WESTERN REGION

2012

HIGHER SCHOOL CERTIFICATE

TRIAL EXAMINATION

Mathematics

Extension 1

General Instructions

- Reading Time - 5 minutes
- Working Time - 2 hours
- Write using a blue or black pen
- Board approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- Show all necessary working in Questions 11 - 14
- Begin each question on a new sheet of paper.

Total marks (70)

Section I

10 marks

- Attempt Questions 1-10
- Answer on the Multiple Choice answer sheet provided.
- Allow about 15 minutes for this section

Section II

60 marks

- Attempt questions 11 – 14
- Answer on the blank paper provided, unless otherwise instructed. Start a new page for each question.
- Allow about 1 hour 45 minutes for this section

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\quad n\neq -1;\quad x\neq 0,\ \text{if } n<0$$

$$\int \frac{1}{x}\,dx = \ln x,\quad x>0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax},\quad a\neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax,\quad a\neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax,\quad a\neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax,\quad a\neq 0$$

$$\int \sec ax\tan ax\,dx = \frac{1}{a}\sec ax,\quad a\neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\quad a\neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a},\quad a>0,\ -a<x<a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right),\quad x>a>0$$

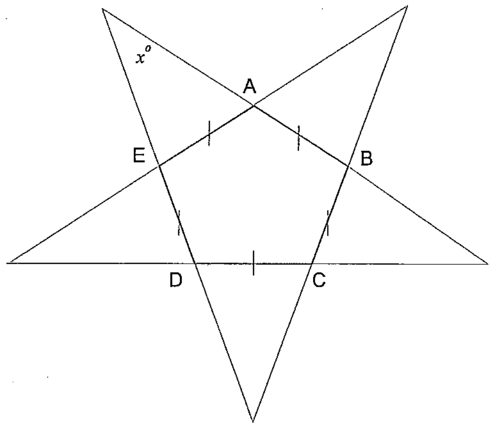
$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x,\quad x>0$

1. Calculate to 3 significant figures $\sqrt[5]{\dfrac{18.7+3.65}{\sqrt{(4.25)^3}}} =$

(A) 1.20 (B) 1.206 (C) 1.21 (D) 12.6

2. In the diagram, ABCDE is a regular pentagon. The value of x is:



(A) 90° (B) 36° (C) 108° (D) 72°

3. Find $\lim\limits_{x\to 0}\dfrac{\sin 7x}{5x}$

(A) $\dfrac{7}{5}$

(B) 1

(C) $\dfrac{5}{7}$

(D) 0

4. $\dfrac{d}{dx}[\cos(\ln x)] =$

(A) $-\sin(\ln x)$

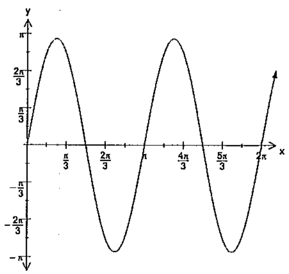
(B) $\dfrac{\cos(\ln x)}{x}$

(C) $\sin(\ln x)$

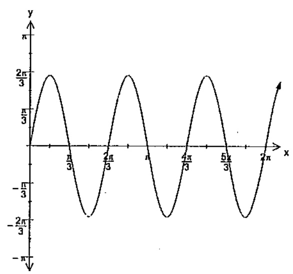
(D) $\dfrac{-\sin(\ln x)}{x}$

5. Which graph represents the curve $y = 3\sin 2x$

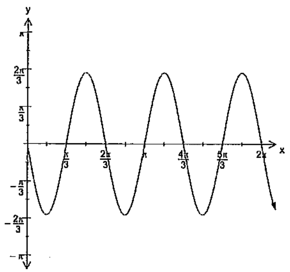
(A)



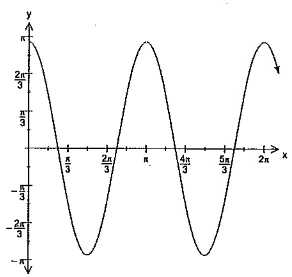
(B)



(C)



(D)



6. Find an approximation of the root of $y = e^x - 3x^2$ by using Newton's Method once and substituting with an approximation of $x = 3.8$ (answer correct to 2 decimal places)

(A) 3.74
(B) 4.22
(C) -12.06
(D) 3.7

7. $4+\frac{3}{x^2}-\frac{2}{x^3}+\frac{7}{x^5}$ can also be written as:

