WILLIAM CLARKE COLLEGE

2004
TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

FRIDAY, 13th August

Mathematics

General Instructions

- Reading time - 5 minutes
- Working Time- 3 hours
- Write using blue or black pen
- Board-approved calculators may be used
- A table of standard integrals is provided
- Start each question on a new sheet of paper
- All necessary working should be shown in every question
- Write your student number at the top of every answer sheet submitted

Total marks **120**

- Attempt Questions 1 - 10
- All questions are of equal value

Total marks 120
Attempt Questions 1–10.
All questions are of equal value.

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

QUESTION 1. (12 marks) Use a SEPARATE Writing Booklet. **Marks**

(a) Evaluate, correct to three significant figures: 2

$$\frac{\log_e 5.8}{\pi}.$$

(b) Factorise $2x^2 - 10x - kx + 5k$. 2

(c) Find a primitive function of $2 + \frac{1}{x}$. 1

(d) Solve the pair of simultaneous equations: 2

$$5x - 3y + 8 = 0$$
$$y = 3x - 4.$$

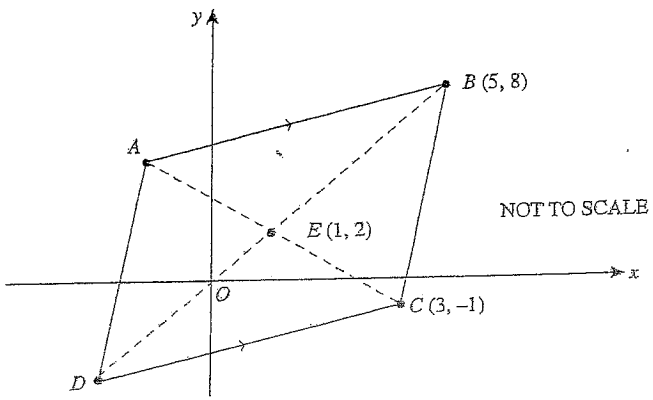
(e) The exchange rate of one Australian dollar, \$A 1.00, is 76 American cents, i e. \$US 0.76. Find the cost in Australian dollars, correct to the nearest cent, of a meal that cost \$US 13.35. 2

(f) Solve $|3x - 10| > 2$ and graph the solution on a number line. 3

QUESTION 2. (12 marks) Use a SEPARATE writing booklet. Marks

(a) Shade the region in the Cartesian plane for which the inequalities $(x-2)^2+y^2 \le 4$ and $x+y \ge 0$ hold simultaneously. 3

(b)



The diagram shows a parallelogram $ABCD$. The points E (1, 2), B (5, 8) and C(3, –1) are shown on the number plane.
Copy the diagram into your Writing Booklet.

(i) E is the midpoint of AC. Show that the coordinates of A are (–1, 5). 2

(ii) Find the gradient of the line AB. 1

(iii) Show that the equation of DC is $x-2y-5=0$. 2

(iv) Calculate the length of AB in the form $k\sqrt{5}$ units. 1

(v) Calculate the perpendicular distance between the lines DC and AB. 2

(vi) Hence, or otherwise, calculate the area of the parallelogram $ABCD$. 1

QUESTION 3. (12 marks) Use a SEPARATE writing booklet. Marks

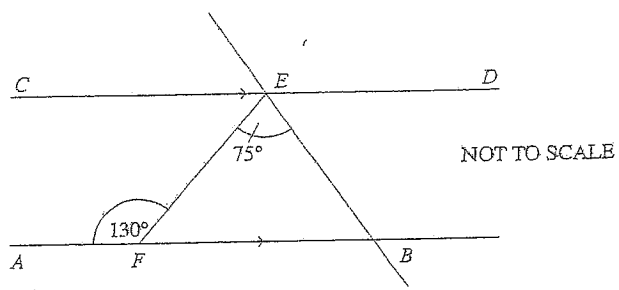
(a) Find the equation of the tangent to the curve $y = 4\sin 2x$ at the point on the curve where $x = \frac{\pi}{8}$. 3

(b) Differentiate:

(i) $(5+e^{3x})^4$ 2

(ii) $\frac{\log_e x}{x}$. 2

(c)



In the diagram, AB is parallel to CD, $\angle AFE = 130°$ and $\angle FEB = 75°$. 2
Copy or trace this diagram into your Writing Booklet.
Find the size of $\angle DEB$. Give reasons for your answer.

(d) Find:

(i) $\int e^{3x-1}dx$ 1

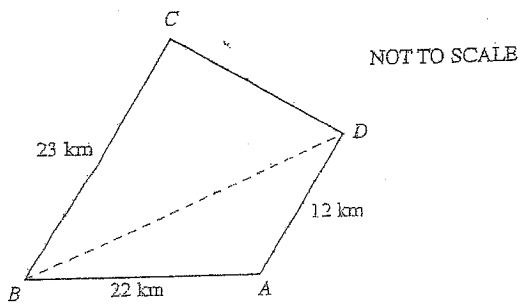
(ii) $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} 2\sec^2 x\, dx$. 2

QUESTION 4. (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) State the domain and range of the function $y = 3\sqrt{4-x}$ — 2

(b) Two dice, each with six faces, and each with the numbers 1, 1, 3, 3, 5, 5 marked on their faces, are rolled. — 2
Find the probability that the sum of the uppermost faces is 6.

(c)



NOT TO SCALE

The diagram represents a cattle station $ABCD$ in the shape of a quadrilateral.
A is 22 km due east of B and D is 12 km from A at a bearing of 030°.
C is 23 km from B at a bearing of 030° and at a bearing of 300° from D.

