STUDENT NUMBER/NAME: ....

Western Region

2012

HIGHER SCHOOL CERTIFICATE

TRIAL EXAMINATION

Mathematics

Extension 2

General Instructions

- Reading Time- 5 minutes
- Working Time - 3 hours
- Write using a blue or black pen
- Board approved calculators may be used
- A Standard Integrals Sheet is provided at the back of this paper which may be detached and used throughout the paper.

Total Marks 100

Section I

10 marks

- Attempt Questions 1-10
- Answer on the Multiple Choice answer sheet provided.
- Allow about 15 minutes for this section.

Section II

90 marks

- Attempt questions 11 – 16
- Answer on the blank paper provided, unless otherwise instructed. Start a new sheet for each question.
- Allow about 2 hours & 45 minutes for this section

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \ -a < x < a$$

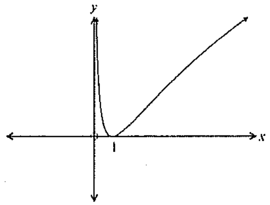
$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

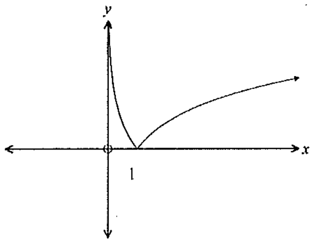
NOTE: $\ln x = \log_e x, \quad x > 0$

1. Which graph best represents $y = |ln\,x|$?

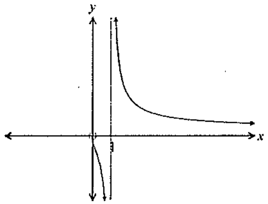
(A)



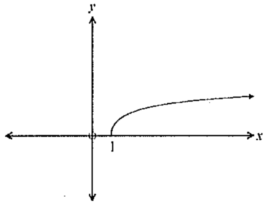
(B)



(C)



(D)



2. A particle is moving in a circular path of radius r, with a constant angular speed ω. The normal component of the acceleration is:

(A) ω

(B) $r\omega$

(C) $r\omega^2$

(D) $(r\omega)^2$

3. The modulus and principle argument of $-2i$ are given by:

(A) $|z| = -2$ and $arg(z) = -\frac{\pi}{2}$.

(B) $|z| = -2$ and $arg(z) = \frac{\pi}{2}$.

(C) $|z| = 2$ and $arg(z) = \frac{\pi}{2}$.

(D) $|z| = 2$ and $arg(z) = -\frac{\pi}{2}$.

4. $\int \frac{2x\,dx}{(x+1)(x+3)} = ?$

(A) $\int \frac{3}{x+3} - \frac{1}{x+1}\,dx$

(B) $\int \frac{1}{x+3} - \frac{3}{x+1}\,dx$

(C) $\int \frac{3}{x+3} + \frac{1}{x+1}\,dx$

(D) $\int \frac{1}{x+3} + \frac{1}{x+1}\,dx$

5. The curve represented by the equation $\frac{x^2}{25} + \frac{y^2}{16} = 1$ isan ellipse with:

(A) eccentricity given by $e = \frac{3}{5}$ and foci at $(\pm 3, 0)$.

(B) eccentricity given by $e = \frac{5}{3}$ and foci at $(\pm 5, 0)$.

(C) eccentricity given by $e = \frac{3}{5}$ and foci at $(\pm 5, 0)$.

(D) eccentricity given by $e = \frac{4}{5}$ and foci at $(\pm 3, 0)$.

6. The polynomial $4x^3 - 27x + k = 0$ has a double root. The possible values of k are:

(A) $k = \pm\frac{3}{2}$

(B) $k = \pm\frac{27}{4}$

(C) $k = \pm 9$

(D) $k = \pm 27$

7. Given that $a > b > 0$, which of the following is not necessarily true?

(A) $a^2 + b^2 > 2ab$

(B) $a^2 + b^2 > 4ab$

(C) $(a+b)^2 > 4ab$

(D) $a + b > 2\sqrt{ab}$

8. The circle $x^2 + y^2 = 4$ is rotated about the line $x = 3$. Using the method of cylindrical shells, the volume could be calculated by:

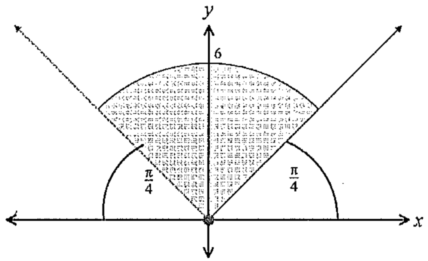
(A) $V = 2\pi \int_{-2}^{2} (3-x)\sqrt{4-x^2}\, dx$.

(B) $V = 4\pi \int_{0}^{2} (4-x)\sqrt{3-x^2}\, dx$.

(C) $V = 4\pi \int_{-2}^{2} (3-x)\sqrt{4-x^2}\, dx$.

(D) $V = 2\pi \int_{0}^{2} (3-x)\sqrt{4-x^2}\, dx$.

9. The shaded region of the Argand plane represents the points where which of the following inequalities hold simultaneously.



(A) $|z| \leq 6$ and $-\frac{\pi}{4} \leq arg(z) \leq \frac{\pi}{4}$

(B) $|z| \leq 6$ and $\frac{\pi}{4} \leq arg(z) \leq \frac{3\pi}{4}$

(C) $|z| \geq \sqrt{6}$ and $-\frac{\pi}{4} \leq arg(z) \leq \frac{\pi}{4}$

(D) $|z| \geq \sqrt{6}$ and $\frac{\pi}{4} \leq arg(z) \leq \frac{3\pi}{4}$

10. Using the recurrence relation $u_n = \int tan^n x\, dx = \frac{tan^{n-1} x}{n-1} - u_{n-2}$,

$\int tan^6 x\, dx = ?$

(A) $\frac{tan^4 x}{4} - \frac{tan^2 x}{2} + x + c$

(B) $\frac{tan^5 x}{5} - \frac{tan^3 x}{3} + tanx + c$

(C) $\frac{tan^6 x}{6} - \frac{tan^4 x}{4} + \frac{tan^2 x}{2} + c$

(D) $\frac{tan^5 x}{5} - \frac{tan^3 x}{3} + tanx - x + c$

End Section I

Section II

Total Marks (90)
Attempt Questions 11 – 16.
Allow about 2 hours&45 minutes for this section.