(A) $\frac{4x+3x-2x+7}{4x}$

(B) $\frac{4x^5+3x^3-2x^2+7}{x^5}$

(C) $\frac{4x^2+3x^2-2x^3-7x^4}{x^2}$

(D) $4+\frac{3x^5-2x^3+7x^2}{x^{10}}$

8. Two dice are rolled and the sum of the numbers is written down. Find the probability of rolling a total less than 6.

(A) $\frac{1}{4}$

(B) $\frac{5}{36}$

(C) $\frac{5}{12}$

(D) $\frac{5}{18}$

9. The correct factorisation of $8x^3 - 27y^3$ is:

(A) $(x-y)(x^2+xy+y^2)$

(B) $(2x-3y)(4x^2+6xy+9y^2)$

(C) $(4x-9y)(x^2+xy+y^2)$

(D) $(2x-3y)(4x^2-6xy+9y^2)$

10. We can express $\sin x$ and $\cos x$ in terms of $\tan \frac{x}{2}$, for all values of x except.......

(A) $x=\frac{\pi}{4},\frac{3\pi}{4},\frac{5\pi}{4}$........

(B) $x=\frac{\pi}{2},\frac{3\pi}{2},\frac{5\pi}{2}$........

(C) $x=\pi,3\pi,5\pi$........

(D) $x=2\pi,6\pi,8\pi$........

End of Section 1

Section II

Total marks (60)
Attempt Questions 11 - 14
Allow about 1 hour 45 minutes for this section.

Answer all questions, starting each question on a new sheet of paper with your name and question number at the top of the page. Do not write on the back of sheets.

Question 11 (15 Marks) Use a Separate Sheet of paper **Marks**

a) Find the obtuse angle between the lines (to nearest degree). **2**

$2x+3y=8$ and $x-2y=-5$.

b) Given that $x=5\sin\theta$ and $y=5\cos\theta+1$.

i) Show the equation relating x and y by eliminating θ is $y^2+x^2-2y-24=0$ **2**

ii) Graph this equation stating major features. **2**

c) A test consists of 10 multiple choice questions. Each question has 4 possible answers. A student guesses the answers to all the questions.

What is the probability that the student:

i) Guesses all questions correctly? **1**

ii) Only guesses 2 right answers? **1**

iii) Gets over 75% on the test? **2**

d) Find $\int 3x\sqrt{4-x}\ dx$ using the substitution $u=4-x$. **3**

e) Solve $\left(3+\frac{1}{x}\right)^2+4\left(3+\frac{1}{x}\right)-21=0$. **2**

End of Question 11

Question 12 (15 Marks) Use a Separate Sheet of paper **Marks**

a) Prove by induction, that **3**

$4^n>1+3n$ for $n>1$, where n is an integer..

b) A mobile phone company has a success rate of 65% when signing up new customers who enter a particular store. If 10 new customers walk into the store:

i) Find the probability as a percentage that 9 of these people sign up. **1**

ii) What is the most likely number of customers to sign up? **2**

c) Evaluate $\int_0^{\frac{\pi}{4}} \sin x\cos^2 x\ dx$. **2**

d) ABCDE are points on a circle radius 4 cm and $\angle DBC=\angle DAE$

i) Draw a diagram to represent this information. **1**

ii) Prove that the triangle formed by the points CDE is isosceles. **2**

e) Sketch the graph of $y=\cos^{-1}(x+3)$. **2**

f) Find the 6^{th} term in the expansion of $\left(3x-\frac{4}{5x^2}\right)^9$. **2**

End of Question 12

Question 13 (15 Marks) Use a Separate Sheet of paper **Marks**

a) Evaluate $\int_0^1 \frac{1}{\sqrt{4-2x^2}}\,dx$ in exact form. **2**

b) Storm is making a toffee dessert. The rate at which the toffee cools is proportional to the difference between the temperature of the toffee (T) and room temperature (R).

$$\text{ie. } \frac{dT}{dt} = -k(T-R)$$

i) Show that $T = R + Ce^{-kt}$, where C is a constant, is a solution of this differential equation. **1**

ii) Storm notices that a 2L pot of toffee initially cools from 540°C to 100°C in 50 minutes in a room whose temperature is 20°C. Storm can not put the toffee into the dessert until it reaches 40°C. How much longer does Storm need to wait to be able to add the toffee and finish her dessert (to the nearest minute)? **3**

iii) Explain or show by calculations, if it would take more or less time to create this dessert if the room temperature was 25°C. Assuming k and C remain the same. **2**

c) Calculate the exact volume generated by the solid formed when $y = \ln x - 1$ is rotated about the y-axis between $y = 0$ and $y = 1$. **2**

d) $P(x) = x^4 - 2x^3 + 5x^2 - 16x + 12$

i) Show that $(x-1)(x-2)$ is a factor of $P(x)$. **1**

ii) Hence find the remaining factor of $P(x)$. **1**

e) i) Show that the area of an equilateral triangle of side length (x-2) is given by: **1**

$$A = \frac{\sqrt{3}(x-2)^2}{4}$$

ii) The sides of the equilateral triangle are increasing at the rate of 5mm/s. At what rate is the area increasing at the instant when the sides are 10cm long? **2**