(i) Copy or trace the diagram into your Writing Booklet and mark the lengths and the bearings on it. — 1

(ii) Explain why $\angle BAD = 120°$ and $\angle BCD$ is a right angle. — 2

(iii) Use the cosine rule to calculate the length of BD, correct to one decimal place. — 2

(iv) Find the area of the cattle station correct to the nearest square kilometre. — 3

QUESTION 5. (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) If α and β are the roots of the quadratic equation $x^2 + 5x - 2 = 0$, find the value of:

(i) $\alpha + \beta$ — 1

(ii) $\alpha\beta$ — 1

(iii) $\alpha^2 + \beta^2$. — 1

(b) Consider the series $\log 3 + \log 6 + \log 12 + \ldots$

(i) Explain why $\log 3 + \log 6 + \log 12 + \ldots$ are the first three terms of an arithmetic series. — 1

(ii) Find the ninth term, expressing your answer in the form $\log k$. — 2

(c) Consider the function $f(x) = -\frac{1}{3}x^3 + x^2 + 3x - 4$.

(i) Show that $f'(x) = -(x^2 - 2x - 3)$. — 1

(ii) Find the coordinates of the stationary points on the curve $y = f(x)$ and determine their nature. — 3

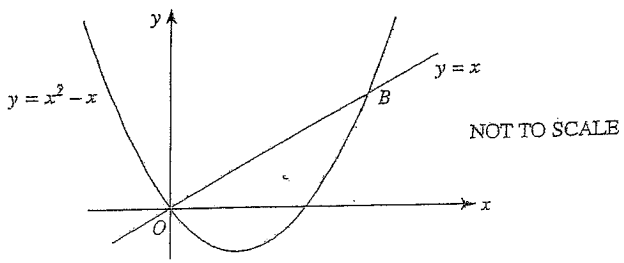
(iii) Sketch the curve $y = f(x)$, clearly labelling all turning points and the y intercept. — 1

(iv) Find the values of x for which the curve is concave down. — 1

QUESTION 6. (12 marks) Use a SEPARATE writing booklet. Marks

(a) Solve $\sin^2 x + \cos x = 1$ for $0 \le x \le 2\pi$ 4

(b)



NOT TO SCALE

The graphs of $y = x^2 - x$ and $y = x$ intersect at the origin and B.

(i) Show that the coordinates of B are (2, 2). 1

(ii) Find the area bounded by $y = x^2 - x$ and $y = x$. 3

(c) Consider a circle of radius 40 cm.

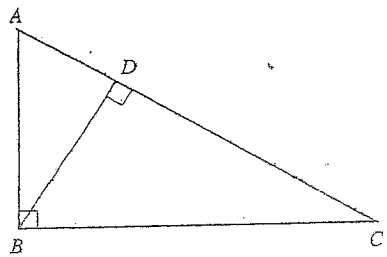
(i) The length of an arc of the circumference of the circle is 1 metre. Find the angle, to the nearest degree, subtended by the arc at the centre of the circle. 2

(ii) Find the area of the minor segment formed by a chord that subtends an angle of 55° at the centre of the circle. Answer correct to 2 decimal places. 2

QUESTION 7. (12 marks) Use a SEPARATE writing booklet. Marks

(a) Show that: $\dfrac{1+\cot\theta}{\operatorname{cosec}\theta} - \dfrac{\sec\theta}{\tan\theta+\cot\theta} = \cos\theta$ 2

(b)



NOT TO SCALE

In the right-angle triangle ABC, $\angle ABC = 90°$, and BD is an altitude.
$AD = 4$ cm, $CD = 9$ cm.

(i) Prove that ΔABD is similar to ΔCDB. 2

(ii) Find the length of BD. 1

(c) A particle is moving in a straight line. Its distance x metres at time t seconds from a fixed point O is given by

$$x = t + 2\sin t \text{ for } 0 \le t \le 2\pi.$$

(i) Find an expression for the velocity, v metres per second, of the particle. 1

(ii) At what times is the particle at rest? 2

(iii) Sketch the graph of x as a function of t. 2

(iv) What important feature on the graph indicates those times when the particle is at rest. Show this feature on the graph. 1

(v) Describe the motion of the particle for the first π seconds. 1

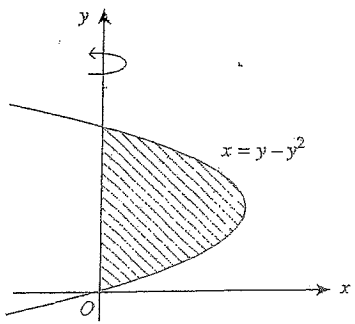
QUESTION 8. (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Consider the parabola $6y = 5 - 8x - x^2$.

(i) Find the coordinates of the vertex. **2**

(ii) Find the coordinates of the focus. **2**

(b)



NOT TO SCALE

The diagram shows the graph of $x = y - y^2$. The shaded region, bounded by $x = y - y^2$ and the y-axis, is rotated about the y-axis to form a solid. **4**

Find the volume of this solid.

(c) Find the range of values of k such that the following simultaneous equations have two solutions: **4**

$$y = x + k$$
$$2x^2 + y^2 = 6.$$

QUESTION 9. (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Consider the geometric series $\frac{6}{\sqrt{3}} + 2 + \frac{2}{\sqrt{3}} + \ldots$

(i) Explain why the series has a limiting sum. **1**

(ii) Find the exact value of the limiting sum. Write your answer with a rational denominator. **2**

(b) A furniture manufacturer makes x chairs each week, which can be sold for \$70 each.
The cost, $C(x)$ dollars, of manufacturing the x chairs is given by:

$$C(x) = 0.04x^2 + 30x + 4000.$$

(i) Explain why the weekly profit, $P(x)$ dollars, from producing x chairs in a week is given by: **1**

$$P(x) = 40x - 4000 - 0.04x^2$$

(ii) How many chairs should be produced each week to maximise the profit? **2**

(iii) What is the maximum weekly profit? **1**

(c) Pentathol is used as an anaesthetic during operations. Pentathol is eliminated from the bloodstream at a rate proportional to the amount present at any time, such that $\frac{dC}{dt} = -kC$, where C milligrams is the amount of Pentathol in the bloodstream t hours after being injected. Pentathol has a half-life of 4 hours, that is, half of its initial dose is eliminated in 4 hours.

