Western Region

Trial Higher School Certificate Examination

1995

MATHEMATICS

2/3 UNIT (COMMON)

Time Allowed - THREE Hours
(Plus 5 minutes reading time)

DIRECTIONS TO CANDIDATES :

* Attempt ALL questions
* ALL questions are of equal value
* The mark value of each question part is typed in BOLD beside the question.

eg : Question 1

2 (a) indicates that the question is worth 2 marks.

* ALL necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
* Standard Integrals are printed on the last page. These may be removed for your convenience
* Board-approved calculators may be used
* Each question should be started on a new page

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$

Question 3. **Start a new Page.** (12 Marks)

(5) (a) Differentiate :

(i) $\sin 2x \div 2\tan x$

(ii) $\ln(x^2 - 5x)$

(iii) $2xe^{3x}$

(5) (b) Find :

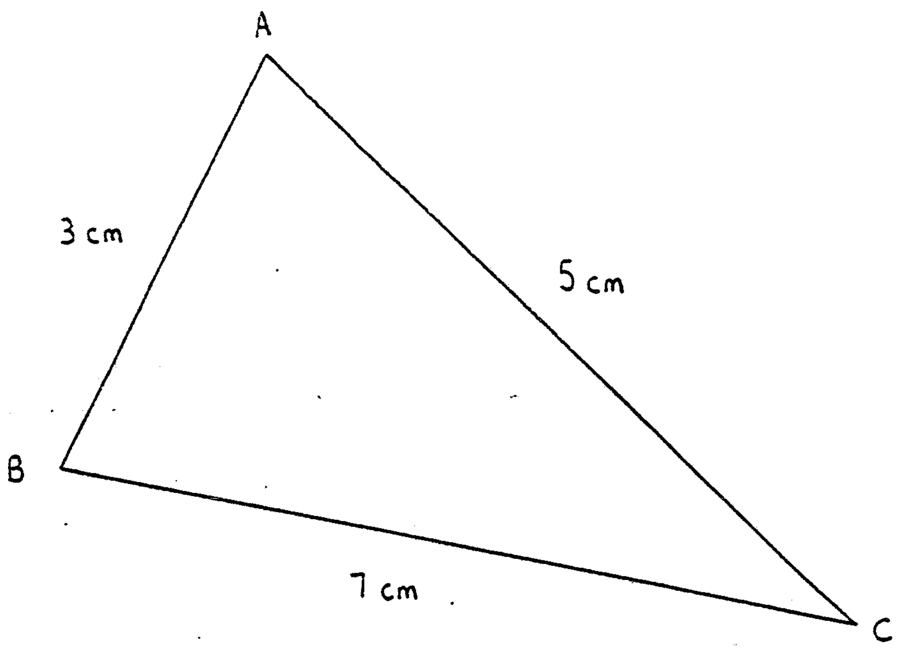
(i) $\int \sqrt{x}\,.\,dx$

(ii) $\int (10x \div 7)^5\,.\,dx$

(iii) $\int \frac{x \div 4}{x^2 \div 8x}\,.\,dx$

(2) (c) In triangle ABC, drawn below, calculate the size of the smallest angle to the nearest whole degree.

Diagram
not to Scale



Question 1. Start a new Page. (12 Marks)

(2) (a) Evaluate $\dfrac{2.7 + 3.4}{5.8 \times 2.7}$ correct to one decimal place.

(2) (b) Factorise $mx - my \div 9x - 9y$

(2) (c) Simplify $\dfrac{3a}{2} \div \dfrac{a}{3}$

(2) (d) Solve $7^{x+2} = 343$

(2) (e) A point P moves on the number plane such that its distance from the x axis is always 3 units. What is the equation of the locus of the point P ?

(2) (f) If the Space Shuttle travels at a constant speed of 1.5×10^4 km / hour. How long would it take to reach the moon, a distance of 3.84×10^5 kilometres from the Earth ?