End of Question 13

2012 Trial HSC Examination — Mathematics Extension 1

Question 14 (15 Marks) Use a Separate Sheet of paper **Marks**

a) Zanthie bought a 'Splat Blaster' that fires paint balls at a velocity of 20ms^{-1}. A target has been placed on a tree, with its centre 2.5m from the ground. The base of the tree is 25m horizontally away from Zanthie. Zanthie holds the Splat Blaster at a height of 1.5m and wants to hit the centre of the target with a paint ball.

i) The equation of horizontal motion is given by $x = 20t\cos\theta$. Derive the equation of vertical motion. **2**

ii) To avoid overhead power lines; Zanthie must fire at an angle less than 45° At what angle should she fire the paint ball to hit the target on the tree? (assume $g = -9.8ms^{-2}$ and give your answer to the nearest degree). **3**

b) By expanding both sides of the identity **2**

$(1+x)^{n+4} = (1+x)^n(1+x)^4$ prove that

$$\binom{n+4}{r} = \binom{n}{r} + 4\binom{n}{r-1} + 6\binom{n}{r-2} + 4\binom{n}{r-3} + \binom{n}{r-4}.$$

c) The chord of contact of the tangents to the parabola $x^2 = 4ay$ from an external point $A(x_1, y_1)$ passes through the point $B(0, 2a)$. Find the equation of the locus of the midpoint of AB. **2**

d) A particle moves in a straight line and its position at time (t) is given by:

$$x = 4 + \frac{\sin 4t}{\sqrt{3}} - \cos 4t$$

i) Express $\frac{\sin 4t}{\sqrt{3}} - \cos 4t$ in the form $R\sin(4t - \alpha)$, where α is in radians. **2**

ii) The particle is undergoing Simple Harmonic Motion, show the equation for acceleration is: **2**

$$\ddot{x} = -16(x-4)$$

iii) When does the particle first reach its maximum speed? **2**

End of Examination

WESTERN REGION

2012
TRIAL HSC
EXAMINATION

Mathematics Extension 1

SOLUTIONS

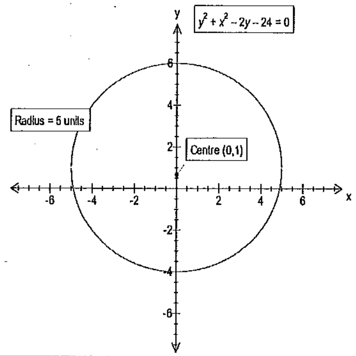
Trial HSC Examination - Mathematics Extension 1
Multiple Choice Answer Sheet

Name ______________________________

Completely fill the response oval representing the most correct answer.