(i) Show that $C = Ae^{-kt}$ satisfies the equation $\frac{dC}{dt} = -kC$. **1**

(ii) Show the value of k is 0.1733 (4 decimal places). **2**

(iii) Bill, who weights 90 kg, is having an operation. During the operation he must have a minimum of 40 milligrams per kilogram of body weight of Pentathol in his bloodstream. **2**

What is this initial amount of Pentathol he should be given for a one-hour operation? (Assume he is not given any additional Pentathol during the operation.)

QUESTION 10. (12 marks) Use a SEPARATE writing booklet. Marks

(a) Consider the function $f(x) = 1 - \cos 2x$.

(i) Sketch the graph of $y = 1 - \cos 2x$ for $0 \le x \le \pi$. 2

(ii) Use the trapezoidal rule with three function values to find an approximation to: 2

$$\int_0^{\frac{\pi}{2}} (1 - \cos 2x)\, dx.$$

(iii) Show by integration that the exact value of $\int_0^{\frac{\pi}{2}} (1 - \cos 2x)\, dx = \frac{\pi}{2}$. 2

Hence explain, with the aid of a diagram, why the use of the trapezoidal rule in (ii) gives the exact value of the integral.

(b) A company buys a yacht for \$250 000, and agrees to pay it off at the same amount each month over 10 years. The interest rate is 9% per annum, reducible monthly.

(i) If the monthly repayments are $\$M$, and $\$A_n$ is the amount owing after n repayments, show that the amount owing (in dollars) after the second repayment is given by: 1

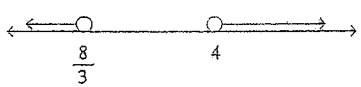
$$A_2 = 250\,000 \times 1.0075^2 - 1.0075M - M.$$

(ii) Hence find the amount of each monthly repayment. 3

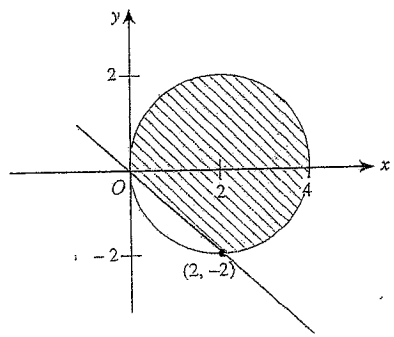
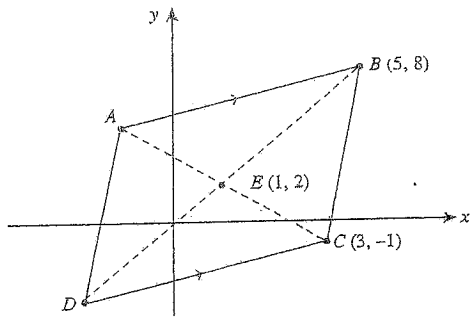
(iii) If the company defaults on their repayments and makes no repayments in the last two years, (that is, no repayments are made after the last repayment of the 8^{th} year), how much does the company then owe at the end of 10 years? Assume there is no bank charge or penalty for the default. 2

End of paper

Question 1

Sample answer	Syllabus outcomes and marking guide
(a) $\frac{\log_e 5.8}{\pi} = 0.55954\ldots$ $= 0.560$ (3 significant figures) must have the zero.	P3 • Gives the correct answer correct to three significant figures. . . . . 2 • Gives the correct answer but not correct to three significant figures 1
(b) $2x^2 - 10x - kx + 5k = 2x(x-5) - k(x-5)$ $= (2x-k)(x-5)$	P3 • Gives the correct answer . . . . . 2 • Takes out one common factor 1
(c) $\int 2 + \frac{1}{x}\,dx = 2x + \log_e x + c$	H5 • Gives a correct answer (c may be omitted). . . . 1
(d) $5x - 3y + 8 = 0 \ldots (1)$ $y = 3x - 4 \ldots (2)$ Substitute (2) into (1) $5x - 3(3x-4) + 8 = 0$ $-4x + 20 = 0$ $x = 5$ Substitute $x = 5$ into (2) $y = 3(5) - 4$ $y = 11$ $\therefore$ solution is $x = 5, y = 11$	P3, P4 • Gives the correct values of both x and y. . . . . 2 • Demonstrates a correct method of solving the simultaneous equations 1
(e) \$US 0.76 = \$A 1.00 \$US 1.00 = \$A $\frac{1.00}{0.76}$ \$US 13.35 = \$A $\frac{13.35}{0.76}$ = \$A 17.57 (nearest cent)	P1, P4, H1 • Gives the correct answer . . . . . 2 • Gives a substantially correct method of calculating the answer 1
(f) $\lvert 3x - 10\rvert > 2$ $3x - 10 < -2$ or $3x - 10 > 2$ $3x < 8$ $\quad$ $3x > 12$ $x < \frac{8}{3}$ or $x > 4$  	P4 • Gives the correct inequality and the correct number line . . . . . 3 • Gives the correct inequality and incorrect number line . . . . . 2 • Gives the correct numerical values of x, but incorrect inequality OR • Uses a correct method to attempt to solve the inequation. 1

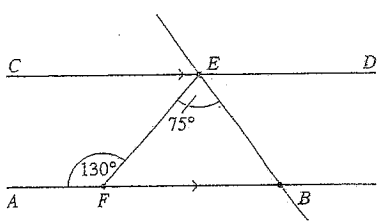
Question 2

Sample answer	Syllabus outcomes and marking guide
(a)  	P4, P5 • Gives the correct graphs and shading . . . 3 • Gives the correct graphs and incorrect shading OR • Gives one correct graph and the correct shading. . . . . 2 • Gives one correct graph OR • Gives the correct shading on incorrect graphs. 1
(b) (i)  $1 = \frac{x+3}{2}$ $\quad$ $2 = \frac{y-1}{2}$ $x = -1$ $\quad$ $y = 5$ Coordinates of A $(-1, 5)$. Alternatively, the midpoint of $AC = \left(\frac{-1+3}{2}, \frac{5-1}{2}\right)$ $= \left(\frac{2}{2}, \frac{4}{2}\right)$ $= (1, 2)$ $= E$ $\therefore A$ is $(-1, 5)$	P3, P4 • Shows a correct method of finding the coordinates of A . . . . . 2 • Demonstrates a correct method of using the midpoint formula to find A 1
(ii) $m_{AB} = \frac{y_B - y_A}{x_B - x_A}$ $= \frac{8-5}{5-(-1)}$ $= \frac{1}{2}$ Gradient of AB is $= \frac{1}{2}$.	P3, P4 • Gives the correct answer 1

