WESTERN REGION

2001
TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 1

General Instructions

- Reading Time- 5 minutes
- Working Time – 2 hours
- Write using a blue or black pen
- Approved calculators may be used
- A table of standard integrals is provided at the back of this paper.
- All necessary working should be shown for every question.
- Begin each question on a fresh sheet of paper.

Total marks (84)

- Attempt Questions 1-7
- All questions are of equal value

Total marks (84)
Attempt Questions 1 – 7
All questions are of equal value

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

Marks

Question 1 (12 marks) Use a SEPARATE writing booklet.

(a) Differentiate $x^2 \cos^{-1} x$ **2**

(b) $x - 3$ divides $x^3 - 3x^2 + px - 14$ with a remainder of 1. **2**

Find the value of p.

(c) Solve the simultaneous equations:- **3**

$|x - 3| < 4$

$|x - 1| > 1$

(d) The point P(5,7) divides the interval joining the points A(-1,1) and B(3,5) <u>externally</u> in the ratio k : 1. **2**

Find the value of k.

(e) (i) Write $x^2 + 6x + 13$ in the form $(ax + b)^2 + c$ **2**

(ii) Hence find **1**

$$\int \frac{dx}{x^2 + 6x + 13}$$

Marks

Question 2 (12 marks) Use a SEPARATE writing booklet.

(a) Find the acute angle, to the nearest minute, between the curve $y = x^2$ and the line $5x - y - 6 = 0$ at the point of intersection $(3,9)$. **2**

(b) (i) Show that the equation $e^x = x + 2$ has a solution in the interval $1 < x < 2$. **2**

(ii) Letting $x_1 = 1.5$, use one application of Newton's Method to approximate that solution, correct to 3 decimal places. **2**

(c) Evaluate $\lim_{x \to 0} \frac{\sin 3x}{5x}$ **1**

(d) Find the maximum value of $3\cos x - 2\sin x$ **2**

(e) Use the substitution $x = \ln u$ to find **3**

$$\int \frac{e^x}{\sqrt{1 - e^{2x}}}\,dx$$

Marks

Question 3 (12 marks) Use a SEPARATE writing booklet.

(a) Show that $\sin^{-1} x$ is an odd function. **2**

(b) Use the method of Mathematical Induction to prove that **3**

$$9^{n+2} - 4^n$$

is divisible by 5, for all positive integers, n.

(c) (i) Using $t = \tan x/2$ **1**
write expressions for $\sin x$ and $\cos x$ in terms of t.

(ii) Hence, or otherwise, solve **3**

$$3\cos x + 5\sin x = 5 \qquad 0 < x < 360^\circ$$

to the nearest degree.

(d) Using the identity **3**

$$(1 + x)^{2n} = (1 + x)^n (1 + x)^n$$

and considering coefficients of x^n show that

$$^{10}C_5 = (^5C_0)^2 + (^5C_1)^2 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots + (^5C_5)^2$$

Marks

Question 4 (12 marks) Use a SEPARATE writing booklet.

(a) By expressing $\cos^2 x$ in terms of $\cos 2x$ find the primitive of $\cos^2 x$. **2**

(b) An 8 person committee is to be formed from a group of 10 women and 15 men.

In how many ways can the committee be chosen if :-

(i) the committee must contain 4 men and 4 women. **1**

(ii) there must be more women than men. **2**

(iii) there must be at least 2 women, **2**

(c) (i) Sketch the function **1**

$$f(x) = |x - 1|$$

over its natural domain.

(ii) Explain why $f(x)$ does not have an inverse over this domain. **1**

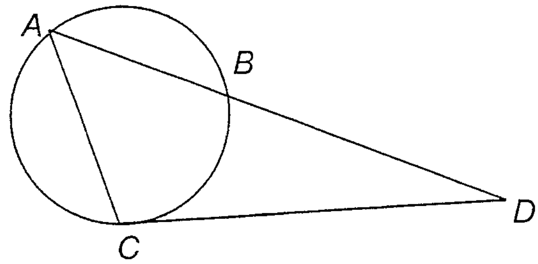
(iii) If $f_1(x)$ is the restriction of $f(x)$ to the domain $x \geqslant 1$ find $f_1^{-1}(x)$, stating its domain and range. **2**

(iv) What is $f_2^{-1}(x)$ if the domain of $f(x)$ is restricted to $x < 1$? **1**

Marks

Question 5 (12 marks) Use a SEPARATE writing booklet.