Question 2. Start a new Page. (12 Marks)

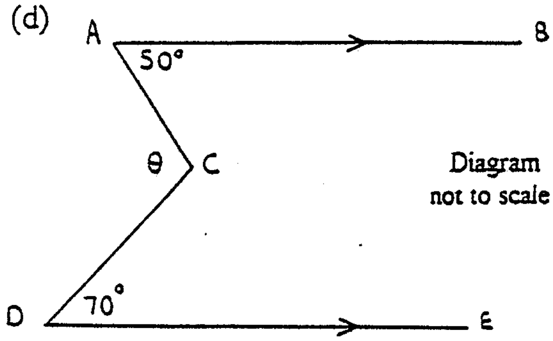
(2) (a) Using a suitable method, show that $0.\dot{5}\dot{3}$ can be written as a common fraction.

(2) (b) Rationalise the denominator of $\dfrac{5}{\sqrt{7} \div 2}$

(4) (c) Solve (i) $|x \div 5| > 8$

(ii) $x^2 < 9$

(2) (d)



In the diagram, AB is parallel to DE
Find the value of θ, giving reasons.

(2) (e) For the parabola $x^2 = 16(y - 1)$,
state (i) the focal length
(ii) the co-ordinates of the focus.

Question 4. Start a new Page. (12 Marks)

(5) (a) A Year 12 Biology class tested to see how much bacteria was present in a variety of food samples at their school canteen. It is known, that after t hours the number of bacteria (N) present in a particular type of food is given by the formula

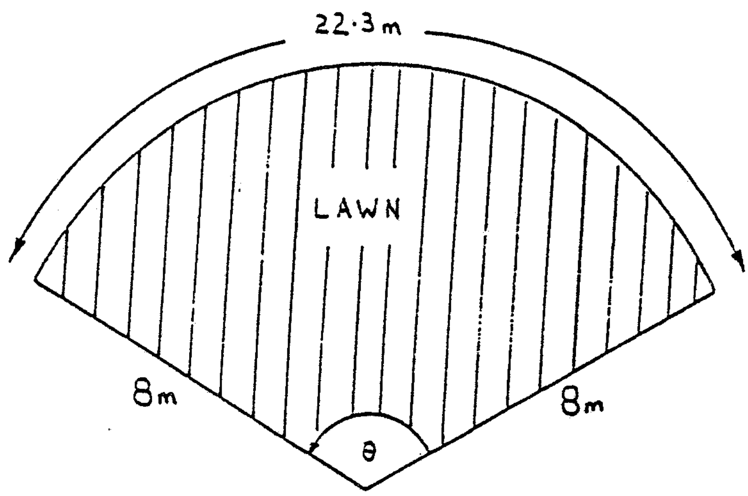
$$N = Ae^{kt}$$

(i) If initially, there were 20 000 bacteria present, calculate the value of A.

(ii) After three hours, there were 45 000 bacteria present. Calculate the value of k correct to 2 decimal places.

(iii) How long would it take for the initial number of bacteria to treble in quantity ?

(7) (b) A lawn Sprinkler sprays over a sector of radius 8 metres, as shown in the diagram below.



(i) If the length of the arc traced out by the end of the sprinkler's spray is 22.3 metres, show that the angle θ, through which the sprinkler turns is 160°.

(ii) Calculate, to the nearest square metre, the area of lawn which is watered by the sprinkler.

(iii) If fertiliser is spread on the lawn at a rate of 1 kg per 25 square metres, how much fertiliser would be required for the lawn.

(iv) The sprinkler can deliver 1 500 ml of water for every 5 degrees of turning. How much water, in litres, is sprayed onto the lawn in one complete sweep of the sprinkler ?

Question 5. Start a new Page. (12 Marks)

(7) (a) The points A(-1,1), B(2,3) and C(3,-2) are the coordinates of the vertices of a triangle.

(i) Draw a number plane on your page and plot the points A, B and C

(ii) Find the length of the side AC

(iii) Show that the equation of the side AC is $3x + 4y - 1 = 0$

(iv) Determine the shortest distance of the point B from the line AC

(v) Calculate the area of the triangle ABC.