Answer all questions, starting each question on a new sheet of paper with your name and question number at the top of the page. Do not write on the back of sheets.
All necessary working should be shown in every question.

Question 11 (15 marks) Start a new sheet of writing paper. **Marks**

(a) (i) Find real numbers a and b such that $\dfrac{7x}{x^2+x-12} \equiv \dfrac{a}{x+4} + \dfrac{b}{x-3}$. **2**

(ii) Hence find $\displaystyle\int \frac{7x\,dx}{x^2+x-12}$ **1**

(b) Find $\displaystyle\int cos^5 x\,dx$. **3**

(c) If $z_1 = 3-2i$ and $z_2 = 3-4i$, find in the form $x+iy$:

(i) $z_1 - z_2$ **1**

(ii) $(z_2)^2$ **1**

(iii) $\dfrac{z_2}{z_1}$ **1**

(iv) $z_1.\overline{z_2}$ **1**

(d) Factorise $x^4 + x^2 - 12$ completely over the field of :

(i) Rational numbers. **1**

(ii) Real numbers. **1**

(iii) Complex Numbers. **1**

(e) Given that α, β and γ are the roots of $x^3+px^2+qx+r=0$, find the equation whose roots are α^2, β^2 and γ^2. **2**

End of Question 11

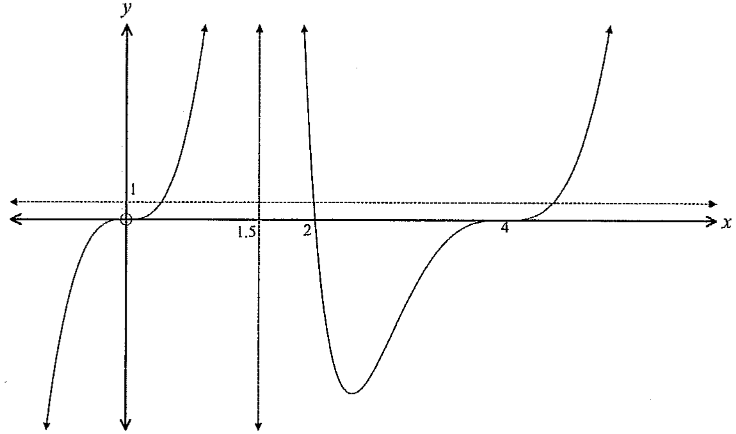
Question 12 (15 marks) Start a new sheet of writing paper. **Marks**

(a) Prove by induction that **4**

$$1^3 - 2^3 + 3^3 - 4^3 + \ldots\ldots - (n-1)^3 + n^3 = \frac{(2n-1)(n+1)^2}{4}$$

where$n$ is a positive odd integer.

(b) A sketch of the curve whose equation is $y = f(x)$ is shown below.



Draw neat, half page sketches of:

(i) $y = |f(x)|$ **2**

(ii) $y = (f(x))^2$ **2**

(iii) $y = \dfrac{1}{f(x)}$ **2**

(c) A particle of mass m, is released to fall vertically under gravity in a medium where the resistance is k times the square of the velocity (v) of the particle.

(i) Show that the distance fallen (x) in terms of v is: **3**

$$x = \frac{1}{2k} ln\left(\frac{g}{g-kv^2}\right)$$

(ii) Show that the velocity in terms of x is: **2**

$$v = \sqrt{\frac{g}{k}\left(1-e^{-2kx}\right)}$$

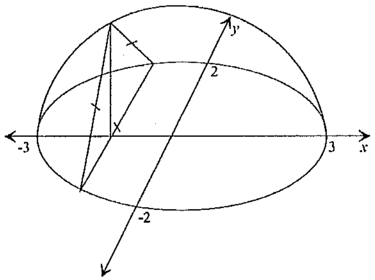
End of Question 12

Question 13 (15 marks) Start a new sheet of writing paper. **Marks**

(a) A solid has as its base, the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$. **3**

Cross sections taken perpendicular to the x axis are equilateral triangles.

A typical cross section is shown on the diagram.



Find the volume of the solid.

(b) Find the square roots of $z=1+i\sqrt{3}$ in the form $\sqrt{z}=\pm(x+iy)$. **3**

(c) For any complex number, z show that $z.\bar{z}=|z|^2$. **2**

(d) (i) Write $z=\sqrt{3}+i$ in the form $r(cos\theta+i\,sin\theta)$. **1**

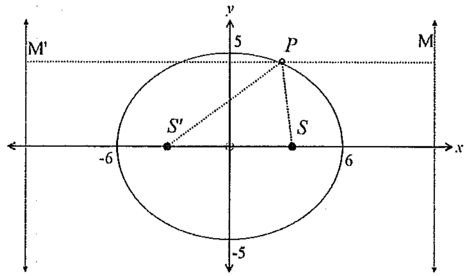
(ii) Hence, or otherwise, find z^5 in the form $x+iy$. **2**

Question 13 continues on the next page

Question 13 continued. **Marks**

(e) The ellipse $\frac{x^2}{36}+\frac{y^2}{25}=1$ is shown below along with its foci and directrices.

By definition, for any point P on the ellipse, $\frac{SP}{PM}=e$.



(i) Find the eccentricity (e) of the ellipse. **1**

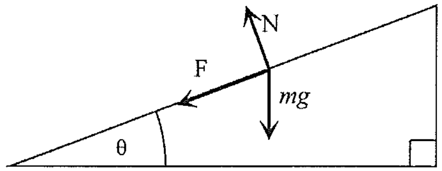
(ii) Give the equations of the directrices of the ellipse. **1**

(iii) Show that for any point P on the ellipse, $PS+PS'=C$, and give the value of the constant C. **2**

End of Question 13

Question 14 (15 marks) Start a new sheet of writing paper. **Marks**

(a) In a V8 supercar race, there is a circular bend or radius r, where the road is banked at an angle of θ to reduce the tendency to slide sideways when rounding the bend. Consider one of the V8 cars to be a point of mass m. Itistravelling at a constant speed v around the bend. The width of the road on the bend is d.



The forces acting on the car are the gravitational force mg, the normal reaction to the road N, and the frictional force F acting down the road.

