Waverley College
Year 12 2 unit Mathematics Examination
Term 4 2006

TIME ALLOWED: 45 MINUTES

INSTRUCTIONS:

Attempt all questions
Approved calculators may be used
Write in blue or black pen only
Show all necessary working
Marks may be deducted for careless or badly arranged work

Section 1	/35	Section 2	/ 10
Total	/45	Percentage	%

Outcomes: A student
P4-chooses and applies appropriate arithmetic, algebraic, graphical, trigonometric and geometric techniques.
P6-relates the derivative of a function to the slope of its graph.
P7-determines the derivative of a function through routine application of the rules of differentiation.
P8-understands and uses the language and notation of calculus.
H2-constructs arguments to prove and justify results.
H9-communicates using mathematical language, notation, diagrams and graphs.

SECTION 1 The Quadratic Polynomial

Question 1

a) Write down the discriminant of $3x^2 - kx + 3 = 0$. 1

b) Find the value(s) of k for which the equation $3x^2 - kx + 3 = 0$ has

(i) exactly one real root. 2

(ii) no real roots. 2

Question 2

Prove the function $y = x^2 - x + 6$ is positive definite. 3

Question 3

Find the axis of symmetry of $y = x^2 - 6x + 10$ and hence find the minimum value of $y = x^2 - 6x + 10$. 2

Question 4

If α and β are the roots of $x^2 - 2x - 7 = 0$, find the values of:

a) $\alpha + \beta$ 1

b) $\alpha\beta$ 1

c) $\frac{1}{\alpha} + \frac{1}{\beta}$ 2

d) $\alpha^2 + \beta^2$ 2

e) $\alpha^2\beta^3 + \alpha^3\beta^2$ 2

Question 5

For what values of k will the equation $x^2 + 4x - 2k = 0$ have roots which are the reciprocal of each other? 2

Question 6

Solve the following equations for all real values of x:

a) $3^{2x} - 4(3^x) + 3 = 0$ 5

b) $4x^4 = 4x^2 + 8$ 5

Question 7

Find the values of A, B and C for which $2x^2 - 7x - 4 \equiv A(x+2)^2 + B(x+2) + C$ 5

SECTION 2 Locus

Question 8

Find the equation of the circle with radius 6 units and centre $(2,-5)$. 2

(Do not expand)

Question 9

Let A and B be the fixed points $(-2,0)$ and $(1,0)$ and let the variable point be $P(x,y)$.

a) The point P moves so that PA is twice the distance of PB. Prove that the locus of P is $x^2-4x+y^2=0$. 5

b) This locus is a circle. Find the centre and radius of this circle. 3

Year 12 2unit : Term 4 2006

Section 1

(a) $\Delta = (-k)^2 - 4 \times 3 \times 3$

$\Delta = k^2 - 36$ (1)

b) (i) For exactly one real root, $\Delta = 0$

$k^2 - 36 = 0$ (1)

$K = \pm 6$ (1)

(ii) For no real roots, $\Delta < 0$

$k^2 - 36 < 0$ (1)

$-6 < k < 6$ (1)

2) $\Delta = (-1)^2 - 4 \times 1 \times 6$

$\Delta = -23$ (1)

$\Delta < 0$

$a = 1$ (1)

$a > 0$

For $\Delta < 0$ and $a > 0$ $y = x^2 - x + 6$ is positive definite (1)

3) $x = \frac{6}{2 \times 1}$

$x = 3$ (1)

$y = (3)^2 - 6(3) + 10$

$y = 1$

$\therefore$ min value = 1 (1) $\frac{1}{2}$ off if vertex given rather than value.

4) a) $\alpha + \beta = 2$ (1)

b) $\alpha\beta = -7$ (1)

c) $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta}$ (1)

$= -\frac{2}{7}$ (1)

d) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$ (1)

$= 2^2 - 2(-7)$

$= 18$ (1)

e) $\alpha^2\beta^3 + \alpha^3\beta^2 = \alpha^2\beta^2(\beta+\alpha)$

$= (\alpha\beta)^2(\alpha+\beta)$ (1)

$= (-7)^2 \times 2$

$= 98$ (1)

5) Let the roots be α and $\frac{1}{\alpha}$

$\alpha \times \frac{1}{\alpha} = -2k$ (1)

$1 = -2k$

$k = -\frac{1}{2}$ (1)

6) a) $3^{2x} - 4(3^x) + 3 = 0$

$(3^x)^2 - 4(3^x) + 3 = 0$

Let $m = 3^x$

$m^2 - 4m + 3 = 0$ (1)

$(m-1)(m-3) = 0$ (1)

$m = 1$ or $m = 3$ (1)

$\therefore 3^x = 1$ $\therefore 3^x = 3$

$x = 0$ $x = 1$ (2)

6b) $4x^4 - 4x^2 - 8 = 0$ (1)

Let $m = x^2$

$4m^2 - 4m - 8 = 0$ (1)

$4(m^2 - m - 2) = 0$

$4(m+1)(m-2) = 0$ (1)

$\therefore m = -1$ or $m = 2$

$x^2 = -1$ $x^2 = 2$

no real soln. $x = \pm\sqrt{2}$ (2)

7) $2x^2 - 7x - 4 \equiv A(x^2 + 4x + 4) + B(x+2) + C$

$2x^2 - 7x - 4 \equiv Ax^2 + 4Ax + 4A + Bx + 2B + C$

$2x^2 - 7x - 4 \equiv Ax^2 + (4A + B)x + 4A + 2B + C$ (2)

$\therefore A = 2$ (1)

and $4A + B = -7$

$4(2) + B = -7$

$B = -15$ (1)

and $4A + 2B + C = -4$

$4(2) + 2(-15) + C = -4$

$8 - 30 + C = -4$

$C = 18$ (1)

Section 2

8) $(x-2)^2 + (y+5)^2 = 36$ (2)

9) a) $PA = 2PB$ (1)

$PA^2 = 4PB^2$

$(x+2)^2 + y^2 = 4\left[(x-1)^2 + y^2\right]$ (2)

$x^2 + 4x + 4 + y^2 = 4x^2 - 8x + 4 + 4y^2$

$0 = 3x^2 - 12x + 3y^2$ (1)

$x^2 - 4x + y^2 = 0$ (1)

b) $x^2 - 4x + 4 + y^2 = 0 + 4$

$(x-2)^2 + y^2 = 4$ (1)

$\therefore$ Centre $(2,0)$ and radius $= 2$ units (2)