(3) (b) (i) Sketch the graph of $y = \cos 2\theta$ for $0 \leqslant \theta \leqslant \pi$

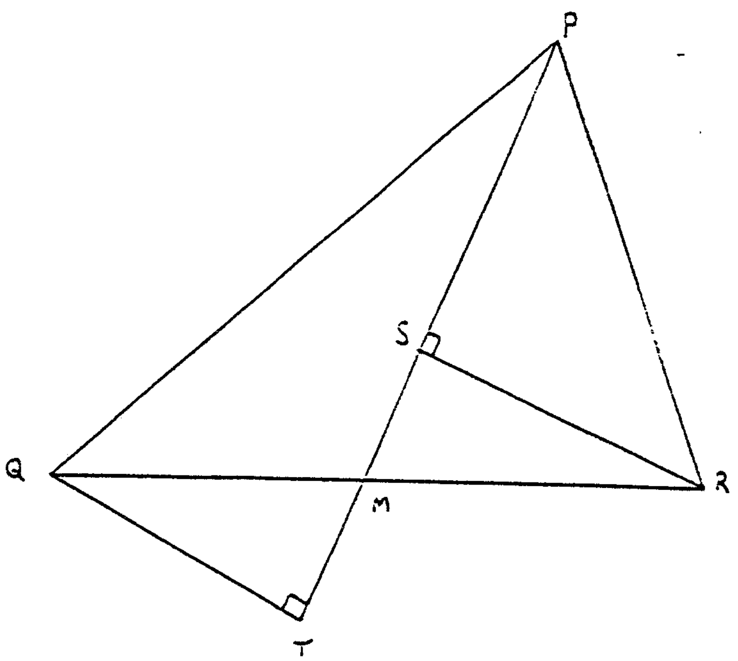
(ii) How many solutions over the domain $0 \leqslant \theta \leqslant \pi$ does $\cos 2\theta = 1/2$ have ?

(iii) Accurately, determine the solutions to $\cos 2\theta = 1/2$ over this domain.

(2) (c) An amount of $7 000 is invested with a building society at an interest rate of 8% p.a. compounded quarterly. What would the original investment amount to after a period of 3 years ?

(3) (b) In the diagram below, QT and RS are perpendicular to the line PT.
M is the midpoint of the interval QR.

Diagram not to scale

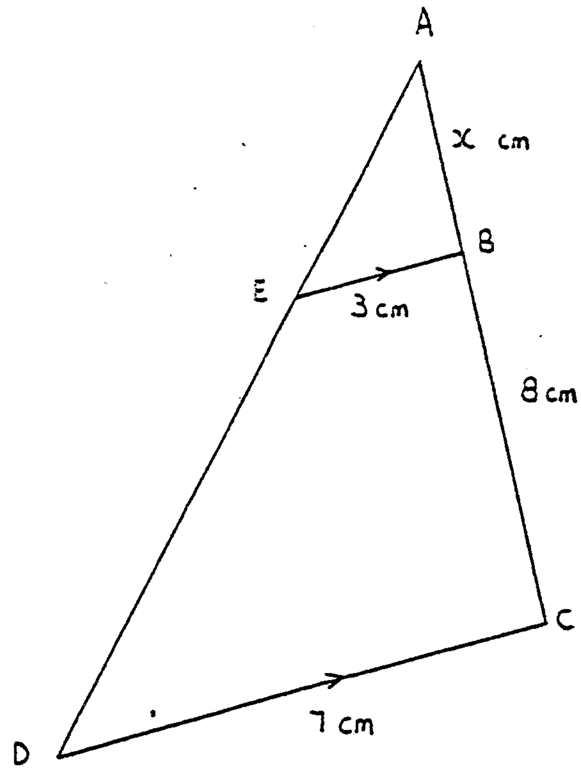


(i) Prove, giving reasons, that triangles QMT and RMS are congruent.

(ii) If PT = 21 cm and SP = 12 cm, what is the length of TM ?
Give reasons for your answer.