(i) By resolving the horizontal and vertical components of force, find expressions for $N \sin\theta$ and $N \cos\theta$. **3**

(ii) Give an expression for the constant speed which will produce no friction against the road. **2**

(iii) Show that when the car is travelling at a constant speed which does produce friction against the road that the friction is given by the expression; **3**

$$F = \frac{mv^2}{r}\cos\theta - mg\sin\theta$$

(b) Find $\int e^x \sin x \, dx$ by the method of integration by parts. **3**

(c) (i) Show that the points P $\left(cp, \frac{c}{p}\right)$ and Q $\left(cq, \frac{c}{q}\right)$ lie on the hyperbola $xy = c^2$. **1**

(ii) Show that the equation of the chord PQ is $x + pqy = c(p+q)$. **2**

(iii) By considering the equation of the chord (and corresponding secant) as Q approaches P, determine the equation of the tangent at P. **1**

End of Question 14

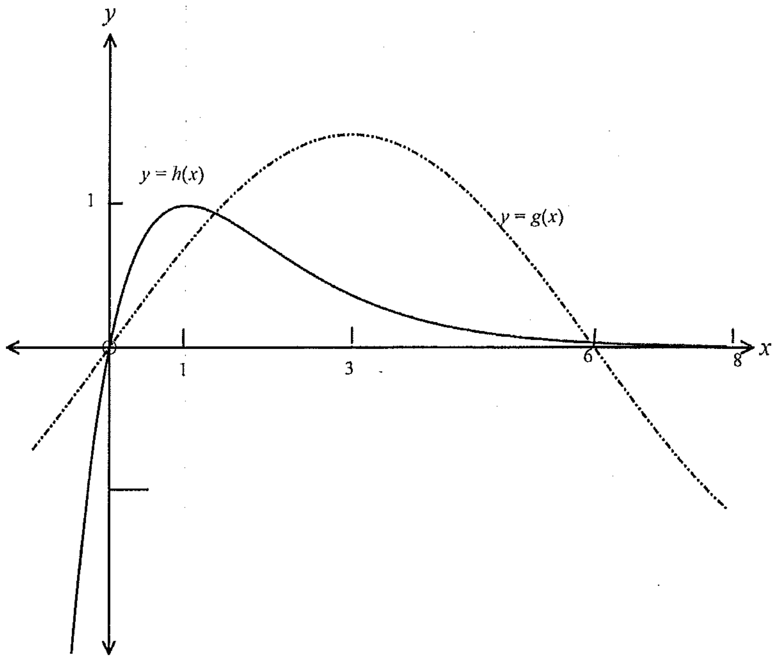
Question 15 (15 marks) Start a new sheet of writing paper. **Marks**

(a) Use the method of cylindrical shells to find the volume formed when the area bounded by $y = x^2 - 4x^4$ and the x axis is rotated about the y axis. **3**

(b) For training, Debbie is having shots at a target. On any one shot, the probability of her hitting the target is 60%. What is the minimum number of shots she needs to make at the target so that the probability that she hits with at least one of the shots is 99%? **3**

(c) Find $\int_0^{\frac{\pi}{4}} \tan x \sec^2 x \, dx$ **2**

(d) The diagram below shows the graphs of $y = g(x)$ and $y = h(x)$ for the domain, Draw a half page sketch showing the graph of $y = g(x).h(x)$. **4**



(e) If $a > b > 0$, show that $1 + \frac{b}{a} > 2\sqrt{\frac{b}{a}}$. **3**

End of Question 15

Question 16 (15 marks) Start a new sheet of writing paper. **Marks**

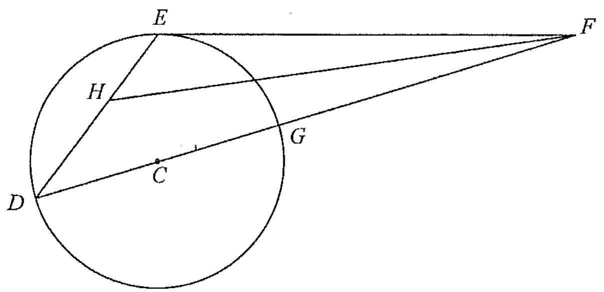
(a) Find the fourth roots of $2 + 2\sqrt{3}\ i$. **3**

(b) The complex number $z = x + iy$ satisfies the relation $(z - \bar{z})^2 + 18(z + \bar{z}) = 36$. Show that the locus of z on the Argand plane is a parabola, and give its focal length and the coordinates of its vertex. **3**

(c) Given that the polynomial $P(x) = 2x^4 + 9x^3 + 6x^2 - 20x - 24$ has a triple zero, find all the zeros of $P(x)$. **3**

(d) For the curve with equation $x^2 + 6xy - 4y^2 = 10$, determine the gradient of the tangent at the point (2, 1) on the curve. **3**

(e) A tangent is drawn from the point E on the circle centre C. The diameter DG is produced to meet the tangent at F. $\angle EFD$ is bisected, so as to meet the chord ED at H. **3**



Find, giving reasons, the size of $\angle DHF$.

End of Examination

The Standard Integrals are on the next page.

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$

Trial HSC Examination
Mathematics Extension 22012

Multiple Choice Answer Sheet

Name ______________________________

Completely fill the response oval representing the most correct answer.

1. A ○ B○ C○ D○
2. A ○ B○ C○ D○
3. A ○ B○ C○ D○
4. A ○ B○ C○ D○
5. A ○ B○ C○ D○
6. A ○ B○ C○ D○
7. A ○ B○ C○ D○
8. A ○ B○ C○ D○
9. A ○ B○ C○ D○
10. A ○ B○ C○ D○