(a)

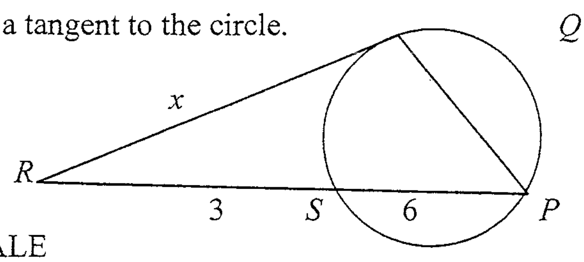


(i) Copy the diagram above and prove that $\Delta BCD \parallel\!| \Delta CAD$. **3**

(ii) Hence prove that $CD = \sqrt{BD.AD}$ **2**

(iii) Use this result to find the value of x in the diagram below. **1**

QR is a tangent to the circle.



NOT TO SCALE

(b) When a particle is x metres from the origin, its velocity, v ms^{-1}, is given by **3**

$$v = \sqrt{8 - 2x^2}$$

Find the acceleration when the particle is 2 metres from the origin.

Question 5 is continued on the next page.

Question 5 (continued)

(c) 30 girls, including Miss Australia, enter a Miss World competition.

The first 6 places are announced.

(i) How many different announcements are possible? **1**

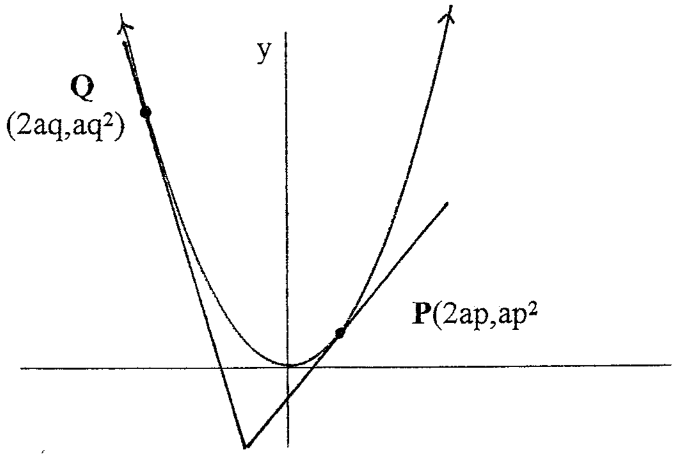
(ii) How many different announcements are possible if Miss Australia is assured of a place in the first 6 ? **2**

End of Question 5

Marks

Question 6 (12 marks) Use a SEPARATE writing booklet.

(a)



The points $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ lie on the parabola $x^2 = 4ay$.

(i) Show that the equation of the tangent at P is given by **2**

$$y = px - ap^2$$

(ii) If the tangent at P and the tangent at Q intersect at $45°$ show that **1**

$$|p - q| = |1 + pq|$$

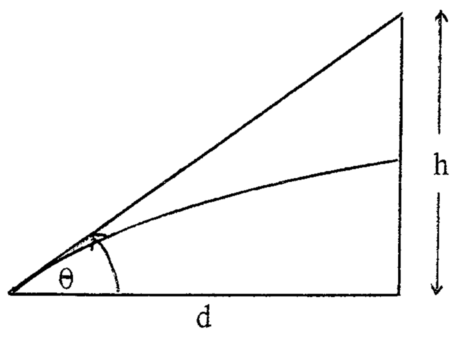
(iii) If $q = 2$ find p, using the result above. **2**

Question 6 continues on the next page

Marks

Question 6 (continued)

(b)



A target is hung on a wall at a height of h metres.

A small cannon, which fires a lead slug, is located on the floor, d metres from the wall.

The muzzle velocity, V, of the cannon is adjustable.

The cannon is aimed at the bulls-eye on the target, at an angle of elevation of θ degrees.

At the instant the cannon is fired the target is released and falls vertically downwards under the force of gravity, g.

Given that $\ddot{x} = 0$ and $\ddot{y} = -g$

(i) Show that after time t **2**

$$x = tV\cos\theta \quad \text{and} \quad y = \frac{-gt^2}{2} + tV\sin\theta$$

(ii) Show that the slug hits the wall at a vertical height of **2**

$$H = \frac{-g\,d^2\sec^2\theta}{2V^2} + d\tan\theta$$

(iii) Experiments with the cannon show that the slug always hits the bulls-eye regardless of the muzzle velocity.

