2

a) $|3-2x| > 5$

$3 - 2x > 5$ or $3 - 2x < -5$

$-2x > 2$ $-2x < -8$

$x < -1$ (1) $x > 4$ (1)

$\therefore x < -1$ or $x > 4$

b) $\sin\beta - \frac{1}{2}\sqrt{3} = 0$

$\sin\beta = \frac{\sqrt{3}}{2}$

$\beta = 60°, 120°$ (1) (1)

c) $\frac{2}{3-\sqrt{7}} = a + b\sqrt{7}$

$\frac{2}{3-\sqrt{7}} \times \frac{3+\sqrt{7}}{3+\sqrt{7}} = a + b\sqrt{7}$

$\frac{2(3+\sqrt{7})}{9-7}$ (1) $= a + b\sqrt{7}$

$3 + \sqrt{7} = a + b\sqrt{7}$

$\therefore a = 3, b = 1$ (1)

d) In ΔADE and ΔABC

$\angle ADE = \angle ABC$ (corresponding angles on || lines)

$\angle AED = \angle ACB$ (" " ")

$\therefore \Delta ADE ||| \Delta ABC$ (equiangular) (1)

DE : BC = 3 : 5 (1) (DE : BC = AD : AB : corresponding sides of similar Δ

e) $2x - y = -8$ (1)

$3x + 2y = -5$ (2)

Multiply (1) by 2

$4x - 2y = -16$ (1) +

$3x + 2y = -5$ (2)

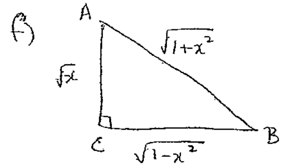
$7x = -21$

$x = -3$ (1)

sub $x = -3$ in (1)

$2(-3) - y = -8$

$y = 2$ (1) $\therefore \left.\begin{matrix} x = -3 \\ y = 2 \end{matrix}\right\}$

f) 

Since ΔABC is right-angled $\left(\sqrt{1+x^2}\right)^2 = \left(\sqrt{x}\right)^2 + \left(\sqrt{1-x^2}\right)^2$ (1)

$1+x^2 = x + 1-x^2$

$2x^2 - x = 0$

$x(2x-1) = 0$

$x = 0, \frac{1}{2}$

but if $x=0$ then $AC = \sqrt{0} = 0$, must be >0

$\therefore x = \frac{1}{2}$ (1) ($\frac{1}{2}$ off for not eliminating 0)

Question 3 (12)

a) (i) $\frac{d}{dx} \frac{3x}{x^2-2x} = \frac{(x^2-2x).3 - 3x.(2x-2)}{(x^2-2x)^2}$ (1) (2)

$= \frac{3x^2-6x-6x^2+6x}{(x^2-2x)^2}$

$= \frac{-3x^2}{(x^2-2x)^2}$ (1)

(ii) $\frac{d}{dx}(2-3x)^7 = 7(2-3x)^6.-3$ (1)

$= -21(2-3x)^6$ (1)

(iii) $\frac{d}{dx}\, x(1-x)^{\frac{1}{2}} = x.\frac{1}{2}(1-x)^{-\frac{1}{2}}.-1 + (1-x)^{\frac{1}{2}}.1$ (1)

$= \frac{-x}{2\sqrt{1-x}} + \sqrt{1-x}$ (2)

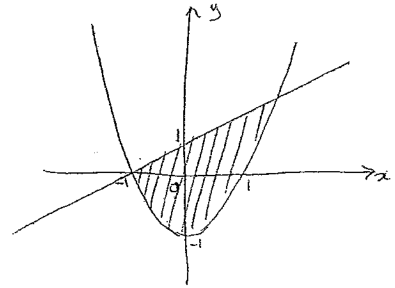
$= \frac{-x + 2(1-x)}{2\sqrt{1-x}}$

$= \frac{2-3x}{2\sqrt{1-x}}$ (1) (Can take earlier answer simplified)

(iv) $\frac{d}{dx} \sqrt[3]{x} - \frac{1}{x} = \frac{d}{dx}\left(x^{\frac{1}{3}} - x^{-1}\right)$ (1)

$= \frac{1}{3}x^{-\frac{2}{3}} + x^{-2}$ OR (1) (2)

$= \frac{1}{3\sqrt[3]{x^2}} + \frac{1}{x^2}$

b) 

(1) for line + parabola

(1) for correct shading (2)

c) In ΔABC

$2a + 2b + 62 = 180$ (Angle sum Δ) (1)

$a+b = 59$ (1) (3)