	A	B	C	D
1.	○	○	●	○
2.	○	●	○	○
3.	●	○	○	○
4.	○	○	○	●
5.	●	○	○	○
6.	●	○	○	○
7.	○	●	○	○
8.	○	○	○	●
9.	○	●	○	○
10.	○	○	●	○

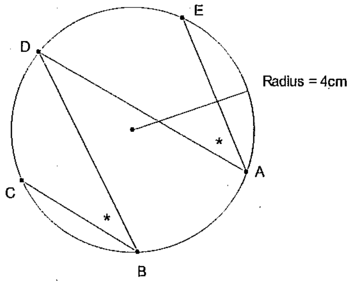
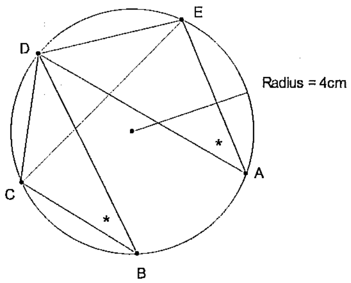
Question 11	Trial HSC Examination - Mathematics Extension 1		2012
Part	**Solution**	**Marks**	**Comment**
a)	$2x+3y=8$ $\quad$ $x-2y=-5$ $3y=8-2x$ $\quad$ $2y=x+5$ $y=\frac{8}{3}-\frac{2x}{3}$ $\quad$ $y=\frac{x}{2}+\frac{5}{2}$ $m_1=-\frac{2}{3}$ $\quad$ $m_2=\frac{1}{2}$ $\tan\alpha=\left\|\frac{m_1-m_2}{1+m_1m_2}\right\|$ $\tan\alpha=\left\|\frac{-\frac{2}{3}-\frac{1}{2}}{1+-\frac{2}{3}\times\frac{1}{2}}\right\|$ $\tan\alpha=\left\|\frac{-\frac{7}{6}}{\frac{2}{3}}\right\|$ $\tan\alpha=\frac{7}{4}$ $\alpha=60°$ So the obtuse angle is $180°-60°=120°$	2	1 for the gradients 1 for correct angle
b)i)	$x=5\sin\theta$..........(1) $y=5\cos\theta+1$................(2) $x=5\sin\theta$........(*squaring x*) $x^2=25\sin^2\theta$ $x^2=25(1-\cos^2\theta)$ $x^2=25-25\cos^2\theta$ $25\cos^2\theta=25-x^2$..........(3) $y=5\cos\theta+1$.........(*squaring y*) $y^2=25\cos^2\theta+10\cos\theta+1$......(4) From (1) $\cos\theta=\frac{y-1}{5}$...........(5) *sub*(3) & (5)into(4) $y^2=25-x^2+10\left[\frac{y-1}{5}\right]+1$ $y^2=25-x^2+2y-2+1$ $y^2+x^2-2y-24=0$	2	Alternate solution by making sin θ and cos θ the subject and using $\sin^2\theta+\cos^2\theta=1$ 1 for doing some substitution of x into y or into $\sin^2\theta+\cos^2\theta=1$ 1 for correctly getting the equation

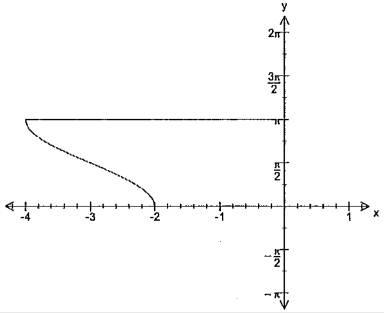
Question 11	Trial HSC Examination - Mathematics Extension 1		2012
Part	**Solution**	**Marks**	**Comment**
ii)	$y^2+x^2-2y-24=0$ $y^2+x^2-2y+1=24+1$ $x^2+(y-1)^2=25$ *Circle* *Centre*(0,1) *Radius* = 5*units* 	2	1 for realising it is a circle and completing the square 1 for correct graph with centre & radius
c)i)	P(correct) = 0.25 P(incorrect) = 0.75 ${}^{10}_{10}C(0.25)^{10}(0.75)^1$ $=\frac{1}{1048576}$	1	
ii)	${}^{10}_{2}C(0.25)^2(0.75)^8$ $=0.282$	1	
iii)	${}^{10}_{8}C(0.25)^8(0.75)^2+{}^{10}_{9}C(0.25)^9(0.75)^1+{}^{10}_{10}C(0.25)^{10}(0.75)^0$ $=4.15\times10^{-4}$	2	

Question 11	Trial HSC Examination - Mathematics Extension 1		2012
Part	**Solution**	**Marks**	**Comment**
d)	$\int 3x\sqrt{4-x}dx$ $u=4-x \quad x=4-u$ $du=-1dx$ $\int 3x\sqrt{4-x}dx$ $=3\int x\sqrt{4-x}dx$ $=-3\int(4-u)\sqrt{u}du$ $=-3\int(4-u)u^{\frac{1}{2}}du$ $=-3\int 4u^{\frac{1}{2}}-u^{\frac{3}{2}}du$ $=-3\left[\frac{8}{3}u^{\frac{3}{2}}-\frac{2}{5}u^{\frac{5}{2}}\right]+C$ $=-8\sqrt{(4-x)^3}+\frac{6\sqrt{(4-x)^5}}{5}+C$	3	1 1 1
e)	$\left(3+\frac{1}{x}\right)^2+4\left(3+\frac{1}{x}\right)-21=0$ $Let\ y=\left(3+\frac{1}{x}\right)$ $y^2+4y-21=0$ $(y+7)(y-3)=0$ $y=-7\ or\ 3$ $\therefore 3+\frac{1}{x}=-7$ or $3+\frac{1}{x}=3$ $\frac{1}{x}=-10$ $\quad \frac{1}{x}=0$ $x=-\frac{1}{10}$ $\quad$ No Solution So $x=-\frac{1}{10}$ is the only solution.	2	1 for using a substitution and finding values 1 for correct solution after resubstitution
		/15	