WESTERN REGION

2012
HSC Course
TRIAL EXAMINATION

Mathematics Extension 2

SOLUTIONS

Mathematics Extension 2 Trial Examination –2012

Multiple Choice Answer Sheet

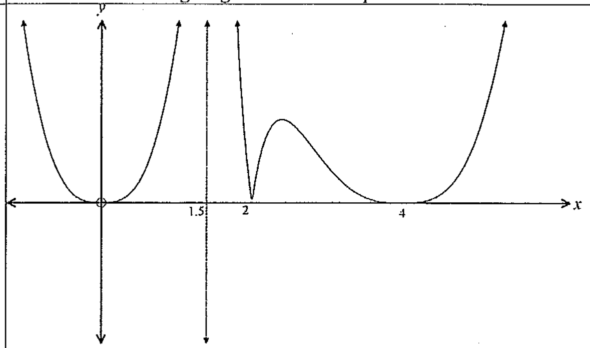
Name ______________________________

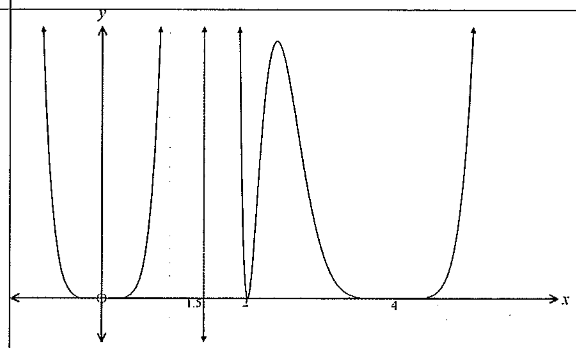
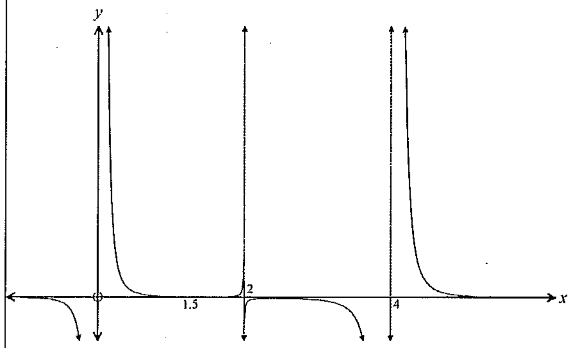
Completely fill the response oval representing the most correct answer.

	A	B	C	D
1.	○	●	○	○
2.	○	○	●	○
3.	○	○	○	●
4.	●	○	○	○
5.	●	○	○	○
6.	○	○	○	●
7.	○	●	○	○
8.	○	○	●	○
9.	○	●	○	○
10.	○	○	○	●

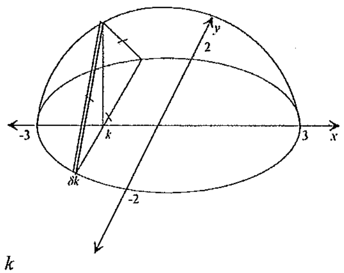
Question 11	Trial HSC Mathematics Extension 2	2012	
Part	Solution	Marks	Comment
(a) (i)	$\dfrac{7x}{x^2+x-12} \equiv \dfrac{a}{x+4}+\dfrac{b}{x-3}$ $\dfrac{7x}{(x+4)(x-3)} \equiv \dfrac{a}{x+4}+\dfrac{b}{x-3}$ $7x = a(x-3) + b(x+4)$ Let $x = 3 \Rightarrow 21 = 7b$ $b = 3$ Let $x = -4 \Rightarrow -28 = -7a$ $a = 4$	2	1 mark for a 1 mark for b
(ii)	$\displaystyle\int \frac{7x}{x^2+x-12}dx = 4\int \frac{dx}{x+4} + 3\int \frac{dx}{x-3}$ $= 4\ln(x+4) + 3\ln(x-3) + C$	1	1 for answer
(b)	$\displaystyle\int \cos^5 x\,dx = \int \cos^4 x \,.\, \cos x\,dx$ $\displaystyle= \int (1-\sin^2 x)^2 .\ \cos x\,dx$ Use $u = \sin x$ $du = \cos$ $\displaystyle= \int (1-2\sin^2 x + \sin^4 x).\ \cos x\,dx$ $\displaystyle= \int (1-2u^2+u^4)\,du$ $= u - \dfrac{2u^3}{3} + \dfrac{u^5}{5} + c$ $= \sin x - \dfrac{2\sin^3 x}{3} + \dfrac{\sin^5 x}{5} + c$	3	1 for substitute 1 for integration of u 1 for answer
(c) (i)	$z_1 - z_2 = 3-2i - (3-4i)$ $= 3-3-2i+4i$ $= 2i$	1	1 for answer
(ii)	$(z_2)^2 = (3-4i)^2$ $= 9-24i+16(i)^2$ $= 9-24i-16$ $= -7-24i$	1	1 for answer

Question 11	Trial HSC Mathematics Extension 2	2012	
Part	Solution	Marks	Comment
(iii)	$\dfrac{z_2}{z_1} = \dfrac{3-4i}{3-2i} \times \dfrac{3+2i}{3+2i}$ $= \dfrac{9+6i-12i+8}{9+4}$ $= \dfrac{17-6i}{13}$	1	1 for answer
(iv)	$z_1.(\overline{z}_2) = (3-2i)(3+4i)$ $= 9+12i-6i+8$ $= 17+6i$	1	1 for answer
(d) (i)	$x^4+x^2-12 = (x^2+4)(x^2-3)$	1	1 for answer
(ii)	$(x^2+4)(x^2-3) = (x^2+4)(x+\sqrt{3})(x-\sqrt{3})$	1	1 for answer
(iii)	$(x^2+4)(x+\sqrt{3})(x-\sqrt{3}) = (x+2i)(x-2i)(x+\sqrt{3})(x-\sqrt{3})$	1	1 for answer
(e)	$x^3+px^2+qx+r=0$ Let $u = x^2$ So when $x = \alpha$, $u = \alpha^2$, etc. Re write equation as $x^3+qx = -px^2-r$ $x(x^2+q) = -px^2-r$ Square both sides $x^2(x^4+2qx^2+q^2) = p^2x^4+2prx^2+r^2$ Substitute $u = x^2$ $u(u^2+2qu+q^2) = p^2u^2+2pru+r^2$ Expand and simplify $u^3+2qu^2+q^2u = p^2u^2+2pru+r^2$ $u^3+2qu^2-p^2u^2+q^2u-2pru-r^2=0$ $u^3+(2q-p^2)u^2+(q^2-2pr)u-r^2=0$ Equation is $x^3+(2q-p^2)x^2+(q^2-2pr)x-r^2=0$	2	1 for sub^n exp^n 1 for answer