In ΔBXC

$\angle BXC + a+b = 180$ (Angle sum Δ)

$\angle BXC + 59 = 180$

$\angle BXC = 121°$ (1)

Question 4 (~~12~~) (9)

a) $x^6 + 6x^3 - 16 = 0$

Let $m = x^3$

$m^2 + 6m - 16 = 0$ (1) 3/ out

$(m-2)(m+8) = 0$

$m = 2$ or $m = -8$ (1)

$\therefore x^3 = 2$ $x^3 = -8$

$x = \sqrt[3]{2}$ $x = -2$ (1)

b) $y = x^3 - 5x$

$\frac{dy}{dx} = 3x^2 - 5$ (1) (3)

at $x = 1$

m of tangent $= 3(1)^2 - 5$

$= -2$

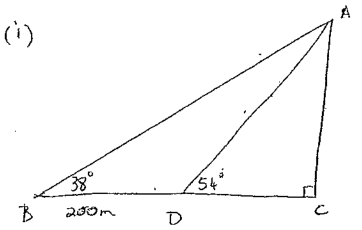
$\therefore$ normal gradient $= \frac{1}{2}$ (1)

$y+2 = \frac{1}{2}(x-1)$

$y = \frac{1}{2}x - 2\frac{1}{2}$ (1)

OR $x - 2y - 5 = 0$

c) (i)



(1)

(ii) $\angle BAD = 54^\circ - 38^\circ$ (exterior angle sum Δ)
$= 16^\circ$ (1)

In ΔBAD

$$\frac{AD}{\sin 38^\circ} = \frac{200}{\sin 16^\circ}$$

$$AD = \frac{200\sin 38^\circ}{\sin 16^\circ} \quad (1)$$

(iii) In ΔACD

$$\sin 54^\circ = \frac{AC}{AD} \quad \left(\tfrac{1}{2}\right)$$

$$\therefore AC = AD \times \sin 54^\circ$$

$$= \frac{200\sin 38^\circ \sin 54^\circ}{\sin 16^\circ}$$

$$\doteqdot 361\text{ m} \quad \left(\tfrac{1}{2}\right)$$

d) $x^2 - 2x + 1 + y^2 + 6y + 9 = 6 + 10$

$(x-1)^2 + (y+3)^2 = 16$ (1)

$\therefore$ centre $(1, -3)$ and radius $= 4$ units (1)

1

$\frac{1}{2}$ marks

2

1

2

Question 5

a) LHS $= \dfrac{\sin\theta}{1-\cos\theta} - \dfrac{\sin\theta}{1+\cos\theta}$

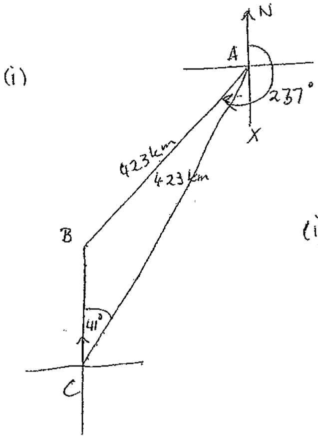
$$= \frac{\sin\theta(1+\cos\theta) - \sin\theta(1-\cos\theta)}{(1-\cos\theta)(1+\cos\theta)} \quad (1)$$

$$= \frac{\sin\theta + \sin\theta\cos\theta - \sin\theta + \sin\theta\cos\theta}{1-\cos^2\theta} \quad (1)$$

$$= \frac{2\sin\theta\cos\theta}{\sin^2\theta} \quad (1)$$

$$= \frac{2\cos\theta}{\sin\theta} \quad (1)$$

b) (i)



(1)

(ii) $\angle XAC = \angle BCA$ (Alternate angles on // lines)
$= 41^\circ$

$\therefore \angle BAC = 237^\circ - 180^\circ - 41^\circ$
$= 16^\circ$ (1)

(iii)
$$\frac{BC}{\sin 16^\circ} = \frac{423}{\sin 41^\circ}$$

$$BC = \frac{423\sin 16^\circ}{\sin 41^\circ}$$

$BC \doteqdot 177.7$ km (1)

Distance sailed $\doteqdot 177.7 + 423$
$\doteqdot 600.7$ km (1)

c) Let $f(x) = x^2 + x$

$f(x+h) = (x+h)^2 + (x+h)$
$= x^2 + 2xh + h^2 + x + h$ (1)

$$f'(x) = \lim_{h\to 0} \frac{x^2 + 2xh + h^2 + x + h - (x^2 + x)}{h}$$

$$= \lim_{h\to 0} \frac{2xh + h^2 + h}{h} \quad (1)$$

$$= \lim_{h\to 0} \frac{h(2x + h + 1)}{h} \quad (1)$$

$$= \lim_{h\to 0} 2x + h + 1$$

$$= 2x + 1 \quad (1)$$

Question 6

a) (i) $AB = \sqrt{(3-0)^2 + (5+1)^2}$

$AB = \sqrt{9+36}$

$AB = \sqrt{45}$

$AB = 3\sqrt{5}$ units (1) ($\frac{1}{2}$ off if not simplified).