Question 12	Trial HSC Examination - Mathematics Extension 1		2012
Part	**Solution**	**Marks**	**Comment**
a)	$4^n>1+3n$ for $n>1$ $ie\ 4^n-3n-1>0$ Step 1 Show true for $n>1$ $ie\ n=2$ $4^2-3\times2-1>0$ $9>0\ \therefore true$ Step 2 Assume true for $n=k$ $4^k-3k-1>0$ Step 3 Prove true for $n=k+1$ $4^{k+1}-3(k+1)-1>0$ $4\times4^k-3k-3-1>0$ $4\times4^k-3k-4>0$ $4(4^k-3k-1)+9k>0$ since $4^k-3k-1>0$ and $9k>0$ Hence if the statement is true for $n=k$, then it is also true for $n=k+1$ The statement is true for $n=2$ and so it is true for $n=3$ and so on. Hence true for all $n>1$	3	1 mark for steps 1 and 2 2 marks for step 3, including conclusion.
b)i)	P(success) = (0.65) P(notsuccess) = (0.35) ${}^{10}_{9}C(0.65)^9(0.35)^1$ $=0.072$ $=7\%$	1	

Question 12	Trial HSC Examination - Mathematics Extension 1		2012
Part	**Solution**	**Marks**	**Comment**
ii)	We need the greatest term in the expansion $\frac{n-r+1}{r}\times\frac{b}{a}\geq 1$ $\frac{10-r+1}{r}\times\frac{0.35}{0.65}\geq 1$ $r\leq 3.85$ $\therefore r=3$ ${}^{10}_{3}C(0.65)^7(0.35)^3$ $\therefore$ 7 will be the most likely number of people to sign up.	2	1 for finding the greatest term 1 for correct number of sign ups.
c)	$\int_0^{\frac{\pi}{4}}\sin x\cos^2 x\ \ dx$ $=\left[\frac{\cos^3 x}{3}\right]_0^{\frac{\pi}{4}}$ $=-\frac{1}{3}\left[\left(\cos\frac{\pi}{4}\right)^3-(\cos 0)^3\right]$ $=-\frac{1}{3}\left[\left(\frac{1}{\sqrt{2}}\right)^3-1\right]$ $=-\frac{1}{3}\left[\frac{1}{2\sqrt{2}}-1\right]$ $=-\frac{1}{6\sqrt{2}}+\frac{1}{3}$ $=-\frac{-1+2\sqrt{2}}{6\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}$ $=\frac{4-\sqrt{2}}{12}$	2	Can also be done by substitution 1 correct integration 1 for answer in any form

Question 12	Trial HSC Examination - Mathematics Extension 1		2012
Part	**Solution**	**Marks**	**Comment**
d)i)	 	1	Must show all information, including radius
ii)	 $\angle DBC=\angle DAE(given)$ $ArcCD=ArcDE$ (converse of angles on the same arc) $\therefore CD=DE$ (equal arcs subtend equal chords) $\therefore \Delta CDE$ is isosceles as $CD=DE$	2 1 1	

Question 12	Trial HSC Examination - Mathematics Extension 1		2012
Part	**Solution**	**Marks**	**Comment**
e)		2	
f)	$\left(3x-\frac{4}{5x^2}\right)^9$ $T_{k+1}={}^nC_k a^{n-k}x^k$ $T_6={}^9C_5(3x)^{9-5}(-\frac{4}{5x^2})^5$ $=126(81x^4)-\frac{1024}{3125x^{10}}$ $=-\frac{10450944}{3125x^6}$	2	1 1
		/15	

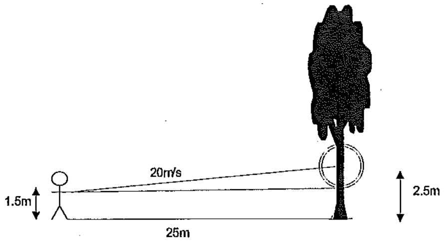
Question 13	Trial HSC Examination - Mathematics Extension 1		2012
Part	**Solution**	**Marks**	**Comment**
a)	$\int_0^1 \frac{1}{\sqrt{4-2x^2}}\,dx$ $\sqrt{4-2x^2}$ $=\sqrt{2(2-x^2)}$ $=\sqrt{2}\sqrt{2-x^2}$ $\therefore \int_0^1 \frac{1}{\sqrt{4-2x^2}}\,dx$ $=\frac{1}{\sqrt{2}}\int_0^1 \frac{1}{\sqrt{2-x^2}}$ $=\frac{1}{\sqrt{2}}\left[\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\right]_0^1$ $=\frac{1}{\sqrt{2}}\left(\left[\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)-\sin^{-1}\left(\frac{0}{\sqrt{2}}\right)\right]\right)$ $=\frac{1}{\sqrt{2}}\left[\frac{\pi}{4}-0\right]$ $=\frac{\pi}{4\sqrt{2}}$ $=\frac{\sqrt{2}\pi}{8}$	2	1 for getting in correct form and using standard integrals 1 answer
b)i)	$T=R+Ce^{-kt}$ $\frac{dT}{dt}=kCe^{-kt}$ and $T-R=Ce^{-kt}$ $\therefore \frac{dT}{dt}=-k(T-R)$ $\therefore T=R+Ce^{-kt}$ is a solution to the differential equation	1	