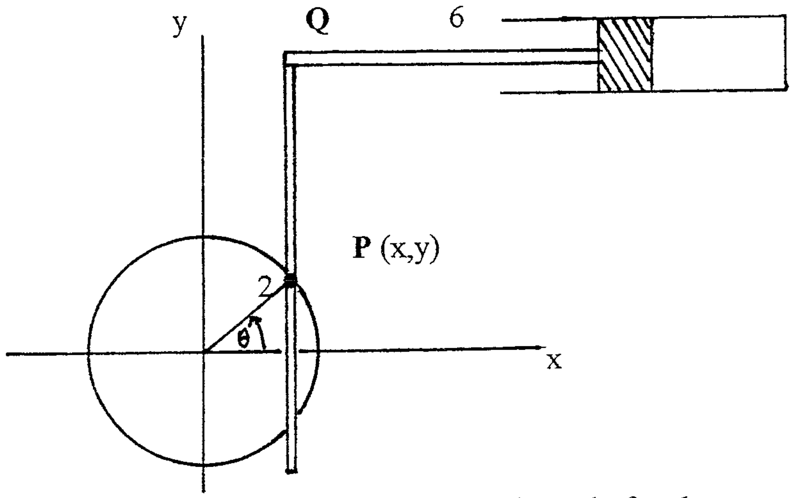
Explain why this is always so. **3**

End of Question 6

Marks

Question 7 (12 marks) Use a SEPARATE writing booklet.

(a)



A piston moves back and forth on the end of a 6metre shaft.

The other end is attached at Q to a vertical slotted arm fitted to a peg P on the rim of a wheel of radius 2metres.

Suppose the wheel begins with point P at $\theta = \frac{\pi}{4}$

when $t = 0$ and rotates anticlockwise at 5 radians per second.

(i) Show that $\theta = 5t + \frac{\pi}{4}$ **1**

(ii) Hence find an expression for x as a function of t and show that the motion of the piston is simple harmonic. **2**

(iii) State the amplitude and period of the motion. **2**

(iv) Find the initial velocity of the piston. **1**

Question 7 continues on the next page

Marks

Question 7 (continued)

(b) A water tank is generated by rotating the curve

$$y = \frac{x^4}{16}$$

around the y - axis.

(i) Show that the volume of water , V as a function of its depth h, is given by: **2**

$$V = \frac{8}{3}\pi . h^{\frac{3}{2}}$$

(ii) Water drains from the tank through a small hole at the bottom. **4**

The rate of change of the volume of water in the tank is proportional to the square root of the water's depth.

Use this fact to show that the water level in the tank falls at a constant rate.

End of paper

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}}\,dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}}\,dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

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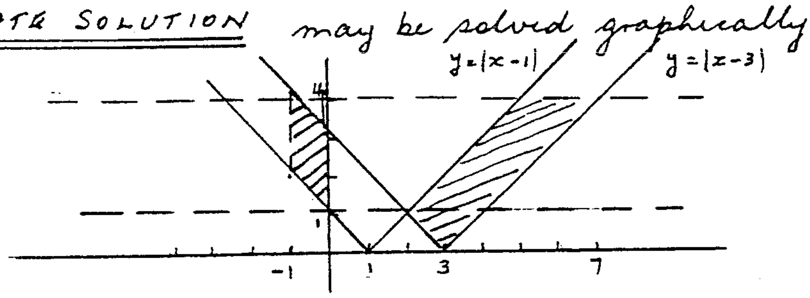
Course: MATHEMATICS EXTENSION 1 **Page No.** 1