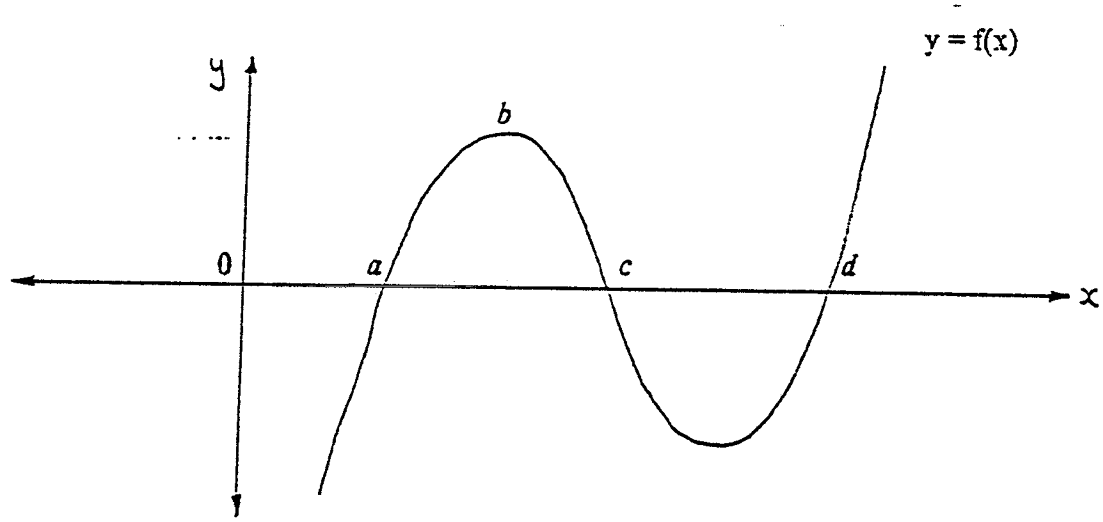
(2) (c) In the diagram below it is known that AB = x cm, BC = 8 cm, EB = 3 cm and DC = 7 cm. The triangles ABE and ACD are similar. Find the value of x.

Diagram not to scale.



Question 6. Start a new Page. (12 Marks)

(7) (a) The diagram below shows the graph of $y = f(x)$. The curve has point symmetry about the point c. Use this graph to answer the following questions.



(i) For what values of x is $f(x) < 0$?

(ii) If a tangent is drawn at any point on the curve $y = f(x)$ between the points a and c, explain how the sign of the derivative varies in moving from the point a through to c.

(iii) Copy this graph neatly onto your own paper.
On the same set of axes, draw a sketch of the derivative $y = f'(x)$ of the function.

(iv) To calculate the area enclosed between the curve $y = f(x)$ and the x axis, the following integral was used ;

$$\int_a^d f(x)\,.\,dx$$

Would this integral achieve the correct result ? Explain your answer.

Question 8. Start a new page. (12 Marks)

(6) (a) A TV games show allows a contestant to win a prize if they select a correct combination of coloured cardboard stars from a small barrel. The contestants are told that the barrel contains 5 red stars, 7 yellow stars and 4 green stars. They are asked to choose up to two stars from the barrel. They must nominate beforehand, the colours they think they will draw out and place them on the games board in front of them as they go.
Using a tree diagram or otherwise, find the probability that a contestant will win if they nominate to select the following ;

(i) a single red star with their first selection.

(ii) two stars of the same colour.

