Western Region
Trial Higher School Certificate Examination
1994

MATHEMATICS
2/3 UNIT

Time Allowed - THREE Hours
(Plus 5 minutes reading time)

DIRECTIONS TO CANDIDATES:

- All questions may be attempted.
- All questions are of equal value.
- All necessary working should be shown in every question.
  Marks may not be awarded for careless or badly arranged work.
- Standard integrals are printed on the last page which may be removed for your convenience.
- Board approved calculators may be used.

Question 1

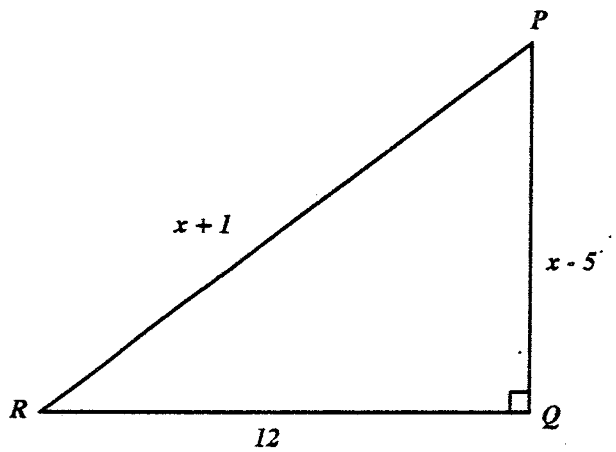
a. Find the value of $\frac{256}{\sqrt{a+b}}$ if $a = 4 \cdot 2$ and $b = 19 \cdot 54$.

Give your answer correct to 2 decimal places.

b. Solve for x:

$|3-2x|<5$

c. In the diagram, angle PQR is a right angle and the lengths of the sides are 12cm, $(x + 1)$cm and $(x - 5)$cm as shown.



i. Use Pythagora's Theorem to find the value of x.

ii. Find the area of the triangle.

d. A manufacturer increases the price of a car by 15% to a new selling price of \$23 000. What was the selling price of the car before the increase?

e. i. Factorise x^3+1

ii. Hence evaluate $\lim_{x \to -1} \frac{x^3+1}{x+1}$

Question 2

a. The vertices of the triangle ABC are the points $A(0,5)$, $B(3,6)$ and $C(4,0)$.

i. Draw a sketch diagram of the triangle to show this information.

ii. Find the gradient (slope) of the side AB.

iii. Find the equation of the line AB giving your answer in the form $ax + by + c = 0$.

iv. A line through C, parallel to AB cuts the y axis at M. Find the co-ordinates of M.

v. Is triangle ABC right angled at B? Justify your answer.

b. i. A gambler has two custom made dice.

One die has only the odd numbers (1, 1, 3, 3, 5 and 5) on its six faces while the other die has only the even numbers (2, 2, 4, 4, 6 and 6) on its six faces. By drawing a tree diagram, or otherwise, determine the probability of getting a total of seven when the two dice are rolled together.

ii. In a certain town, the probability of an adult catching a cold during 1994 is 0·4. Two adults in the town are chosen at random.

What is the probability that at least one of them will catch a cold in 1994?

Question 3

a. Differentiate, with respect to x:

i. $(2x+1)^2$

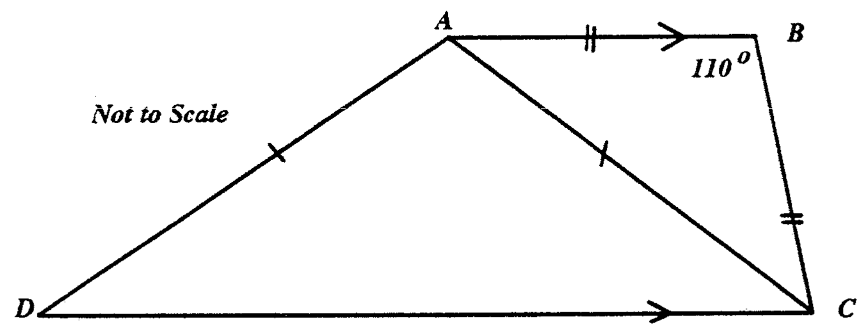
ii. $\dfrac{\ln 2x}{x}$

iii. e^{2x+1}

b. Find $\displaystyle\int \frac{2x}{x^2+1}\,dx$

c. Evaluate $\displaystyle\int_0^{\frac{\pi}{3}} \sin 2x \, dx$

d. In the diagram, $AB \parallel DC$, $AB = BC$, $AD = AC$, and $\angle ABC = 110°$ as shown.

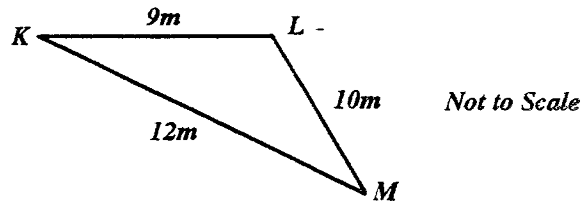


i. Copy the diagram onto your answer sheet.

ii. Find, giving reasons, the size of $\angle DAC$.

Question 4

a.



KLM is a triangle with $KL = 9$m, $LM = 10$m and $MK = 12$m as in the above figure.

i. Use the cosine rule to find the size of angle MKL to the nearest degree.

ii. Calculate the area of the triangle KLM to the nearest square metre.

b. It is assumed that the number $N(t)$ of fleas on a camel at time $t \geq 0$ is given by $N(t) = \dfrac{A}{1+e^{-t}}$, where A is a constant and t is measured in weeks.

i. At time $t = 0$, $N(t)$ is estimated at 2000 fleas. What is the value of A?

ii. What is the value of $N(t)$ after 2 weeks?

iii. How many fleas would you expect to find on the camel when time t is large?

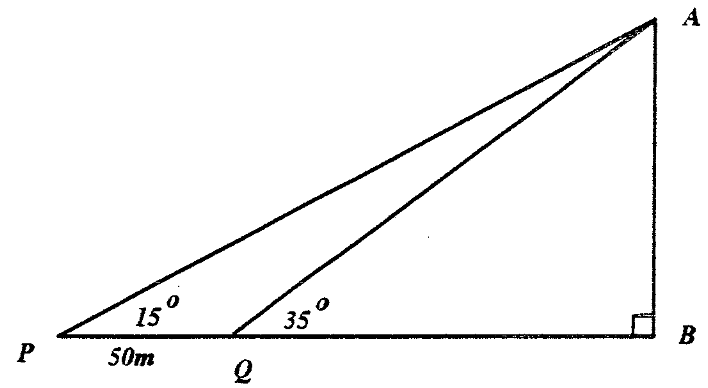
iv. After how many days does the number of fleas first exceed 2500?

v. Find an expression for the rate at which the number of fleas increases at any time t.

Question 5

a. Find all solutions to the equation $2^{2n} - 5.2^n + 4 = 0$

b. The angles of elevation of the top of a vertical tower, AB, from two points, P and Q, on the same horizontal plane as the base of the tower are $15°$ and $35°$ respectively.



i. Find the size of angle PAQ.

ii. If P and Q are 50 metres apart, use the Sine Rule to find the length of AQ correct to one decimal place.

iii. Hence, find the height of the tower to the nearest metre.

c. If α and β are the roots of the equation $2x^2 + 6x + 3 = 0$, find the value of:

i. $\alpha + \beta$

ii. $\alpha\beta$

iii. $\dfrac{\alpha}{\beta} + \dfrac{\beta}{\alpha}$

Question 6

a. Express $\frac{1}{3-\sqrt{7}}$ in the form $a+b\sqrt{7}$,

where a and b are rational numbers.