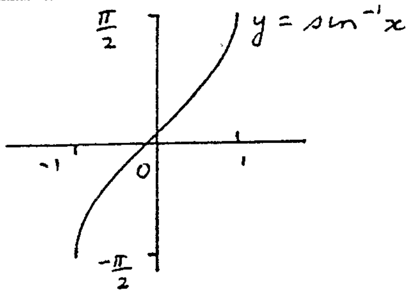
Solutions	Marks	Comments
QUESTION 1		
(a) $\frac{d}{dx} x^2 \cos^{-1} x = \cos^{-1} x \cdot 2x + x^2 \cdot \frac{-1}{\sqrt{1-x^2}}$ $= 2x \cos^{-1} x - \frac{x^2}{\sqrt{1-x^2}}$	2	1 mark for product rule 1 mark for correct $\frac{d}{dx} \cos^{-1}$
(b) remainder of 1 $\Rightarrow P(3) = 1$ $\Rightarrow 27 - 27 + 3p - 14 = 1$ $\Rightarrow p = 5$	1 1	
(c) $\lvert x-3 \rvert < 4 \Rightarrow -4 < x-3 < 4$ $\Rightarrow -1 < x < 7$ $\lvert x-1 \rvert > 1 \Rightarrow x-1 < -1$ OR $x-1 > 1$ $\Rightarrow x < 0$ OR $x > 2$ $\therefore$ solution is $\{x : -1 < x < 0\} \cup \{x : 2 < x < 7\}$	1 1 1	SET NOTATION NOT ASSESSED
ALTERNATE SOLUTION may be solved graphically  		1 MARK for each graph 1 mark solution
(d) $(m,n) = (k,1)$ $P(5,7)$ $A(-1,1)$ $B(3,5)$ $5 = \frac{1.(-1) + k.3}{k+1}$ $\Rightarrow 5k+5 = 3k-1$ $\Rightarrow k = -3$ since we are dividing externally ignore the sign $\therefore k = 3$ ALTERNATIVELY $(m,n) = (k,-1) \Rightarrow 5 = \frac{(-1)(-1) + 3k}{k+1} \Rightarrow k = 3$	1 1	
(e) (i) completing the square $x^2 + 6x + 13 = (x+3)^2 - 9 + 13$ $= (x+3)^2 + 4$ (ii) $\int \frac{dx}{x^2+6x+13} = \int \frac{dx}{(x+3)^2 + 2^2}$ using the standard integrals $= \frac{1}{2} \tan^{-1}\left(\frac{x+3}{2}\right) + C$	1 1 1	

TOTAL 12

Solutions	Marks	Comments
QUESTION 2		
(a) $y = x^2 \Rightarrow y' = 2x$ gradient of $5x - y - 6 = 0$, $m_2 = 5$ at $x = 3$, $y' = 6$ $\tan\theta = \left\lvert \frac{m_1 - m_2}{1 + m_1 m_2} \right\rvert$ $= \left\lvert \frac{6-5}{1+30} \right\rvert$ $= \frac{1}{31}$ ie $\theta = 1^\circ 51'$ (to nearest minute)	1 1	
(b) (i) let $f(x) = e^x - x - 2$ $f(1) = e - 3$ $= -0.282$ $f(2) = e^2 - 4$ $= 3.389$ Since $f(x)$ changes from negative to positive in the interval $(1,2)$ there is a value of $x : f(x) = 0$ in this interval (ii) $f(x) = e^x - x - 2$ $f'(x) = e^x - 1$ let $x_1 = 1.5$ $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$ $= 1.5 - \frac{e^{1.5} - 1.5 - 2}{e^{1.5} - 1}$ $\doteqdot 1.218$ (3 dp)	1 1 1 1	
(c) $\lim_{x \to 0} \frac{\sin 3x}{5x} = \frac{3}{5} \lim_{x \to 0} \frac{\sin 3x}{3x}$ $= \frac{3}{5}$ since $\lim_{x \to 0} \frac{\sin x}{x} = 1$	1 OR 1	← FOR CORRECT ANSWER
(d) $3\cos x - 2\sin x = \sqrt{13}\cos(x - \alpha)$ $= \sqrt{13}$ (since max value of $\cos(x-\alpha)$ is 1) ALTERNATE SOLUTION by calculus $y' = -3\sin x - 2\cos x$ $= 0$ $\Rightarrow \tan x = -2/3$ $\Rightarrow x = 146^\circ 19'$ OR $326^\circ 31'$ ← by substitution max value $= 3.61$	1 1 OR 1 1	

Solutions	Marks	Comments
ESTION 2 (continued)		
) $x = \ln u \Rightarrow u = e^x$	1	
$dx = \frac{1}{u} \cdot du$		
$\int \frac{e^x}{\sqrt{1-e^{2x}}}\,dx = \int \frac{u}{\sqrt{1-u^2}} \cdot \frac{1}{u}\,du$	1	
$= \int \frac{du}{\sqrt{1-u^2}}$		
$= \sin^{-1} u + c$		
$= \sin^{-1}(e^x) + c$	1	
	OR	
ALTERNATIVELY		
$\int \frac{e^x}{\sqrt{1-e^{2x}}}\,dx = \int \frac{e^{\ln u}}{\sqrt{1-e^{2\ln u}}} \times \frac{du}{u}$	1	
$= \int \frac{u}{\sqrt{1-e^{\ln u^2}}} \times \frac{du}{u}$		
$= \int \frac{u}{\sqrt{1-u^2}} \cdot \frac{du}{u}$		
$= \int \frac{du}{\sqrt{1-u^2}}$	1	
$= \sin^{-1} u + c$		
$= \sin^{-1}(e^x) + c$	1	