Question 12	Trial HSC Mathematics Extension 2		2012
Part	Solution	Marks	Comment
(a)	When $n = 1$, $LHS = 1^3 = 1$ and $RHS = \frac{(2\times 1-1)(1+1)^2}{4} = \frac{1\times 2^2}{4} = 1$ $\therefore$ True for n = 1 Assume true for $n = k$ where k is an odd integer. i.e $1^3 - 2^3 + 3^3 - 4^3 + \ldots\ldots - (k-1)^3 + k^3 = \frac{(2k-1)(k+1)^2}{4}$ Prove true for the next odd number (k+2) $LHS = 1^3 - 2^3 + 3^3 - 4^3 + \ldots\ldots k^3 - (k+1)^3 + (k+2)^3$ $= \frac{(2k-1)(k+1)^2}{4} - (k+1)^3 + (k+2)^3$ $= \frac{2k^3+3k^2-1}{4} - \left(k^3+3k^2+3k+1\right) + \left(k^3+6k^2+12k+8\right)$ $= \frac{2k^3+15k^2+36k+27}{4}$ $= \frac{(k+3)^2(2k+3)}{4}$ $= \frac{((k+2)+1)^2(2(k+2)+1)}{4}$ Therefore true for $n = k+2$. So if the statement is true for a given odd integer, it is true for the next odd integer, and since true for $n = 1$, is true for all odd integers greater than or equal to 1.	4	1 mark for show n= 1 and assumption for $n = k$ 2 for proving case for $n = k+1$ 1 for conclusion
(b) (i)		2	1 mark off for each error

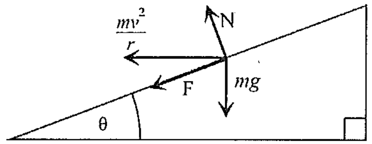
Question 12	Trial HSC Mathematics Extension 2		2012
Part	Solution	Marks	Comment
(ii)		2	1 mark off for each error
(iii)		2	1 mark off for each error

Question 12	Trial HSC Mathematics Extension 2		2012
Part	Solution	Marks	Comment
(c) (i)	Resistance $= kv^2$ This is acting against gravity so $\ddot{x} = g - kv^2$ $v\dfrac{dv}{dx} = g - kv^2$ $\dfrac{dv}{dx} = \dfrac{g-kv^2}{v}$ $\dfrac{dx}{dv} = \dfrac{v}{g-kv^2}$ $= -\dfrac{1}{2k}\dfrac{-2kv}{g-kv^2}$ $x = -\dfrac{1}{2k}ln(g-kv^2) + C$ When $x = 0, v = 0$ $C = \dfrac{ln\,g}{2k}$ $x = \dfrac{ln\,g}{2k} - \dfrac{1}{2k}ln(g-kv^2)$ $= \dfrac{1}{2k}(ln\,g - ln(g-kv^2))$ $x = \dfrac{1}{2k}ln\left(\dfrac{g}{g-kv^2}\right)$	3	1 for equation in x 1 for value of C 1 for answer
(ii)	$x = \dfrac{1}{2k}ln\left(\dfrac{g}{g-kv^2}\right)$ $2kx = ln\left(\dfrac{g}{g-kv^2}\right)$ $e^{2kx} = \left(\dfrac{g}{g-kv^2}\right)$ $(g-kv^2)e^{2kx} = g$ $(g-kv^2) = \dfrac{g}{e^{2kx}}$ $kv^2 = g - \dfrac{g}{e^{2kx}}$ $v^2 = \dfrac{g}{k}\left(1 - \dfrac{1}{e^{2kx}}\right)$ $v^2 = \dfrac{g}{k}(1 - e^{-2kx})$ $v = \sqrt{\dfrac{g}{k}(1 - e^{-2kx})}$	2	1 mark for writing as exponential 1 mark for answer

Question 13	Trial HSC Mathematics Extension 2		2012
Part	Solution	Marks	Comment
(a)	A thin slice of thickness δk is taken at a point $x = k$.  At $x = k$ $\dfrac{k^2}{9} + \dfrac{y^2}{4} = 1$ $\dfrac{y^2}{4} = \dfrac{9-k^2}{9}$ $y^2 = \dfrac{4}{9}(9-k^2)$ $y = \dfrac{2}{3}\sqrt{9-k^2}$ Area Triangle $= \dfrac{1}{2}ab\sin C$ $= \dfrac{1}{2}\left(\dfrac{4}{3}\sqrt{9-k^2}\right)^2 \sin 60^\circ$ $= \dfrac{4}{9}\sqrt{3}\,(9-k^2)$ (ΔV) would be given by: The volume of the slice is given by: $\Delta V = \dfrac{4\sqrt{3}}{9}(9-k^2)\delta k$ Volume of solid given by: $V = 2\displaystyle\int_0^3 \dfrac{4\sqrt{3}}{9}(9-x^2)\,dx$ $= \dfrac{8\sqrt{3}}{9}\displaystyle\int_0^3 (9-x^2)\,dx$ $= \dfrac{8\sqrt{3}}{9}\left[9x - \dfrac{x^3}{3}\right]_0^3$ $= \dfrac{8\sqrt{3}}{9}(18-0)$ $= 16\sqrt{3}$ Cubic units.	3	1 mark for equation in y 1 mark for area equation 1 mark for final answer