Question 2 (Continued)

Sample answer	Syllabus outcomes and marking guide
(iii) $y-y_1=m(x-x_1)$ where $m=\frac{1}{2}$ $y-(-1)=\frac{1}{2}(x-3)$ $(x_1,y_1)=(3,-1)$ $2y+2=x-3$ $x-2y-5=0$ $\therefore$ equation of DC is $x-2y-5=0$.	P3, P4 • Gives the correct substitution into the correct formula and shows expansion ... 2 • Demonstrates a correct method of finding the equation of DC ... 1
(iv) $AB=\sqrt{(x_B-x_A)^2+(y_B-y_A)^2}$ $=\sqrt{6^2+3^2}$ $=\sqrt{45}$ $=3\sqrt{5}$ $\therefore$ length of AB is $3\sqrt{5}$ units.	P3, P4 • Gives the correct answer ... 1
(v) $d=\frac{\lvert ax_1+by_1+c\rvert}{\sqrt{a^2+b^2}}$ $a=1, b=-2, c=-5$ $x_1=-1, y_1=5$ $=\frac{\lvert 1(-1)-2(5)-5\rvert}{\sqrt{1+4}}$ Alternatively, $x_1=5, y_1=8$ etc. $=\frac{16}{\sqrt{5}}$ $\therefore$ perpendicular distance is $\frac{16}{\sqrt{5}}$ units.	P3, P4 • Gives the correct answer ... 2 • Gives the correct substitution into the correct formula ... 1
(vi) $A=b\times h$ $=3\sqrt{5}\times\frac{16}{\sqrt{5}}$ $=48$ $\therefore$ area of $ABCD$ is 48 square units.	P3, P4 • Gives the correct answer ... 1

Question 3

Sample answer	Syllabus outcomes and marking guide
(a) Use $y-y_1=m(x-x_1)$ Now $y=4\sin 2x$ $\frac{dy}{dx}=8\cos 2x$ When $x=\frac{\pi}{8}$ $\frac{dy}{dx}=8\cos\frac{\pi}{4}$ $y=4\sin\frac{\pi}{4}$ $=\frac{8}{\sqrt{2}}$ $=\frac{4}{\sqrt{2}}$ $\therefore y-\frac{4}{\sqrt{2}}=\frac{8}{\sqrt{2}}\left(x-\frac{\pi}{8}\right)$ $\sqrt{2}y-4=8x-\pi$ $8x-\sqrt{2}y+4-\pi=0$ $\therefore$ the equation of the tangent is $8x-\sqrt{2}y+4-\pi=0$ OR $8\sqrt{2}x-2y+4\sqrt{2}-\sqrt{2}\pi=0$	P4, P7, H5 • Gives the correct answer in any form ... 3 • Finds the equation of the tangent, but with a minor error ... 2 • Finds the correct gradient of the tangent at $x=\frac{\pi}{8}$ OR • Demonstrates the correct method of finding the equation of the tangent ... 1
(b) (i) $\frac{d}{dx}(5+e^{3x})^4=4(3e^{3x})(5+e^{3x})^3$ $=12e^{3x}(5+e^{3x})^3$	P7, H5 • Gives the correct answer in any form ... 2 • Uses the function of a function rule or chain rule, with an error ... 1
(ii) $\frac{d}{dx}\left(\frac{\log_e x}{x}\right)=\frac{x\left(\frac{1}{x}\right)-\log_e x(1)}{x^2}$ $=\frac{1-\log_e x}{x^2}$	P7, H5 • Gives the correct answer in any form ... 2 • Uses the quotient rule formula OR • Rewrites the expression with x^{-2} and uses the product rule ... 1
(c)  $\angle DEF=130°$ (alternate angles of $\parallel$ lines AB, CD) $\angle DEB+75°=130°$ (adjacent angles) $\therefore\angle DEB=55°$	P2, H2, H9 • Gives a correct answer with correct reasons ... 2 • Demonstrates a correct use of parallel lines cut by a transversal and angles associated with it ... 1

Question 3 (Continued)

Sample answer	Syllabus outcomes and marking guide
(d) (i) $\int e^{3x-1}dx = \frac{1}{3}e^{3x-1} + c$	H5 • Gives the correct answer, the + c may be omitted 1
(ii) $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} 2\sec^2 x\ dx = \left[2\tan x\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}}$ $= 2\tan\frac{\pi}{3} - 2\tan\frac{\pi}{4}$ $= 2\sqrt{3} - 2$ or $2(\sqrt{3}-1)$	H5 • Gives the correct answer in any form ... 2 • Gives a correct integration OR • Performs the correct working with a minor error 1

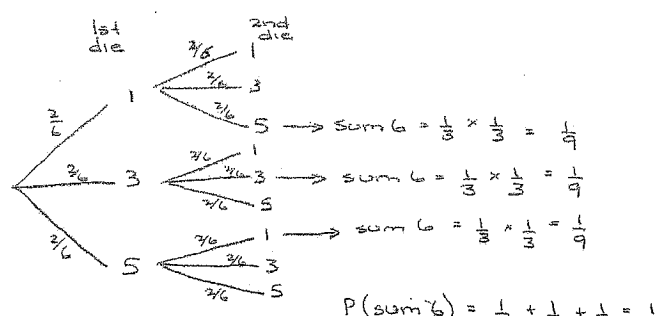
Question 4

Sample answer	Syllabus outcomes and marking guide
(a) $y = 3\sqrt{4-x}$ domain: $4 - x \geq 0$ $4 \geq x$ $x \leq 4$ when $x = 4$ $y = 0$ $\therefore$ range $y \geq 0$ $x = -5$ $y = 9$	P3, P5 • Gives the correct domain and range..... 2 • Gives the correct domain OR • Gives the correct range 1
(b) Die 1 / Die 2 table (see below) P(sum of 6) $= \frac{12}{36}$ or $\frac{1}{3}$	H5 • Gives a correct answer ..... 2 • Uses an appropriate diagram or list to demonstrate an understanding of the question 1

Die 2 \ Die 1	1	1	3	3	5	5
1					×	×
1					×	×
3			×	×		
3			×	×		
5	×	×				
5	×	×				

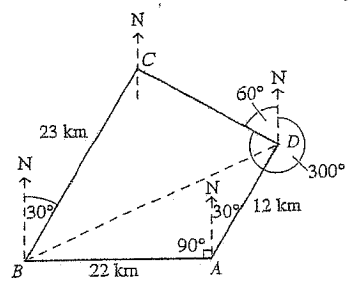
Alternate diagram:

(b)



same scale as above.