TOTAL 12

Solutions	Marks	Comments
QUESTION 3		
(a) graphically  an odd function since graph has point symmetry about O	1	(graph)
	1	(statement)
	OR	
ALTERNATIVE algebraically		
let $y = \sin^{-1}(-x)$		
$\Rightarrow -x = \sin(y)$		
$\Rightarrow x = \sin(-y)$	1	
$\Rightarrow -y = \sin^{-1}(x)$		
$\Rightarrow y = -\sin^{-1}(x)$	1	
ie $\sin^{-1}(-x) = -\sin^{-1}(x)$		
$\therefore$ ODD		
(b) The statement is true for $n=1$ since $9^3 - 4 = 725$ is divisible by 5.	1	
Assume the statement true for some integer, k		
ie let $9^{k+2} - 4^k = 5J$		
SHOW 5 divides $9^{(k+1)+2} - 4^{k+1}$		
$9^{(k+1)+2} - 4^{k+1} = 9.9^{k+2} - 4.4^k$	1	
$= 4(9^{k+2} - 4^k) + 5.9^{k+2}$		
$= 4.5J + 5.9^{k+2}$		
$= 5(4J + 9^{k+2})$	1	
$\therefore$ statement is true for $k+1$ whenever true for k. Since statement is true for $n=1$, it is true for $n = 2, 3, \ldots$ & hence all positive integers, n		
(c) (i) $\sin x = \dfrac{2t}{1+t^2}$ $\quad \cos x = \dfrac{1-t^2}{1+t^2}$	1	
(ii) $3 . \dfrac{1-t^2}{1+t^2} + 5 . \dfrac{2t}{1+t^2} = 5$	1	
$\Rightarrow 3 - 3t^2 + 10t = 5 + 5t^2$		
$\Rightarrow 8t^2 - 10t + 2 = 0$		
$\Rightarrow 4t^2 - 5t + 1 = 0$		
$\Rightarrow (4t-1)(t-1) = 0$		
$\Rightarrow t = 1/4$ OR $t = 1$ (CONTINUED OVER)	1	

Solutions	Marks	Comments
QUESTION 3 (continued)		
(c) (ii) continued		
$\Rightarrow \frac{x}{2} = 14^\circ, 194^\circ, 45^\circ$ OR 225°		
$\Rightarrow x = 28^\circ$ OR 90° since $0^\circ < x < 360^\circ$	1	
OTHERWISE	OR	
$3\cos x + 5\sin x = 5$		
may be written in the form		
$\sqrt{34}\cos(x - 59^\circ) = 5$	1	
$\Rightarrow \cos(x - 59^\circ) = \frac{5}{\sqrt{34}}$		
$\Rightarrow x - 59^\circ = 31^\circ$ OR -31°	1	
$\Rightarrow x = 90^\circ$ OR 28°	1	
(d) $(1+x)^{10} = {}^{10}C_0 + {}^{10}C_1 x + \ldots + {}^{10}C_5 x^5 + \ldots + {}^{10}C_{10} x^{10}$	1	
$(1+x)^5 = {}^5C_0 + {}^5C_1 x + {}^5C_2 x^2 + {}^5C_3 x^3 + {}^5C_4 x^4 + {}^5C_5 x^5$		
multiplying by $(1+x)^5$ and taking coefficients of x^5		
${}^{10}C_5 = {}^5C_0\,{}^5C_5 + {}^5C_1\,{}^5C_4 + {}^5C_2\,{}^5C_3 + \ldots + {}^5C_4\,{}^5C_1 + {}^5C_5\,{}^5C_0$	1	
BUT ${}^5C_0 = {}^5C_5$, ${}^5C_1 = {}^5C_4$ etc	1	
$\therefore {}^{10}C_5 = ({}^5C_0)^2 + ({}^5C_1)^2 + \ldots + ({}^5C_5)^2$		