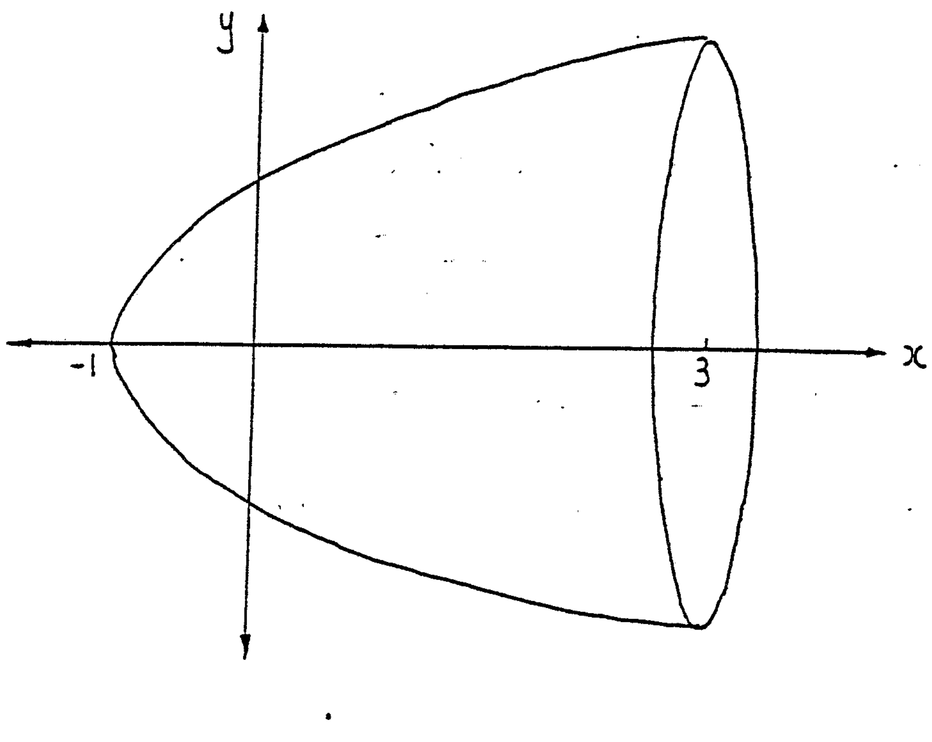
(iii) exactly one green star from a selection of two.

(iv) The games show organisers provide prizes according to the level of difficulty in selecting stars. Which combination of two coloured stars do you think would attract the most expensive prize ? Explain your answer .

(3) (b) The limiting sum of the geometric series $1 \div 5^{x} \div 5^{2x} \div 5^{3x} \div \ldots\ldots$ is 5/4
Find the value of x.

(3) (c) The region enclosed by the curve $y = \sqrt{x \div 1}$ and the x axis between $x = -1$ and $x = 3$ is rotated about the x axis as shown.

Find the volume of the solid of revolution formed.



Question 7. Start a new Page. (12 Marks)

(5) (a) The depth of a river 10 metres wide has been measured at different distances from a point on the bank . The results to these measurements are shown in the table below ;

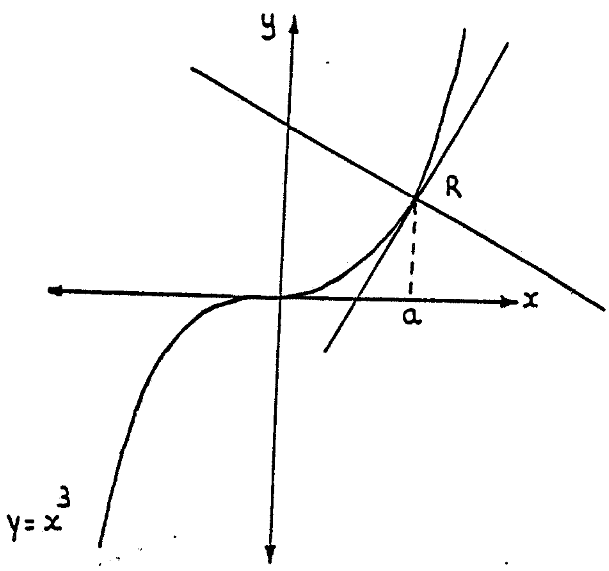
Distance from the bank (metres)	0	2	4	6	8	10
Depth of the water (metres)	0	3.2	3.6	4.1	2.8	1.0

(i) Draw a diagram to show the cross section of the river.

(ii) Using the trapezoidal rule with six function values, estimate the area of the river's cross section.

(iii) If the river is flowing at a rate of 3 metres / second, what volume of water flows past this point on the bank in 1 hour ?