Question 13	Trial HSC Examination - Mathematics Extension 1		2012
Part	**Solution**	**Marks**	**Comment**
ii)	*When* $t=0$ $T=540°$ $R=20°$ $540=20+Ce^0$ $520=Ce^0$ $C=520$ $\therefore T=20+520e^{-kt}$ *When* $t=50$ $T=100°$ $R=20°$ $100=20+520e^{-50k}$ $\frac{80}{520}=-50k$ $k=\frac{\ln\left[\frac{8}{52}\right]}{-50}$ $k=0.037436043$ $\therefore T=20+520e^{-0.037436043t}$ *When* $T=40°$ $R=20°$ $40=20+520e^{-0.037436043\times t}$ $t=\frac{\ln\left[\frac{1}{26}\right]}{0.037436043}$ $t=87$ *minutes* Extra time $87-50=37$ So Storm must wait another 37 minutes for the Toffee to cool enough	3	1 for C 1 for k 1 for correct answer must have used full value of k
iii)	$40=25+520e^{-0.037436043\times t}$ $t=\frac{\ln\left[\frac{3}{104}\right]}{0.037436043}$ $t=95$minutes $\therefore$ it would take longer for the toffee to cool in a room at 25°	2	1 for the calculation or other explanation 1 for a statement stating the result

Question 13	Trial HSC Examination - Mathematics Extension 1		2012
Part	**Solution**	**Marks**	**Comment**
c)	$y=\ln x-1$ $y+1=\ln x$ $e^{y+1}=x$ $x^2=\left(e^{y+1}\right)^2$ $=e^{2y+2}$ $V=\pi\int_a^b x^2dy$ $=\pi\int_0^1 e^{2y+2}dy$ $=\pi\left(\frac{e^{2y+2}}{2}\right)_0^1$ $=\pi\left\{\left[\frac{e^4}{2}\right]-\left[\frac{e^2}{2}\right]\right\}$ $=\frac{\pi}{2}\left(e^4-e^2\right)units^3$	2	1 for finding x^2 1 for answer
d)i)	$P(x)=x^4-2x^3+5x^2-16x+12$ $P(1)=1^4-2\times1^3+5\times1^2-16\times1+12$ $P(1)=1-2+5-16+12$ $P(1)=0$ $\therefore(x-1)$ is a factor $P(2)=2^4-2\times2^3+5\times2^2-16\times2+12$ $P(2)=16-16+20-32+12$ $P(2)=0$ $\therefore(x-2)$ is a factor $\therefore(x-1)(x-2)$ is a factor of $P(x)=x^4-2x^3+5x^2-16x+12$	1	

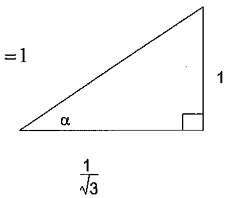
Question 13	Trial HSC Examination - Mathematics Extension 1		2012
Part	**Solution**	**Marks**	**Comment**
ii)	$(x-1)(x-2)=x^2-3x+2$ Long division: $(x^4-2x^3+5x^2-16x+12)\div(x^2-3x+2)$ gives quotient x^2+x+6 $x^4-3x^3+2x^2$ x^3+3x^2-16x x^3-3x^2+2x $6x^2-18x+12$ $6x^2-18x+12$ 0 $\therefore$ The remaining factor is (x^2+x+6) which cannot be reduced to linear factors.	2	1 1
e)i)	$A=\frac{1}{2}ab\sin C$ $A=\frac{1}{2}\times(x-2)\times(x-2)\times\sin 60°$ $A=\frac{1}{2}(x-2)^2\times\frac{\sqrt{3}}{2}$ $A=\frac{\sqrt{3}(x-2)^2}{4}$	1	