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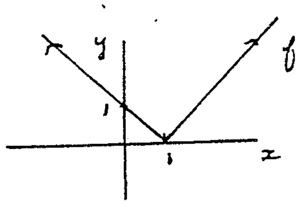
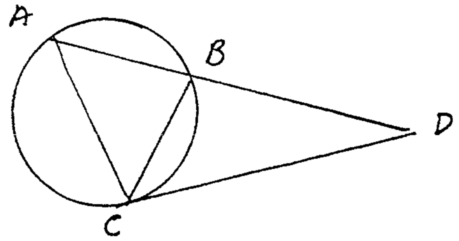
Course: MATHEMATICS EXTENSION 1 Page No. 6

Solutions	Marks	Comments
QUESTION 4		
(a) $\cos^2 x = \frac{1}{2}(1+\cos 2x)$	1	
$\int \cos^2 x \, dx = \frac{1}{2}\int (1+\cos 2x)\,dx$		
$= \frac{x}{2} + \frac{\sin 2x}{4} + C$	1	
(b) (i) ${}^{10}C_4 \times {}^{15}C_4 = \frac{10!}{4!\,6!} \times \frac{15!}{4!\,11!} = 286650$	1	ANSWERS may be left in ${}^nC_r$ format
(ii) there must be 5, 6, 7 or 8 women	1	
ie ${}^{10}C_5\,{}^{15}C_3 + {}^{10}C_6\,{}^{15}C_2 + {}^{10}C_7\,{}^{15}C_1 + {}^{10}C_8$	1	
$=$ ~~32715~~ 138 555		
(iii) at least 2 women is the complement of (no women or 1 woman)	1	
ie no. of ways is ${}^{25}C_8 - {}^{15}C_8 - 10\,{}^{15}C_7 = 1010790$	1	
(c) (i)  $f(x) = \lvert x-1 \rvert$	1	
(ii) horizontal line cuts graph in more than 1 place ie for each y there corresponds more than 1 x value	1	
<u>OR</u> specific example eg: $f(0)=1 \quad f(2)=1$		
(iii) By reflection about $y=x$ $\quad f_1^{-1}(x) = x+1$	1	
Domain $\{x : x \geq 0\}$	1	
Range $\{y : y \geq 1\}$		
(iv) $f_2^{-1}(x) = 1 - x$	1	

TOTAL 12

Solutions	Marks	Comments
QUESTION 5		
(a) (i) 		
In ΔBCD and ΔCAD $A\hat{D}C$ is common	1	
$B\hat{C}D = C\hat{A}D$ (angle between a tangent & its chord equals angle in alternate segment)	1	
$\therefore \Delta BCD \parallel\!\mid \Delta CAD$ (equiangular)	1	
(ii) $\therefore \frac{CD}{AD} = \frac{BD}{CD}$ (ratio of sides in similar triangles)	1	
ie $CD^2 = BD.AD$ $\therefore CD = \sqrt{BD.AD}$	1	
(iii) $x = \sqrt{3 \times 9} = 3\sqrt{3}$ (OR 5.2)	1	
(b) $a = \frac{d}{dx}\frac{1}{2}v^2$	1	
$= \frac{d}{dx}(4 - x^2)$ $= -2x$	1	
when $x = 2$, $a = -4\,ms^{-2}$	1	
(c) (i) ${}^{30}P_6 = \frac{30!}{24!}$	1	
(ii) subtract from total the number of placings which <u>do not</u> include Miss Australia	1	
ie $\frac{30!}{24!} - \frac{29!}{23!} = \frac{6.29!}{24!}$	1	

TOTAL <u>12</u>

ourse: MATHEMATICS EXTENSION I Page No. 8

QUESTION 6 Solutions	Marks	Comments
a) (i) $y = \frac{x^2}{4a} \Rightarrow y' = \frac{x}{2a}$		
at $x = 2ap$ $y' = p$	1	
quation of tangent is given by $y - ap^2 = p(x - 2ap)$	1	
ie $y = px - ap^2$		
(ii) $\tan 45^\circ = \left\lvert \frac{p-q}{1+pq} \right\rvert$	1	
$\Rightarrow \left\lvert \frac{p-q}{1+pq} \right\rvert = 1$		
ie $\lvert p-q \rvert = \lvert 1+pq \rvert$		
(iii) if $q = 2$, $\lvert p-2 \rvert = \lvert 1+2p \rvert$		
$\Rightarrow p-2 = 1+2p$ OR $p-2 = -1-2p$		
$\Rightarrow p = -3$ OR $p = 1/3$	2	1 each solution
b) (i) $\ddot{x} = 0$		
$\Rightarrow \dot{x} = c_1$		
hen $t = 0$ $\dot{x} = V\cos\theta$	1	
$\therefore x = tV\cos\theta$		
$\ddot{y} = -g$		
$\Rightarrow \dot{y} = -gt + c_2$		
when $t = 0$, $\dot{y} = V\sin\theta$		
$\therefore \dot{y} = -gt + V\sin\theta$	1	
$\therefore y = -\frac{gt^2}{2} + tV\sin\theta$		