Question 13	Trial HSC Mathematics Extension 2	2012	
Part	Solution	Marks	Comment
(b)	Let $x + iy$ be the square root of z. $(x+iy)^2 = 1 + i\sqrt{3}$ Then $x^2 + 2xyi - y^2 = 1 + i\sqrt{3}$ Equating real and imaginary parts. $x^2 - y^2 = 1$ and $2xy = \sqrt{3}$ Solve simultaneously $y = \frac{\sqrt{3}}{2x}$ $x^2 - \left(\frac{\sqrt{3}}{2x}\right)^2 = 1$ $x^2 - \frac{3}{4x^2} = 1$ $4x^4 - 3 = 4x^2$ $4x^4 - 4x^2 - 3 = 0$ $4u^2 - 4u - 3 = 0$ $4u^2 - 6u + 2u - 3 = 0$ $2u(2u-3) + (2u-3) = 0$ $(2u-3)(2u+1) = 0$ $u = \frac{3}{2}$ or $u = -\frac{1}{2}$ $x^2 = \frac{3}{2}$ or $x^2 = -\frac{1}{2}$ $x = \pm\left(\sqrt{\frac{3}{2}}\right)$ $y = \pm\frac{\sqrt{2}}{2}$ Roots are $\sqrt{\frac{3}{2}} + \frac{\sqrt{2}\,i}{2}$ or $-\sqrt{\frac{3}{2}} - \frac{\sqrt{2}\,i}{2}$ or in rationalised form $\frac{\sqrt{6}+i\sqrt{2}}{2}$ or $\frac{-\sqrt{6}-i\sqrt{2}}{2}$ $\sqrt{z} = \pm\left(\frac{\sqrt{6}+i\sqrt{2}}{2}\right)$	3	1 mark for real and imaginary parts 1 mark for correct x in answer 1 mark for correct y in answer
(c)	$z.\bar{z} = (x+iy)(x-iy)$ $= x^2 - i^2y^2$ $= x^2 + y^2$ $= \sqrt{(x^2+y^2)}^2$ $= \lvert z\rvert^2$	2	1 mark for expansion 1 mark for final answer

Question 13	Trial HSC Mathematics Extension 2	2012	
Part	Solution	Marks	Comment
(d) (i)	$r = \sqrt{(\sqrt{3})^2 + (1)^2} = \sqrt{4} = 2$ $\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ $\theta = 30^\circ$ $\theta = \frac{\pi}{6}^c$ $z = 2\left(cos\frac{\pi}{6} + isin\frac{\pi}{6}\right)$	1	1 mark for final answer
(ii)	$z^5 = \left(2\left(cos\frac{\pi}{6} + isin\frac{\pi}{6}\right)\right)^5$ $= 2^5\left(cos\frac{5\pi}{6} + isin\frac{5\pi}{6}\right)$ $= 32\left(-\frac{\sqrt{3}}{2} + \frac{i}{2}\right)$ $= -16\sqrt{3} + 16i$	2	1 mark for De Moivre step 1 mark for final answer
(e) (i)	$a = 6$ and $b = 5$ $b^2 = a^2(1-e^2)$ $25 = 36(1-e^2)$ $\frac{25}{36} = 1 - e^2$ $e^2 = 1 - \frac{25}{36}$ $e^2 = \frac{11}{36}$ $e = \frac{\sqrt{11}}{6}$	1	1 mark for final answer
(ii)	Directrices $x = \pm\frac{a}{e} = \pm 6.\frac{6}{\sqrt{11}} = \pm\frac{36}{\sqrt{11}}$	1	1 mark for final answer
(iii)	By definition $\frac{PS}{PM} = e$ and $\frac{PS'}{PM'} = e$ $PS + PP' = ePM + ePM'$ $= e(PM + PM')$ $= e(MM')$ $= e\left(2.\frac{36}{\sqrt{11}}\right)$ $= \frac{\sqrt{11}}{6}.\frac{72}{\sqrt{11}}$ $= 12$	2	1 mark for simplifying SP + SP' 1 mark for final answer

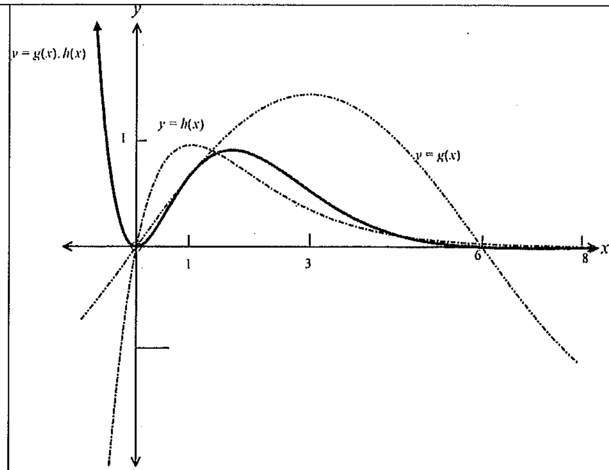
Question 14	Trial HSC Mathematics Extension 2	2012	
Part	**Solution**	**Marks**	**Comment**
(a) (i)	 Horizontally the centripetal force is $\frac{mv^2}{r}$ and vertically the force is zero as the car remains at the same position on the bank. ∴ The resultant vertical force is zero. $N\cos\theta - F\sin\theta = mg$ $N\cos\theta = mg + F\sin\theta$ The resultant horizontal force = $\frac{mv^2}{r}$ $N\sin\theta + F\cos\theta = \frac{mv^2}{r}$ $N\sin\theta = \frac{mv^2}{r} - F\cos\theta$	3	1 mark for mg 1 mark for $\frac{mv^2}{r}$ 1 mark for final answer
(ii)	If there is no friction then $F = 0$ $N\cos\theta = mg$ $N\sin\theta = \frac{mv^2}{r}$ $\tan\theta = \frac{mv^2}{r} \times \frac{1}{mg}$ $\tan\theta = \frac{v^2}{rg}$ $v^2 = rg\tan\theta$ $v = \sqrt{rg\tan\theta}$	2	1 mark for $\tan\theta$ 1 mark for final answer

Question 14	Trial HSC Mathematics Extension 2	2012	
Part	**Solution**	**Marks**	**Comment**
(iii)	$\frac{N\sin\theta}{N\cos\theta} = \frac{\frac{mv^2}{r} - F\cos\theta}{mg + F\sin\theta}$ $\tan\theta(mg + F\sin\theta) = \frac{mv^2}{r} - F\cos\theta$ $mg\tan\theta + F\sin\theta\tan\theta = \frac{mv^2}{r} - F\cos\theta$ $F(\cos\theta + \sin\theta\tan\theta) = \frac{mv^2}{r} - mg\tan\theta$ $F\,\frac{\cos^2\theta + \sin^2\theta}{\cos\theta} = \frac{mv^2}{r} - mg\tan\theta$ $\frac{F}{\cos\theta} = \frac{mv^2}{r} - mg\tan\theta$ $F = \frac{mv^2}{r}\cos\theta - mg\sin\theta$	3	1 mark for initial setup. 1 mark for simplifying and eliminating N 1 mark for final answer
(b)	Let $u = e^x \quad \frac{dv}{dx} = \sin x$ $\frac{du}{dx} = e^x \quad v = -\cos x$ $\int u.\frac{dv}{dx}\,dx = u.v - \int v.\frac{du}{dx}\,dx$ $\int \sin x\, e^x\,dx = -e^x\cos x - \int(-\cos x).e^x\,dx$ $= -e^x\cos x + \int \cos x.e^x\,dx$ $= -e^x\cos x + e^x.\sin x - \int \sin x.e^x\,dx$ $2\int \sin x.e^x\,dx = -e^x\cos x + e^x.\sin x$ $\int \sin x.e^x\,dx = \frac{e^x(\sin x - \cos x)}{2} + C$	3	1 mark for initial integration by parts 1 for 2nd integration by parts 1 mark for final answer