(7) (b) The diagram below shows the graph of the curve $y = x^3$.
The tangent and normal at the point R where $x = a$ are drawn.



(i) Determine the coordinates of the point R

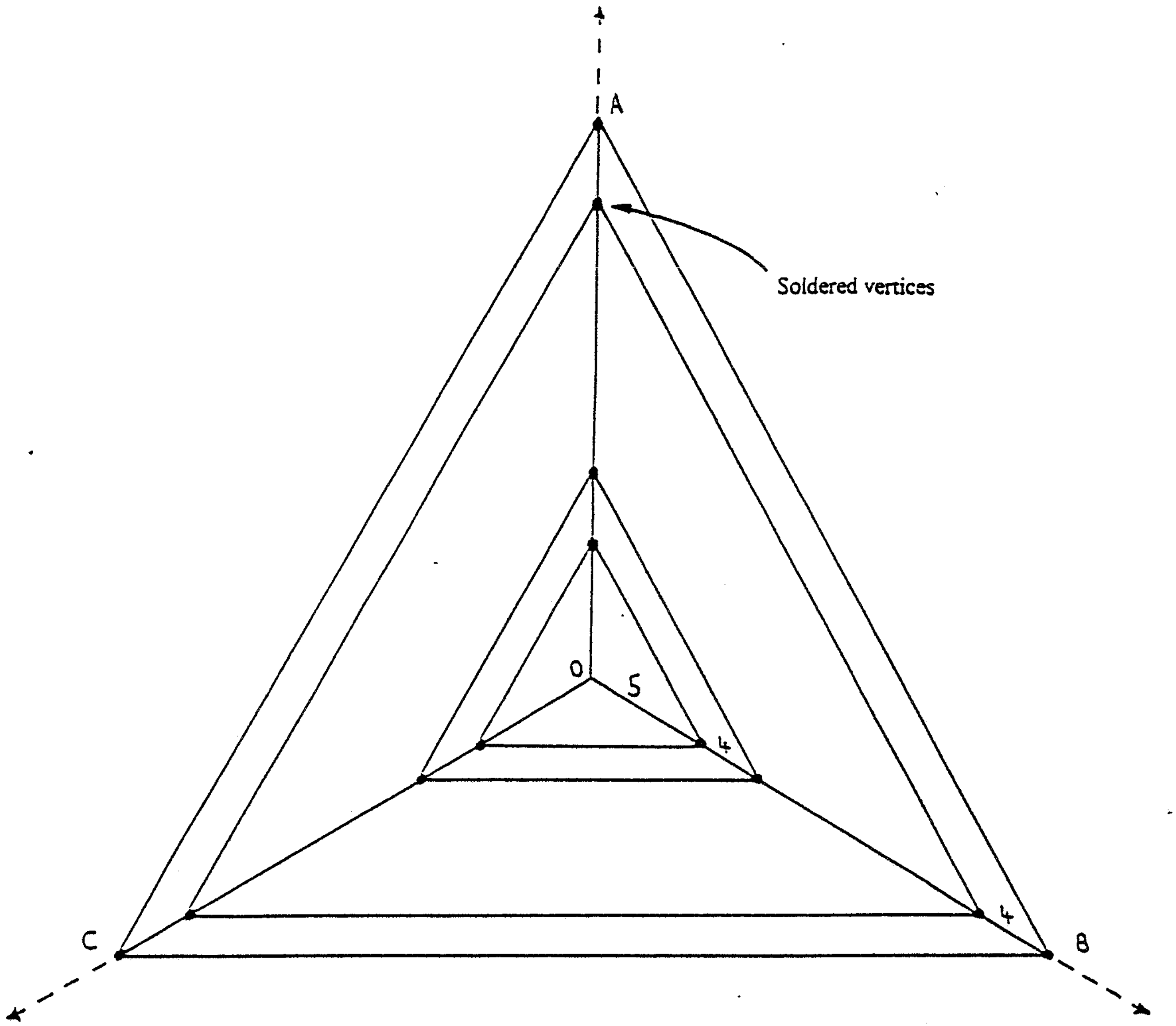
(ii) Find the gradient of the tangent at $x = a$

(iii) Find the equation of the tangent at the point where $x = a$

(iv) Show that the equation of the normal at the point where $x = a$ is $x + 3a^2y = a(3a^4 + 1)$

(v) Another line, with equation $y = a^2x$ intersects the normal at the point Q. What are the coordinates of Q, this point of intersection ?