b. Show that the volume, obtained by rotating about the x axis, the area bounded by the curve $y=e^x$, the co-ordinate axes, and the line $x=2$, has magnitude $\frac{\pi}{2}\left(e^4-1\right)$.

c. The numbers m and n are such that m, n and 12 form a geometric progression, while $(m+1)$, $(n+2)$ and 12 form an arithmetic progression.

i. Show that

$$12m=n^2$$

and $$2n-m=9$$

ii. Solve these equations to find all possible values of m and n.

iii. One of these geometric progressions so defined has a limiting sum. Find this sum.

Question 7

a. Given that $5x^2 - 2x + 3 \equiv A(x-1)(x-2) + B(x-1) + C$ for all values of x, find the value of A, B and C.

b. Consider the curve given by $y = x^3 - 3x - 1$ for $-3 \leq x \leq 2$.

i. Find all the stationary points and determine their nature.

ii. Find the point of inflexion.

iii. Sketch the curve for $-3 \leq x \leq 2$ showing all its essential features.

Question 8

a. Find the values of m for which the equation $4x^2 - mx + 9 = 0$ has

i. exactly one real root.

ii. real roots.

b. On a number plane a region is defined by the two conditions:

$$y \geq (x-3)^2$$

and $$y \leq 4$$

i. Show by shading on a sketch, the region given by these two conditions.

ii. Calculate the area of this shaded region.

Question 9

a. A parabola P has equation $x^2 = 16(y-1)$.

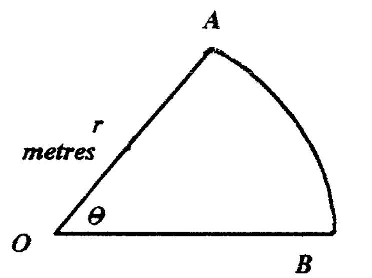
Draw a neat sketch of P and clearly indicate on it:

i. the co-ordinates of the vertex.

ii. the co-ordinates of the focus.

iii. the equation of the directrix.

b.



In the figure AO and BO are radii of length r metres, of a circle centre O. The arc AB of the circle subtends and angle θ radians at O.

i. Write down the formulae for

α. the length of the arc AB.

β. the area if the sector AOB.

ii. The perimeter of the above figure AOB is 6 metres. Show that the area, A square metres, of the sector AOB is given by

$$A = \frac{18\theta}{(\theta+2)^2}$$

iii. Show that the maximum area of the sector occurs when θ is 2 radians.

Question 10

a. A point P moves such that it is always 3 units from a fixed point $Q(0, 3)$.

i. Draw a sketch of the locus of P.

ii. Show that the equation of this locus is given by $x^2 + y^2 - 6y = 0$.

b. Copy and complete the table of values for $y = \ln\left(\frac{1+x}{1-x}\right)$ on your answer sheet.

x	0	0·2	0·4	0·6	0·8
y	0		0·847		2·197

i. By using Simpson's rule with 5 function values, estimate the integral:

$$\int_0^{0\cdot8} \ln\left(\frac{1+x}{1-x}\right) dx.$$

Give your answer to two significant figures.

ii. Prove that the derivative of $y = \ln\left(\frac{1+x}{1-x}\right)$ is $\frac{2}{1-x^2}$.

iii. Show that $y = \ln\left(\frac{1+x}{1-x}\right)$ has no stationary points.

iv. State the domain and range of $y = \ln\left(\frac{1+x}{1-x}\right)$.