(c) (i)	Sub $P\left(cp, \frac{c}{p}\right)$ into $xy = c^2$ $LHS = cp.\frac{c}{p}$ $= c^2$ $= RHS$ $\therefore$ P lies on $xy = c^2$ Similarly Q lies on $xy = c^2$	1	1 mark for final answer
(ii)	Grad of PQ $= \dfrac{\frac{c}{q} - \frac{q}{p}}{cq - cp}$ $= \dfrac{\frac{1}{q} - \frac{1}{p}}{q-p}$ $= \dfrac{p-q}{pq} \cdot \dfrac{1}{q-p}$ $= -\dfrac{1}{pq}$ Equation of PQ $y - \frac{c}{p} = -\frac{1}{pq}(x - cp)$ $pqy - cq = -x + cp$ $x + pqy = cp + cq$ $x + pqy = c(p+q)$	2	1 mark for gradient PQ 1 mark for final equation
(iii)	As Q approaches P $q \Rightarrow p$ Equation of PQ approaches $x + p.py = c(p+p)$ $x + p^2y = 2cp$	1	1 mark for final answer

Question 15	Trial HSC Mathematics Extension 2	2012	
Part	**Solution**	**Marks**	**Comment**
(a)	$x^2 - 4x^4 = 0$ $x^2(1 - 4x^2) = 0$ $x^2(1-2x)(1+2x) = 0$ $x = -\frac{1}{2},\ x = 0,\ x = \frac{1}{2},$ Volume of Shell $= 2\pi xy\,\Delta x$ Volume of Solid $\approx \lim_{\Delta x \to 0} \sum_0^{\frac{1}{2}} 2\pi xy\,\Delta x$ Volume of Solid $= \int_0^{\frac{1}{2}} 2\pi xy\,dx$ $= \int_0^{\frac{1}{2}} 2\pi x(x^2 - 4x^4)\,dx$ $= 2\pi \int_0^{\frac{1}{2}} x^3 - 4x^5\,dx$ $= 2\pi\left[\frac{x^4}{4} - \frac{2x^6}{3}\right]_0^{\frac{1}{2}}$ $= 2\pi\left[\left(\frac{1}{64} - \frac{1}{96}\right) - (0)\right]$ $= \frac{\pi}{96}$ cubic units	3	1mark for limits 1 mark for integral for volume 1 mark for final answer

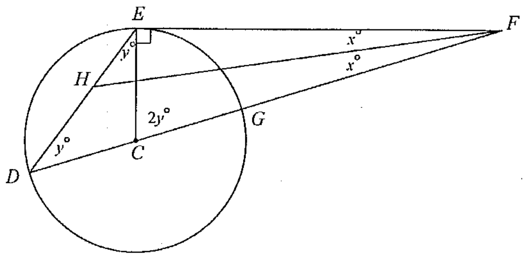
(b)	For any one shot, the probability of hitting the target is $P(H) = 0.6$ and hence $P(\overline{H}) = 0.4$ If n shots are made at the target, the probability that none hit is $p(\overline{H}\,\overline{H}\,\overline{H}\,\overline{H}\ldots\ldots) = 0.4^n$ So Probability of hitting with at least one shot is $P(\text{at least 1 H}) = 1 - 0.4^n$ We want $P(\text{at least 1 H}) > 0.99$ $1 - 0.4^n > 0.99$ $-0.4^n > -0.01$ $0.4^n < 0.01$ $ln(0.4^n) < ln(0.01)$ $n\,ln(0.4) < ln(0.01)$ $n > \frac{ln(0.01)}{ln(0.4)}$ (reverse sign as dividing by negative) $n > 5.02$ So the least number of shots is 6.	3	1 mark for at least being complement. 1 mark for $0.4^n < 0.01$ 1 mark for final answer
(c)	Use $u = \tan x$ $\quad du = \sec^2 x\,dx$ $x = 0 \quad u = \tan 0 = 0$ $x = \frac{\pi}{4} \quad u = \tan\frac{\pi}{4} = 1$ $\int_0^{\frac{\pi}{4}} \tan x \sec^2 x\,dx = \int_0^1 u\,du$ $= \left[\frac{u^2}{2}\right]_0^1$ $= \frac{1}{2} - 0$ $= \frac{1}{2}$	2	1 for substitution 1 mark for final answer Alternates possible.

(d)		4	1 mark for graph for $x < 0$ 1 mark for graph for $0 < x < P$ 1 mark for graph for $P < x < 6$ 1 mark for graph for $x > 6$ *Where P is x value of the point of intersection of the original 2 graphs*
(e)	If $a > b$, then $a - b > 0$ and $(a-b)^2 > 0$ $a^2 - 2ab + b^2 > 0$ $a^2 + b^2 > 2ab$ $a^2 + 2ab + b^2 > 4ab$ $(a+b)^2 > 4ab$ $a + b > 2\sqrt{ab}$ $\frac{a+b}{a} > \frac{2\sqrt{ab}}{\sqrt{a^2}}$ $\frac{a+b}{a} > \left(2\sqrt{\frac{ab}{a^2}}\right)$ $1 + \frac{b}{a} > 2\sqrt{\frac{b}{a}}$	3	1 mark for $a^2 + b^2$ 1 mark for $a + b$ 1 mark for final result