Question 13	Trial HSC Examination - Mathematics Extension 1		2012
Part	**Solution**	**Marks**	**Comment**
ii)	*Let* $s=x-2$ (*side length*) $\frac{ds}{dx}=1$ and $\frac{ds}{dt}=5mm/s=0.5cm/s$ $\frac{ds}{dt}=\frac{ds}{dx}\times\frac{dx}{dt}$ $0.5=1\times\frac{dx}{dt}$ $\frac{dx}{dt}=0.5$ $A=\frac{\sqrt{3}(x-2)^2}{4}$ $\frac{dA}{dx}=\frac{\sqrt{3}(x-2)}{2}$ $\frac{dA}{dt}=\frac{dA}{dx}\times\frac{dx}{dt}$ $\frac{dA}{dt}=\frac{\sqrt{3}(x-2)}{2}\times 0.5$ *when* $s=10, x=12$ $\frac{dA}{dt}=\frac{\sqrt{3}(12-2)}{2}\times 0.5$ $=\frac{5\sqrt{3}}{2}cm^2/s$ So the area of the triangle is increasing at $4.33cm^2/s$	2	1 Both exact or decimal answer OK 1
		/15	

Question 14	Trial HSC Examination - Mathematics Extension 1		2012
Part	**Solution**	**Marks**	**Comment**
a)		2	1 for each
i)	Vertical Motion $\ddot{y} = -g = -9.8ms^{-2}$ $\therefore \dot{x} = \int -9.8dt$ $= -9.8t + C_2$ *When* $t = 0$ $\dot{y} = V\sin\theta$ *So* $C = V\sin\theta$ $\dot{y} = V\sin\theta$ $20\sin\alpha = -9.8t + C_2$ $C_2 = 20\sin\alpha$ $\dot{y} = -9.8t + 20\sin\alpha$ $y = \int -9.8t + 20\sin\alpha dt$ $= -4.9t^2 + 20t\sin\alpha + C_3$ *When* $t = 0$ $y = 1.5$ $0 = 0 + C_1$ $\therefore C_3 = 1.5$ $\therefore y = -4.9t^2 + 20t\sin\alpha + 1.5$		1 for using correct integrations 1 for final result.

Question 14	Trial HSC Examination - Mathematics Extension 1		2012
Part	**Solution**	**Marks**	**Comment**
ii)	$x = 20t\cos\alpha ....(1)$ $y = -4.9t^2 + 20t\sin\alpha + 1.5 ....(2)$ *From*(1) $t = \frac{x}{20\cos\alpha} ....(3)$ *sub* (3) into (2) $y = -4.9\left[\frac{x}{20\cos\alpha}\right]^2 + 20\left[\frac{x}{20\cos\alpha}\right]\sin\alpha + 1.5$ $= \frac{-4.9x^2}{400\cos^2\alpha} + \frac{x\sin\alpha}{\cos\alpha} + 1.5$ $= \frac{-4.9}{400}x^2\sec^2\alpha + x\tan\alpha + 1.5$ $= \frac{-4.9}{400}x^2(1+\tan^2\alpha) + x\tan\alpha + 1.5$ For the ball to hit the target $x = 25$ *and* $y = 2.5$ $2.5 = \frac{-4.9}{400}25^2(1+\tan^2\alpha) + 25\tan\alpha + 1.5$ $2.5 = -7.65625(1+\tan^2\alpha) + 25\tan\alpha + 1.5$ $0 = -7.65625\tan^2\alpha - 25\tan\alpha + 8.65625$	3	 1
ii)	Let $x = \tan\alpha$ Solve using quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \frac{25 \pm \sqrt{25^2 - (4 \times 7.65625 \times 8.65625}}{2 \times 7.65625}$ $x = \frac{25 \pm 18.97}{15.3125}$ $x = 2.87$ or 0.39 $\tan\alpha = 2.87$ *or* $\tan\alpha = 0.39$ $\alpha = 70°47'$ *or* $21°29'$ However to avoid the air resistance Zanthie must shoot at an angle of 21°		1 1