(6) (b) A three dimensional piece of artwork made from wire, is constructed by soldering the vertices of a series of equilateral triangles to three wire axes which are perpendicular to each other. The axes originate from the point O as shown in the diagram. Only some of the equilateral triangles are shown.



The vertices of the smallest equilateral triangle are 5 cm from O. The next equilateral triangle has its vertices 4cm further away and they continue in this way until the vertices of the last equilateral triangle ABC are 81cm from O.

(i) How many vertices are soldered to the axes ?

(ii) What is the perimeter, in exact form, of the smallest equilateral triangle ?

(iii) Show that the total length of wire required to construct the piece of artwork is $(243 + 2580\sqrt{2})$ cm.

Question 9. Start a new Page. (12 Marks)

(3) (a) (i) Differentiate $\log_e(\cos 2x)$

(ii) Hence evaluate $\displaystyle\int_0^{\pi/6} \tan 2x \, . \, dx$ giving your answer correct to 3 decimal places.

(9) (b) A particle P, is moving along the x axis. It's velocity, v m/sec, at time t is given by $v = 9 + 6t - 3t^2$ m/sec. Initially, the particle is 1 metre to the right of the origin.

(i) What is the position of the particle P, after 4 seconds ?
(ii) What is the position of the particle when it is momentarily at rest ?
(iii) What is the maximum velocity that the particle achieves ?
(iv) Sketch the graph of the velocity versus time for the first 4 seconds of motion.
(v) Describe, in your own words, the motion of the particle P over the first 4 seconds.

Question 10. Start a new Page. (12 Marks)

(6) (a) A company manufactures computers.

The cost, in dollars, of producing x computers in a 3 month period is given by the expression $5x + \dfrac{8000}{x}$.

Market research has shown that the demand for a particular new model of computer over the next 9 months will be quite high.

In order to meet this demand the company have hired an additional piece of production machinery at a cost of \$6 000 for the 9 month period.

(i) Show that the total cost (\$C) of producing x computers in the 9 month period is

$$C = 15x + \frac{24\,000}{x} + 6000$$

(ii) If the company cannot manufacture more than 50 computers in the 9 month period, calculate the number of computers they could produce when their total costs are \$8 550 ?

(iii) How many computers should the company produce in order to minimise their costs ?

(1)(a) 0.4

(b) $(m+9)(x-y)$

(c) $\frac{11a}{6}$

(d) $x=1$

(e) $y=-3$

(f) 25.6

(2) (a) $x=\frac{53}{99}$

(b) $\frac{5\left(\sqrt{7}-2\right)}{3}$

(c) (i) $x>3$ or $x<-13$

(ii) $-3<x<3$

(d) $\theta=120^0$

(e)(i) $a=4$ (ii) Focus = (0, 5)

(3) (a) (i) $2\cos 2x+2\sec^2 x$

(ii) $\frac{2x-5}{x^2-5x}$

(iii) $2e^{3x}(3x+1)$

(b) (i) $\frac{2}{3}x^{\frac{2}{3}}+c$

(ii) $\frac{1}{60}(10x+7)^6+c$

(iii) $\frac{1}{2}\ln(x^2+8x)+c$

(c) $c\approx 22^0$

(4) (a) (i) $A=20\ 000$

(ii) $k\approx 0.27$

(iii) $t=4$ hrs 4 mins

(b) (i) $\theta\approx 160^0$

(ii) $A\approx 89\ \text{m}^2$

(iii) 3.56 kg

(iv) 48 Litres

(5) (a) (i) Diagram

(ii) $AC=5$ units

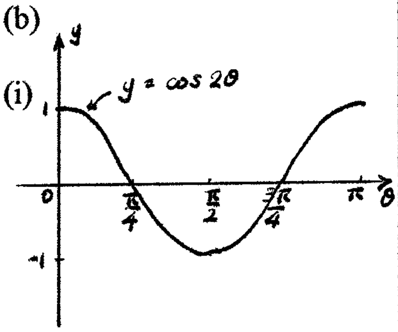
(iii) Proof

(iv) 3.4 units

(v) Area = 8.5 units^2

(b)