Question 16	Trial HSC Mathematics Extension 2	2012	
Part	**Solution**	**Marks**	**Comment**
(a)	Express in Mod arg form $r=\sqrt{2^2+\left(2\sqrt{3}\right)^2}=\sqrt{4+12}=4$ and $\theta=\tan^{-1}\sqrt{3}=\frac{\pi}{3}$ Let z be the fourth root of $2+2\sqrt{3}\,i$ So $z^4=2+2\sqrt{3}\,i$ $=4\text{cis}\,\frac{\pi}{3}$ $z=\left(4\text{ cis}\frac{\pi}{3}\right)^{\frac{1}{4}}$ $z=4^{\frac{1}{4}}\left(\text{cis }\frac{\pi}{12}\right)$ by De Moivre's Thm $z=\sqrt{2}\left(\text{cis }\frac{\pi}{12}\right)$ This is the first root. Since increasing argument of $2+2\sqrt{3}\,i$ by 2π will give the same value, the roots will be spaced at $\frac{2\pi}{4}=\frac{\pi}{2}$. So roots will be $z_1=\sqrt{2}\left(\text{cis }\frac{\pi}{12}\right)$, $z_2=\sqrt{2}\left(\text{cis }\frac{\pi}{12}+\frac{\pi}{2}\right)=\sqrt{2}\left(\text{cis }\frac{7\pi}{12}\right)$ $z_3=\sqrt{2}\left(\text{cis }\frac{\pi}{12}+2\times\frac{\pi}{2}\right)=\sqrt{2}\left(\text{cis }\frac{13\pi}{12}\right)=\sqrt{2}\left(\text{cis }-\frac{11\pi}{12}\right)$ $z_4=\sqrt{2}\left(\text{cis }\frac{\pi}{12}+3\times\frac{\pi}{2}\right)=\sqrt{2}\left(\text{cis }\frac{19\pi}{12}\right)=\sqrt{2}\left(\text{cis }-\frac{5\pi}{12}\right)$	3	1 mark for mod arg 1 mark for finding z 1 for stating all the roots.
(b)	$z=x+iy$ $\bar{z}=x-iy$ $(z-\bar{z})=x+iy-x+iy=2iy$ $(z+\bar{z})=x+iy+x-iy=2x$ $(z-\bar{z})^2=(2iy)^2=-4y^2$ $(z-\bar{z})^2+18(z+\bar{z})=36$ $-4y^2+18(2x)=36$ $4y^2=36x-36$ $y^2=9(x-1)$ $y^2=4\left(\frac{9}{4}\right)(x-1)$ $\therefore\ (z-\bar{z})^2+18(z+\bar{z})=36$ represents a parabola with focal length $\frac{9}{4}$ and vertex at $(1,0)$.	3	1 mark for finding $z+\bar{z}$ and $z-\bar{z}$ 1 for parabola equation 1 for focal length and vertex

Question 16	Trial HSC Mathematics Extension 2	2012	
Part	**Solution**	**Marks**	**Comment**
(c)	$P(x)=2x^4+9x^3+6x^2-20x-24$ If $x=\alpha$ is a triple zero of $P(x)$ it is a zero of $P''(x)$ $P'(x)=8x^3+27x^2+12x-20$ $P''(x)=24x^2+54x+12$ $=6\left(4x^2+9x+2\right)$ $=6\left(4x^2+8x+x+2\right)$ $=6(4x(x+2)+(x+2))$ $=6(x+2)(4x+1)$ Zeros are $x=-2$ and $x=-\frac{1}{4}$ $P\left(-\frac{1}{4}\right)=-18.757\ldots\ P(-2)=0$ So $x=-2$ is the triple zero. $(x+2)^3=x^3+6x^2+12x+8$ Long division of $2x^4+9x^3+6x^2-20x-24$ by $x^3+6x^2+12x+8$: quotient $2x-3$ $2x^4+12x^3+24x^2+16x$ $-3x^3-18x^2-36x-24$ $-3x^3-18x^2-36x-24$ 0 Zeros are $x=-2$ (triple zero) and $x=1\frac{1}{2}$. **Alternative** $P(x)=(x+2)^3\cdot Q(x)$ In $(x+2)^3$ leading term is x^3 and constant term is 8 For $Q(x)$ leading term is $2x$ to give $2x^4$ *in* $P(x)$ For $Q(x)$ constant term is -3 to give -24 in $P(x)$ So $Q(x)=2x-3$ $\therefore$ remaining zero is $x=1\frac{1}{2}$ Zeros are $x=-2$ (triple zero) and $x=1\frac{1}{2}$.	3	1 for stating $P''(x)$ 1 for zeros of $P''(x)$ 1 for all zeros

Question 16	Trial HSC Mathematics Extension 2	2012	
Part	**Solution**	**Marks**	**Comment**
(d)	$x^2 + 6xy - 4y^2 = 10$ $2x + 6x.\frac{dy}{dx} + 6y - 8y.\frac{dy}{dx} = 0$ $2x + 6y = (8y - 6x)\frac{dy}{dx}$ $\frac{dy}{dx} = \frac{2x + 6y}{8y - 6x}$ At the point (2, 1) on the curve. $\frac{dy}{dx} = \frac{4 + 6}{8 - 12}$ $= \frac{10}{-4}$ $= -\frac{5}{2}$ So gradient at the point (2,1) is $-\frac{5}{2}$	3	1 for implicit differentiation 1 for $\frac{dy}{dx}$ 1 for gradient

Question 16	Trial HSC Mathematics Extension 2	2012	
Part	**Solution**	**Marks**	**Comment**
(e)	 Join the radius CE $\angle CEF = 90^\circ$ (angle between a tangent and radius $= 90^\circ$) Let $\angle EFH = \angle HFD = x^\circ$ Let $\angle CED = y^\circ$ CE = CD (radii of circle) $\angle EDC = y^\circ$ (Base angles of isosceles Δ). $\angle ECF = 2y^\circ$ (angle at centre is $2\times$ angle at circumference. In ΔEFC $2x + 2y + 90 = 180$ (Angle sum of Δ) $2x + 2y = 90$ $x + y = 45$ In ΔEHF, $\angle EHF + x + y + 90 = 180$ $\angle EHF = 90 - (x + y)$ $= 90^\circ - 45^\circ$ $= 45^\circ$ $\angle DHF = 180^\circ - \angle EHF$ $= 180 - 45^\circ$ $= 135^\circ$	3	1 for ΔEFC setup. 1 for ΔEHF setup. 1 for final answer