Question 14	Trial HSC Examination - Mathematics Extension 1		2012
Part	**Solution**	**Marks**	**Comment**
b)	$(1+x)^{n+4}=(1+x)^n(1+x)^4$ $\binom{n+4}{r}=\binom{n}{r}+4\binom{n}{r-1}+6\binom{n}{r-2}+4\binom{n}{r-3}+\binom{n}{r-4}$ *LHS* $(1+x)^{n+4}=1+\binom{n+4}{r}x+......+\binom{n+4}{r}x^r+....$ coefficient of $x^r=\binom{n+4}{r}$ *RHS* $(1+x)^n(1+x)^4=\left[1+\binom{n}{1}x+......+\binom{n}{r}x^r+....\right]\times$ $\left[1+\binom{4}{1}x+\binom{4}{2}x^2+\binom{4}{3}x^3+\binom{4}{4}x^4\right]$ coefficient of $x^r=\binom{n}{r}+\binom{n}{r-1}\binom{4}{1}+\binom{n}{r-2}\binom{4}{2}+$ $\binom{n}{r-3}\binom{4}{3}+\binom{n}{r-4}\binom{4}{4}$ $=\binom{n}{r}+4\binom{n}{r-1}+6\binom{n}{r-2}+4\binom{n}{r-3}+\binom{n}{r-4}$ Because coefficients of x^r in both expansions are the same it follows $\binom{n+4}{r}=\binom{n}{r}+4\binom{n}{r-1}+6\binom{n}{r-2}+4\binom{n}{r-3}+\binom{n}{r-4}$	2	1

Question 14	Trial HSC Examination - Mathematics Extension 1		2012
Part	**Solution**	**Marks**	**Comment**
c)	Equation of the chord of contact $xx_1=2a(y+y_1)$ This passes through $B(0,2a)$ $0x_1=2a(2a+y_1)$ $0=4a^2+2ay_1$ $\frac{2ay_1}{2a}=\frac{-4a^2}{2a}$ $y_1=-2a$ Locus of the Midpoint AB $A(x_1,-2a)$ & $B(0,2a)$ $y=\frac{-2a+2a}{2}$ $\quad x=\frac{x_1}{2}$ $y=0$ $\therefore y=0$ is the equation of the locus of the midoint AB	2	1 for finding y_1 1 for the equation of the locus
d)i)	$R\sin(4t-\alpha)=R\sin 4t\cos\alpha-R\cos 4t\sin\alpha$ *If* $R\sin(4t-\alpha)=\frac{1}{\sqrt{3}}\sin 4t-\cos 4t$ *then* $r\cos\alpha=\frac{1}{\sqrt{3}}$ and $r\sin\alpha=1$  continued over	2	

Question 14	Trial HSC Examination - Mathematics Extension 1		2012
Part	**Solution**	**Marks**	**Comment**
	$R=\sqrt{\left(\frac{1}{\sqrt{3}}\right)^2+1}$ $=\sqrt{\frac{1}{3}+1}$ $=\sqrt{\frac{4}{3}}$ $=\frac{2}{\sqrt{3}}$ $\tan\alpha=\frac{R\sin\alpha}{R\cos\alpha}$ $=\frac{1}{\frac{1}{\sqrt{3}}}$ $=\sqrt{3}$ $\therefore\tan\alpha=\sqrt{3}$ $\alpha=\frac{\pi}{3}$ $\therefore\frac{\sin 4t}{\sqrt{3}}-\cos 4t=R\sin(4t-\alpha)$ $\frac{\sin 4t}{\sqrt{3}}-\cos 4t=\frac{2}{\sqrt{3}}\sin(4t-\frac{\pi}{3})$		1 for deriving R 1 for deriving α
ii)	$x=4+\frac{\sin 4t}{\sqrt{3}}-\cos 4t$ *from part(i)* $x=4+\frac{2}{\sqrt{3}}\sin(4t-\frac{\pi}{3})$ $\dot{x}=\frac{8}{\sqrt{3}}\cos(4t-\frac{\pi}{3})$ $\ddot{x}=-\frac{32}{\sqrt{3}}\sin(4t-\frac{\pi}{3})$ $\ddot{x}=-16\left[\frac{2}{\sqrt{3}}\sin(4t-\frac{\pi}{3})\right]$ and from (1) $x-4=\frac{2}{\sqrt{3}}\sin(4t-\frac{\pi}{3})$ $\ddot{x}=-16(x-4)$	2	 1 1

Question 14	Trial HSC Examination - Mathematics Extension 1		2012
Part	**Solution**	**Marks**	**Comment**
iii)	The maximum speed occurs when the particle is at the centre of motion. When $x=4$ $4=4+\frac{2}{\sqrt{3}}\sin(4t-\frac{\pi}{3})$ $\frac{2}{\sqrt{3}}\sin(4t-\frac{\pi}{3})=0$ $\sin(4t-\frac{\pi}{3})=0$ $4t-\frac{\pi}{3}=0,\pi,2\pi,........$ $4t=\frac{\pi}{3},\frac{4\pi}{3},......$ $t=\frac{\pi}{12},\frac{\pi}{3},..........$ The particle first reaches maximum speed when $t=\frac{\pi}{12}$ *seconds*	2	 1 1
		/15	