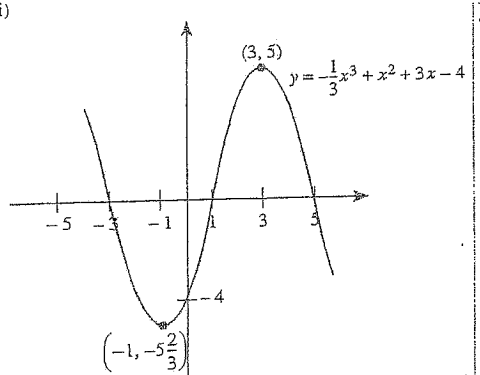
Question 4 (Continued)

Sample answer	Syllabus outcomes and marking guide
(c) (i) 	P4, H4, H5 • Gives the correct bearings and lengths on correct diagram 1
(ii) $\angle BAD = 90° + 30°$ (from diagram) $= 120°$ $\angle BCD = 30° + 60°$ (using alternate angles and north-south line through C) $= 90°$	P4, H4, H5 • Establishes the correct reasons for the size of the angles 1
(iii) $BD^2 = AB^2 + AD^2 - 2(AB)(AD)\cos A$ $= 22^2 + 12^2 - 2(22)(12)\cos 120°$ $= 892$ $BD = 29.9$ km (correct to 1 decimal place)	P4, H4, H5 • Gives the correct answer . . . . . 2 • Gives the correct substitution into the correct formula. 1
(iv) $DC^2 = BD^2 - BC^2$ $= 892 - 23^2$ $= 363$ or 365 (from rounded BD) $DC = 19.1$ km (1 decimal place) $ABCD$ is a trapezium, since $BC \parallel AD$ Area $(ABCD) = \frac{1}{2}(BC + AD)CD$ $= \frac{1}{2}(23 + 21)(19.1)$ $= 334.25$ $= 334$ km^2 (nearest km^2) Alternative Solution: Area $(ABCD) = \frac{1}{2}(AB)(AD)\sin 120° + \frac{1}{2}(BC)(CD)$ $= \frac{1}{2}(22)(12)\sin 120° + \frac{1}{2}(23)(19.1)$ $= 114.315 + 219.65$ $= 334$ km^2 (nearest km^2)	P4, H4, H5 • Gives the correct answer . . . . . 3 • Makes significant progress to establish the correct answer . . . . . 2 • Makes partial progress to establish the correct answer OR • Finds the value of DC 1

Question 5

Sample answer	Syllabus outcomes and marking guide
(a) (i) $x^2 + 5x - 2 = 0$ $\alpha + \beta = \frac{-b}{a}$ $\therefore \alpha + \beta = -5$	P4 • Gives the correct answer 1
(ii) $\alpha\beta = \frac{c}{a}$ $\therefore \alpha\beta = -2$	P4 • Gives the correct answer 1
(iii) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$ $= (-5)^2 - 2(-2)$ $= 25 + 4$ $\therefore \alpha^2 + \beta^2 = 29$ Alternative solution. $\alpha^2 + 5\alpha - 2 = 0$ $\beta^2 + 5\beta - 2 = 0$ $\alpha^2 + \beta^2 + 5(\alpha + \beta) - 4 = 0$ $\alpha^2 + \beta^2 = -5(-5) + 4$ $\therefore \alpha^2 + \beta^2 = 29$	P4 • Gives the correct answer 1
(b) (i) For an AP $d = T_2 - T_1$ $\quad$ $d = T_3 - T_2$ $= \log 6 - \log 3$ $\quad$ $= \log 12 - \log 6$ $= \log\frac{6}{3}$ $\quad$ $= \log\frac{12}{6}$ $= \log 2$ $\quad$ $= \log 2$ Since d is common, $\therefore$ the series is arithmetic.	H3 • Show both values of d are log2 1
(ii) $T_9 = a + 8d$ $= \log 3 + 8\log 2$ $= \log 3 + \log 256$ $= \log 768$	H3 • Gives the correct answer . . . . . 2 • Gives the correct substitution of a and d into the correct formula. 1

Question 5 (Continued)

Sample answer	Syllabus outcomes and marking guide
(c) (i) $f(x) = -\frac{1}{3}x^3 + x^2 + 3x - 4$ $f'(x) = -x^2 + 2x + 3$ $= -(x^2 - 2x - 3)$	H6 • Gives the first line of the derivative 1
(ii) Stationary points are when $f'(x) = 0$. $-(x^2 - 2x - 3) = 0$ $(x-3)(x+1) = 0$ $x = 3$ or $x = -1$ When $x = 3$, $y = 5$ When $x = -1$, $y = -5\frac{2}{3}$ Now $f''(x) = -2x + 2$ $f''(3) = -4 < 0$ (concave down) $\therefore (3, 5)$ is a maximum turning point $f''(-1) = 4 > 0$ (concave up) $\therefore \left(-1, -5\frac{2}{3}\right)$ is a minimum turning point	H6 • Gives the correct stationary points with correct working for their nature . . . . . 3 • Gives the correct stationary points and states their nature without verifying them OR • Gives the correct x-value only for the stationary points and correctly determines their nature . . . . . 2 • Gives the correct x-values OR • Gives one correct stationary point 1
(iii) 	P3, P5, H6, H9 • Gives correct graph with correct labelling (x intercepts not required) 1
(iv) $f''(x) = -2x + 2$ $f''(x) = 0$ for point of inflexion $x = 1$ is the x coordinate of point of inflexion (see graph for change in concavity). Concave down for $x > 1$.	H6 • Gives the correct values of x 1