(ii) m of $AB = \frac{5+1}{3-0}$

$= 2$ ($\frac{1}{2}$)

$y+1 = 2(x-0)$

$y+1 = 2x$

$y = 2x-1$ or $2x-y-1=0$ ($\frac{1}{2}$)

(iii) m of required line $= 2$ (1)

$y-3 = 2(x-8)$

$y-3 = 2x-16$

$y = 2x-13$ (1) OR $2x-y-13=0$

(iv) By inspection $D = (5,-3)$ because ABCD is ||gram (2)

OR m of $BC = \frac{5-3}{3-8}$

$= -\frac{2}{5}$

$\therefore$ m of $AD = -\frac{2}{5}$

$y+1 = -\frac{2}{5}(x-0)$

$y = -\frac{2}{5}x - 1$ OR $2x+5y+5=0$ (1)

$2x-y-13=0$

$2x+5y+5=0$

$-6y-18=0$

$y=-3$

$x=5$ (1)

OR Midpoint of AC = midpoint of BD (diagonals of ||gram bisect each other)

Then find D from midpoint.

(v) Parallelogram because opposite sides are parallel. (1)

(vi) Eqn of AB is $2x-y-1=0$ $C(8,3)$

perp. dist. $= \left|\frac{2(8)-1(3)-1}{\sqrt{2^2+(-1)^2}}\right|$ ($\frac{1}{2}$)

$= \left|\frac{12}{\sqrt{5}}\right|$

$= \frac{12\sqrt{5}}{5}$ units ($\frac{1}{2}$)

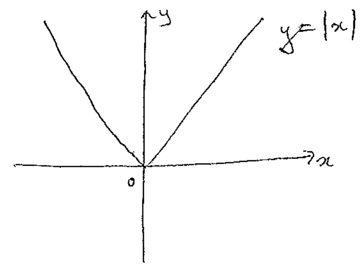
(vii) Area parallelogram $= bh$

$= AB \times$ perp distance to C

$= 3\sqrt{5} \times \frac{12\sqrt{5}}{5}$ ($\frac{1}{2}$)

$= 36$ units^2 ($\frac{1}{2}$)

b)



(1)

Domain: all real x (1)

Range: $y \geq 0$ (1)

Question 7

a) $AB = AC$ (data)

$AX = AY$ (data)

$AB - AX = AC - AY$

$\therefore$ $BX = CY$ (1)

In ΔBXC and ΔCYB

$BX = CY$ (above)

$\angle XBC = \angle YCB$ (base $\angle$s isos. ΔABC) (1)

$BC = BC$ (common) (1)

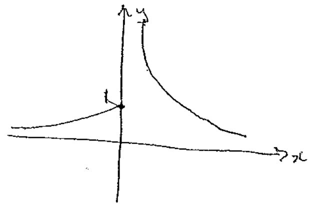
$\therefore$ $\Delta BXC \equiv \Delta CYB$ (SAS)

(ii) $\angle XCB = \angle YBC$ (corresponding angles of congruent Δ's).

$\angle XBC - \angle YBC = \angle YCB - \angle XCB$

$\angle XBY = \angle YCX$ (1)

b) (i)



(2) 1 for each part graph

(ii) $f(2) + f(0) + f(-2)$

$= \frac{1}{2} + 1 + 2^{-2}$ (1)

$= \frac{1}{2} + 1 + \frac{1}{4}$

$= 1\frac{3}{4}$ (1)

c) (i) $2\cos^2 B + 3\sin^2 B - 2$

$= 2(1-\sin^2 B) + 3\sin^2 B - 2$ (1)

$= 2 - 2\sin^2 B + 3\sin^2 B - 2$

$= \sin^2 B$ (1)

(ii) $2\cos^2 A + 3\sin^2 A - 3 = 0$

$2\cos^2 A + 3\sin^2 A - 2 - 1 = 0$ (1)

$\sin^2 A - 1 = 0$

$\sin^2 A = 1$

$\sin A = \pm 1$

$A = 90°, 270°$ (1)