(1)(a) 52.54

(b) $-1 < x < 4$

(c) (i) $x = 14$ (ii) 54 cm^2

(d) \$20 000

(e) (i) $(x+1)(x^2 - x + 1)$

(ii) 3

(2) (a) (i) Sketch

(ii) $\frac{1}{3}$ (iii) $x - 3y + 15 = 0$

(iv) $M\left(0. -\frac{4}{3}\right)$

(v) Yes, because $AB \perp BC$

(b) (i) $\frac{1}{3}$ (ii) 0.64

(3) (a) (i) $4(2x-1)$

(ii) $\dfrac{1 - \ln 2x}{x^2}$

(iii) $2e^{2x+1}$

(b) $\ln(x^2+1) + c$

(c) $\dfrac{3}{4}$

(d) (i) Diagram (ii) 110^0

(4) (a) (i) $\angle MKL = 55^0$

(ii) 44 m^2

(b) (i) 4000 (ii) 3523

(iii) As $t \to \infty, N \to 4000$

(iv) $3\frac{1}{2}$ days

(v) $\dfrac{4000e^{-t}}{(1+e^{-t})^2}$

(5) (a) $n = 0$ and 2

(b) (i) $\angle PAQ = 20^0$

(ii) $AQ = 37.8$m

(iii) $AB = 22$ m (nearest m)

(c) (i) -3 (ii) $\frac{3}{2}$ (iii) 4

(6) (a) $\frac{3}{2} + \frac{1}{2}\sqrt{7}$

(b) $\frac{\pi}{2}(e^4 - 1)$

(c) (i) Proof

(ii) $m = 3, 27$ and $n = 6, 18$

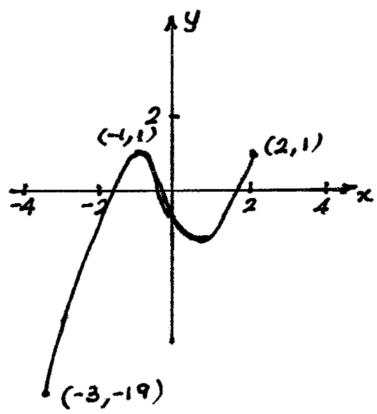
(iii) 81

(7) (a) $A = 5, B = 13, C = 6$

(b) (i) $(1, -3)$ rel. min

$(-1, 1)$ rel. max.

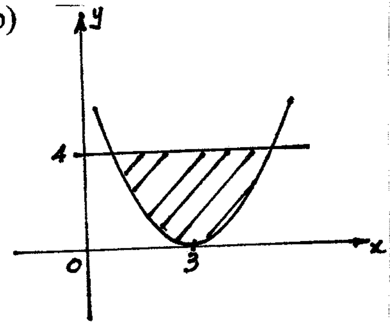
(ii) Inflexion at $(0, -1)$



(8) (a) (i) $m = \pm 12$

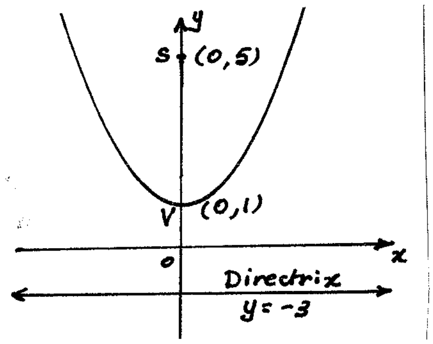
(ii) $m \le -12, m \ge 12$

(b)



(ii) $10\frac{2}{3}$ units^2

(9)(a)

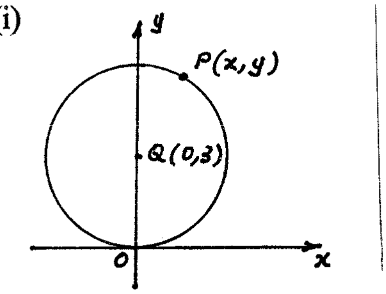


(b) (i) (α) $l = r\theta$

(β) Area $= \frac{1}{2}r^2\theta$

(ii) and (iii) Proofs

(10) (a) (i)



(ii) Proof

(b) (i) 0.74 (2 s.f)

(ii) Proof

(iii) Proof

(iv) $D: -1 < x < 1$

R: All real y