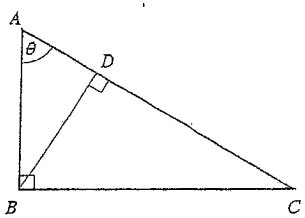
Question 6

Sample answer	P4, H5
(a) $\sin 2x + \cos x = 1$ Let $\sin^2 x = 1 - \cos^2 x$ $1 - \cos^2 x + \cos x = 1$ $0 = \cos^2 x - \cos x$ $0 = \cos x(\cos x - 1)$ $\cos x = 0$ $\quad$ $\cos x - 1 = 0$ $\cos x = 1$ $x = \frac{\pi}{2}, \frac{3\pi}{2}$ $\quad$ $x = 0, 2\pi$ $\therefore x = 0, \frac{\pi}{2}, \frac{3\pi}{2}, 2\pi$	• Gives the correct solutions 4 • Correctly determines the two trig equations for $\cos x$ and at least 1 correct solution for each • Gives correct linear equations for $\cos x$ 2 • Follows through 1 correct trig equation with both solutions for it. 2 • Substitutes $\sin^2 x$ with $1 - \cos^2 x$. 1 • Correct quadratic equation 1
(b) (i) $y = x^2 - x$ and $y = x$ $x = x^2 - x$ $x^2 - 2x = 0$ $x(x-2) = 0$ $\therefore x = 0$ or $x = 2$ When $x = 0$, $y = 0$ When $x = 2$, $y = 2$ $\therefore$ graphs intersect at (2, 2). OR, when $x = 2$, $y = 2^2 - 2 = 2$ and $y = 2$ $\therefore$ graphs intersect at (2, 2).	P4 • Demonstrates a correct method of finding the point of intersection 1
(ii) $A = \int_0^2 (y_{\text{upper}} - y_{\text{lower}})\,dx$ $= \int_0^2 [x - (x^2 - x)]\,dx$ $= \int_0^2 [2x - x^2]\,dx$ $= \left[x^2 - \frac{1}{3}x^3\right]_0^2$ $= 4 - \frac{8}{3}$ $= 1\frac{1}{3}$ $\therefore$ area is $1\frac{1}{3}$ units^2.	H8 • Gives the correct answer . . . . . 3 • Gives substantial progress to find the area. . . . . 2 • Gives partial progress to find the area . . . 1

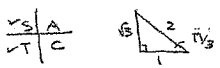
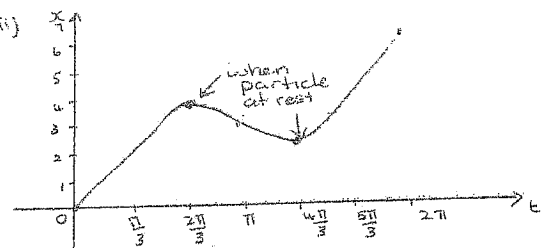
Question 6 (Continued) Sample answer	Syllabus outcomes and marking guide
(c) (i) From $l = r\theta$ $\theta = \frac{l}{r}$ $= \frac{100}{40}$ $= 2.5$ radians $= \frac{2.5 \times 180}{\pi}$ degrees $= 143.239\ldots$ degrees $= 143°$ (to nearest degree)	H4, H5 • Gives the correct answer (disregard rounding) ..... 2 • Gives the correct angle in radians OR Gives the correct conversion of their radian measure to degrees 1
(ii) $A = \frac{1}{2}r^2(\theta - \sin\theta)$ $= \frac{1}{2}(40)^2\left[\frac{55\pi}{180} - \sin\frac{55\pi}{180}\right]$ $= 800(0.140779\ldots)$ $= 112.623\ldots$ $\therefore$ area of segment is 112.6 cm^2 (1 decimal place).	H4, H5 • Gives a correct answer (disregard rounding) ..... 2 • Gives correct substitution into a correct formula 1

Question 7 Sample answer	Syllabus outcomes and marking guide
(a) $\frac{1+\cot\theta}{\operatorname{cosec}\theta} = \left[\frac{\sin\theta}{\sin\theta} + \frac{\cos\theta}{\sin\theta}\right] \div \frac{1}{\sin\theta}$ $= \sin\theta + \cos\theta$ $\frac{\sec\theta}{\tan\theta+\cot\theta} = \frac{1}{\cos\theta} \div \left[\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right]$ $= \frac{1}{\cos\theta} \div \frac{\sin^2\theta+\cos^2\theta}{\cos\theta\sin\theta}$ $= \frac{\cos\theta\sin\theta}{\cos\theta}$ LHS = RHS	P4 • Uses correct substitutions to prove LHS = RHS • Correctly evaluates $\frac{1+\cot\theta}{\operatorname{cosec}\theta}$ or $\frac{\sec\theta}{\tan\theta+\cot\theta}$ • Eliminates all reciprocal ratios • Finds a common denominator 2

Question 7 (Continued)

Sample answer	Syllabus outcomes and marking guide
(b) (i)  Let $\angle BAD = \theta$ $\therefore \angle ABD = 90° - \theta$ (angle sum of ΔABD) $\therefore \angle DBC = \theta$ (complementary angles) In ΔABD and ΔBDC $\angle BAD = \angle CBD$ (both θ) $\angle ADB = \angle BDC$ (both 90° given) $\therefore \Delta ABD \parallel\parallel \Delta CDB$ (two angles equal)	H2, H9 • Gives a correct proof with reasons. . . . . 2 • Gives a correct proof without reasons OR • Gives a substantially correct proof with reasons 1
(ii) From the similar triangles $\frac{BD}{AD} = \frac{CD}{BD}$ $BD^2 = AD \times CD$ $= 4 \times 9$ $= 36$ $\therefore BD = 6$ $\therefore$ the length of BD is 6 cm.	P4, H2 • Gives the correct answer . . . . . 2 • Gives a correct ratio of appropriate sides OR • Demonstrates substantial progress and understanding in finding BD 1