Course: MATHEMATICS EXTENSION I

Solutions	Marks	Comments
QUESTION 6 (continued)		
(b) (ii) time to reach the wall is $t = \frac{d}{\cos\theta}$	1	
$\therefore H = -\frac{1}{2} g \left(\frac{d}{V\cos\theta}\right)^2 + \frac{d}{V\cos\theta} \cdot V\sin\theta$	1	
$= \frac{-g d^2 \sec^2\theta}{2V^2} + d\tan\theta$		
(iii) the target falls under gravity from height, h		
at $t=0$, $\ddot{y} = g$		
$\dot{y} = gt + c_1$		
at $t=0$ $\dot{y} = 0 \Rightarrow c_1 = 0$		
$\therefore \dot{y} = gt$		
hence $y = \frac{1}{2}gt^2 + c_2$		
at $t=0$ $y = 0$		
hence $y = \frac{1}{2}gt^2$	1	
after time $t = \frac{d}{V\cos\theta}$, the target has fallen a distance of $\frac{gd^2\sec^2\theta}{2V^2}$		
$\therefore$ its vertical height is $h - \frac{gd^2\sec^2\theta}{2V^2}$	1	
BUT $h = d\tan\theta$		
ie its vertical height is $-\frac{gd^2\sec^2\theta}{2V^2} + d\tan\theta$	1	

which is the same as the height of the projectile TOTAL 12
$\therefore$ slug will always hit the bulls-eye.

Course: MATHEMATICS EXTENSION I

QUESTION 7 Solutions	Marks	Comments
(a)(i) after t seconds $\theta = 5t + \pi/4$	1	
(ii) $x = 2\cos\theta$ $= 2\cos(5t + \pi/4)$	1	
Now $\dot{x} = -10\sin(5t + \pi/4)$ $\ddot{x} = -50\cos(5t + \pi/4)$ $= -25x$ $= -5^2 x$ $= -n^2 x$	1	
The horizontal motion of the point Q, and therefore of piston, is essentially the motion of the point P, $\ddot{x} = -n^2 x$ which is SHM.		
<u>ALTERNATIVELY</u> we could say that the position function of P, hence Q, hence the piston is in the form $x = a\cos(nt + \alpha$ which is SHM	OR 1	
(iii) from (i) amplitude $a = 2$ period $T = \frac{2\pi}{n}$ $= \frac{2\pi}{5}$	1 1	
(iv) when $t = 0$, $\dot{x} = -10\sin\pi/4$ $= -5\sqrt{2}\ ms^{-1}$	1	

Solutions	Marks	Comments
QUESTION 7 (continued)		
(b) (i) $y = \frac{x^4}{16} \Rightarrow x^2 = 4y^{1/2}$	1	
$V = \pi \int_0^h x^2\, dy$	1	
$= 4\pi \int_0^h y^{1/2}\, dy$		
$= 4\pi \left[\frac{2}{3} y^{3/2} \right]_0^h$		
$= \frac{8}{3} \pi h^{3/2}$		
(ii) now $\frac{dV}{dt} \propto h^{\frac{1}{2}}$		
ie $\frac{dV}{dt} = -kh^{1/2}$	1	No penalty if negative is omitted
the rate of change of the water level is given by $\frac{dh}{dt}$		
by the chain rule $\frac{dh}{dt} = \frac{dh}{dV} \cdot \frac{dV}{dt}$	1	
$\frac{dV}{dh} = 4\pi h^{1/2}$ hence $\frac{dh}{dV} = \frac{h^{-1/2}}{4\pi}$	1	
$\therefore \frac{dh}{dt} = \frac{h^{-1/2}}{4\pi} \cdot -kh^{1/2}$		
$= -\frac{k}{4\pi}$ (which is a constant)	1	
$\therefore$ water level in the tank falls at a constant rate.		

TOTAL 12