Question 7 (Continued)

Sample answer	Syllabus outcomes and marking guide
(c) $x = t + 2\sin t \quad 0 \le t \le 2\pi$	
i) $v = \frac{dx}{dt} = 1 + 2\cos t$	• correct velocity 1
ii) at rest when $v = 0$ $0 = 1 + 2\cos t$ $\cos t = -\frac{1}{2}$  Acute $\angle = \pi/3$ $t = \pi - \pi/3, \ \pi + \pi/3$ $= \frac{2\pi}{3}, \frac{4\pi}{3}$	• correct times when velocity = 0 . . . . . 2 • correct statement of velocity = 0, ie $1 + 2\cos t = 0$ • 1 correct solution only 1
(iii) 	• correct graph showing 2 stationary pts 2 • correct shape, no labelling OR • stationary pts correct, errors on the graph OR • correct start at zero, labelled axes 1

x	0	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	2π
t	0	3.8	3.14	2.4	3.5	6.3

iv) the stationary points are when $v = 0$ ie gradient = 0	• correct solution, must be labelled on graph
v) From 0 to $\frac{2\pi}{3}$ seconds particle moves away from 0. Stops at $\frac{2\pi}{3}$, reverses direction, moves towards zero at $t = \pi$ $x = \pi$ metres.	• correct statement of motion.

Question 8

Sample answer	Syllabus outcomes and marking guide
(a) (i) $6y = 5 - 8x - x^2$ $x^2 + 8x + 16 = -6y + 5 + 16$ $(x+4)^2 = -6\left(y - \frac{7}{2}\right)$ $\therefore$ vertex is $\left(-4, \frac{7}{2}\right)$	P4 • Gives the correct answer . . . . . 2 • Uses an appropriate method to find the vertex 1
(ii) Focal length is $\frac{6}{4}$ or $\frac{3}{2}$. Parabola is concave down. Focus $x = -4$, $y = \frac{7}{2} - \frac{3}{2} = 2$ $\therefore$ the focus is $(-4, 2)$.	P4 • Gives the correct answer OR • Gives a correct answer from their working in (i) . . . . . 2 • Finds a correct focal length from their (i) and a correct concavity OR • Substantially correct working to find the focus using the focal length 1
(b) When $x = 0$, $y = 0$ or $y = 1$ $V = \pi \int_0^1 x^2 \, dy$ $= \pi \int_0^1 (y - y^2)^2 \, dy$ $= \pi \int_0^1 (y^2 - 2y^3 + y^4) \, dy$ $= \pi \left[\frac{1}{3}y^3 - \frac{1}{2}y^4 + \frac{1}{5}y^5\right]_0^1$ $= \pi \left[\frac{1}{3} - \frac{1}{2} + \frac{1}{5}\right]$ $= \frac{\pi}{30}$ $\therefore$ the volume is $\frac{\pi}{30}$ units^3 [or 0.105 units^3 (3 decimal places)]	H8 • Gives the correct answer . . . . . 4 • Demonstrates a correct solution with a minor error . . . . . 3 • Integrates correctly from a correct volume integral . . . . . 2 • Gives a substantially correct expression to be integrated. 1

Question 8 (Continued)

Sample answer	Syllabus outcomes and marking guide
(c) $y = x + k$ $2x^2 + y^2 = 6$ $2x^2 + (x+k)^2 = 6$ $2x^2 + x^2 + 2xk + k^2 = 6$ $3x^2 + 2kx + (k^2 - 6) = 0$ Two solutions if $\Delta > 0$. $\therefore b^2 - 4ac > 0$ $(2k)^2 - 4(3)(k^2 - 6) > 0$ $4k^2 - 12k^2 + 72 > 0$ $8k^2 - 72 < 0$ $k^2 < 9$ $-3 < k < 3$ $\therefore$ there are two solutions if $-3 < k < 3$	P4, H2 • Gives a correct answer . . . . . 4 • Gives a correct quadratic inequality in terms of k . . . . . 3 • Gives a correct quadratic equation in terms of x and states $\Delta > 0$ OR • Demonstrates a clear understanding of the solution with minor errors . . . . . 2 • Gives a correct quadratic equation in terms of x OR • Demonstrates a reasonable understanding of the solution. 1

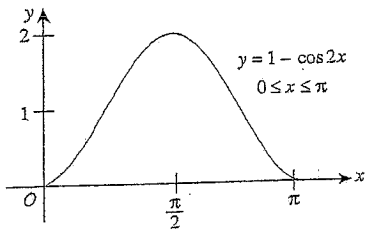
Question 9

Sample answer	Syllabus outcomes and marking guide
(a) (i) G.P. has a limiting sum if $-1 < r < 1$ $r = \frac{T_2}{T_1} = \frac{2}{\left(\frac{6}{\sqrt{3}}\right)}$ $r = \frac{\sqrt{3}}{3}$ or $\frac{1}{\sqrt{3}} = 0.577\ldots$ $\therefore$ the series has a limiting sum.	H5 • Finds the correct value of r and states $-1 < r < 1$ or equivalent 1
(ii) $S_\infty = \frac{a}{1-r}$ $= \frac{\frac{6}{\sqrt{3}}}{1-\frac{1}{\sqrt{3}}}$ $= \frac{\frac{6}{\sqrt{3}}}{\frac{\sqrt{3}-1}{\sqrt{3}}}$ $= \frac{6}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}-1}$ $= \frac{6}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$ $= \frac{6(\sqrt{3}+1)}{3-1}$ $\therefore S_\infty = 3(\sqrt{3}+1)$	H5 • Gives a correct answer with a rational denominator for their value of r ..... 2 • Demonstrates the correct use of 'sum to infinity' formula. 1
(b) (i) Profit = selling price – cost price $P(x) = 70x - (0.04x^2 + 30x + 4000)$ $= 40x - 4000 - 0.04x^2$	P4, H4 • Gives a correct explanation of profit and a substitution into $P = s - c$ 1
(ii) $\frac{dP}{dx} = 40 - 0.08x$ When $\frac{dP}{dx} = 0$, $x = \frac{40}{0.08} = 500$ $\frac{d^2P}{dx^2} = -0.08 < 0$ $\therefore$ concave down $\therefore$ maximum $\therefore$ profit is maximum when 500 stools are produced.	H4, H5 • Gives the correct answer and justifies the answer ..... 2 • Gives an answer of 500 without justification OR • Gives a correct method with a minor error 1
(iii) $P(500) = 40 \times 500 - 4000 - 0.04 \times 500^2$ $= 6000$ $\therefore$ maximum profit is \$6000 per week.	P4, H5 • Gives a correct answer for their value of x 1

Question 9 (Continued)

Sample answer	Syllabus outcomes and marking guide
(c) (i) $C = Ae^{-kt}$ $\frac{dC}{dt} = -kAe^{-kt}$ $= -kC$ $\therefore C = Ae^{-kt}$ satisfies $\frac{dC}{dt} = -kC$.	H3, H5 • Shows the correct reasoning 1
(ii) $C = Ae^{-kt}$ When $t = 0$, $C = A$ When $t = 4$, $C = \frac{1}{2}A$ $\therefore \frac{1}{2}A = Ae^{-4k}$ $e^{-4k} = \frac{1}{2}$ $e^{4k} = 2$ $4k = \log_e 2$ $k = \frac{1}{4}\log_e 2$ $= 0.173286\ldots$ $k = 0.1733$ (4 decimal places)	H3, H4 • Shows the correct working to obtain the correct value of k (disregard rounding) .. 2 • Gives the correct substitution into the correct formula. 1
(iii) $40 \times 90 = 3600$ mg When $t = 1$, requires $C = 3600$ $\therefore 3600 = Ae^{-0.1733}$ $A = 3600 \times e^{0.1733}$ $= 4281.202..$ $\therefore$ the initial dose is 4281 or 4282 mg.	H3, H4 • Gives an answer from 4281 to 4282 inclusive (disregard rounding) ..... 2 • Gives a correct substitution into the correct formula to find A 1

Question 10

Sample answer	Syllabus outcomes and marking guide
(a) (i) 	P5, H5, H9 • Correctly shows the position and shape of the graph and correctly labels the axes . . 2 • Correctly shows position and shape of graph OR • Substantially gives the correct shape and correct label of axes 1
(ii)	H5, H8 • Gives a correct approximation (disregard rounding) . . . . . 2 • Demonstrates a correct use of trapezoidal rule with a minor arithmetic error 1

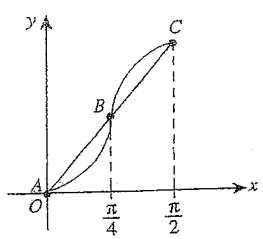
$x =$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$
$y =$	0	1	2

$$\int_0^{\frac{\pi}{2}} (1 - \cos 2x)\,dx \doteqdot \frac{h}{2}[y_0 + 2y_1 + y_2]$$

$$\doteqdot \frac{\pi}{8}[0 + 2(1) + 2]$$

$$\doteqdot \frac{\pi}{2}$$

$$\doteqdot 1.57$$

(iii) $\int_0^{\frac{\pi}{2}} (1 - \cos 2x)\,dx = \left[x - \frac{1}{2}\sin 2x\right]_0^{\frac{\pi}{2}}$ $= \left[\frac{\pi}{2} - \frac{1}{2}\sin\pi\right] - \left[0 - \frac{1}{2}\sin 0\right]$ $= \frac{\pi}{2}$	H5, H9 • Shows the integral with substitution and gives an appropriate explanation why this set of values for trapezoidal rule gives an exact value . . . . . 2 • Shows the integration correctly OR • Gives a correct explanation why (ii) gives an exact value . . . . .
	
The curve has point symmetry at $x = \frac{\pi}{4}$ (point B). Thus the area above the curve and below the line AB is equal to the area below the curve and above the line BC.	

Question 10 (Continued)

Sample answer	Syllabus outcomes and marking guide
(b) (i) Interest rate = 9% p.a. = $\frac{9}{12}\%$ per month $= 0.0075$ Amount owing after n months (A_n): $A_1 = 250\,000 \times 1.0075 - M$ $A_2 = A_1 \times 1.0075 - M$ $= (250\,000 \times 1.0075 - M) \times 1.0075 - M$ $= 250\,000 \times 1.0075^2 - 1.0075M - M$	H4, H5, H9 • Shows $A_2 = A_1 \times 1.0075 - M$ or equivalent. 1
(ii) $A_n = 250\,000 \times 1.0075^n - M(1 + 1.0075 + 1.0075^2 + \ldots)$ $= 250\,000 \times 1.0075^n - \frac{M(1.0075^n - 1)}{1.0075 - 1}$ Given $A_{120} = 0$ $\frac{M(1.0075^{120} - 1)}{1.0075 - 1} = 250\,000 \times 1.0075^{120}$ $M = \frac{250\,000 \times 1.0075^{120} \times 0.0075}{1.0075^{120} - 1}$ $M = 3166.89$ $\therefore$ the monthly payments are \$3166.89	H4, H5, H9 • Gives the correct answer (disregard rounding) . . . . . 3 • Gives a correct expression to find M . . . . 2 • Gives a correct expression for A_n, using the sum of a geometric series. 1
(iii) Amount owing at the end of 8 years $A_{96} = 250\,000 \times 1.0075^{96} - \frac{3166.89(1.0075^{96} - 1)}{0.0075}$ $= \$69\,321.22$ Amount owing after 2 more years $A = 69\,321.22(1.0075)^{24}$ $= 82\,936.846\ldots$ $\therefore$ at the end of 10 years the company owes \$82 936.85	H4, H5, H9 • Gives the correct answer from their value of M . . . . . 2 • Gives the correct amount owing after 8 years from their value of M 1