(i)



(ii) 2 from the graphs

(iii) $\theta=\frac{\pi}{6}$ or $\frac{5\pi}{6}$

(c) $A=\$8\ 877.69$

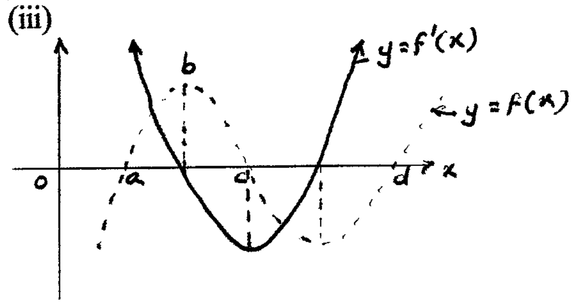
(6) (a) (i) $x<a; c<x<d$

(ii) $a<x<b, \frac{dy}{dx}>0$

At $x=b, \frac{dy}{dx}=0$

$b<x<c, \frac{dy}{dx}<0$

(iii)



(iv) Need to find

$\int_a^c f(x)dx + \left|\int_c^d f(x)dx\right|$

(b) (i) A.A.S. Test (ii) TM = 4·5 cm

(c) $x=6$

(7) (a) (i) Diagram

(ii) Area = 28.4 m^2

(iii) 306.72 KL

(b) (i) $R(a,a^3)$

(ii) $3a^2$

(iii) $y=3a^2x-2a^3$

(iv) Proof

(v) $Q(a,a^3)$

(8) (a) (i) $\frac{5}{16}$

(ii) $\frac{37}{120}$

(iii) $\frac{2}{5}$

(iv) $P(G,G)=\frac{1}{20}$ has the lowest probability.

(b) $x = -1$

(c) 8π units^3 or 25.13 units^3

(9)(a) (i) $-2\tan 2x$

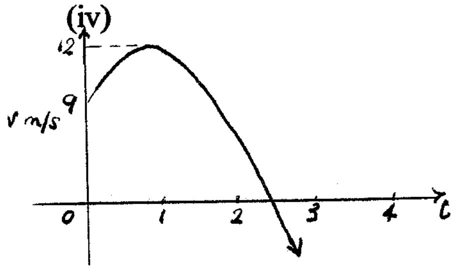
(ii) 0.347 (to 3 d.p)

(b) (i) $x = 21$

(ii) $x = 28$ m

(iii) $v = 12$ m/s

(iv)



(v) Start at $x = 1$, increasing in speed until $t = 1$. Begins to slow down from $t = 1$ coming to rest at $t = 3$ at $x = 28$ m. At $t = 3$ particle moves back to O and at $t = 4$ it is at $x = 21$ m.

(10) (a) (i) Proof

(ii) 10 computers

(iii) $x = 40$

(b) (i) 60

(ii) $15\sqrt{2}$

(iii) Proof

(1)(a) 0.4

(b) $(m+9)(x-y)$

(c) $\frac{11a}{6}$

(d) $x=1$

(e) $y=\pm 3$

(f) 25.6

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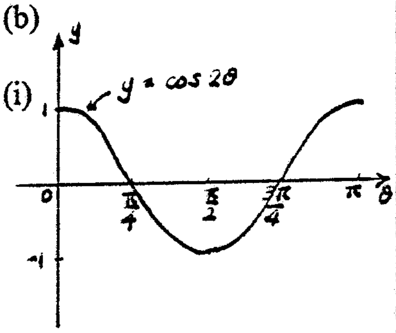

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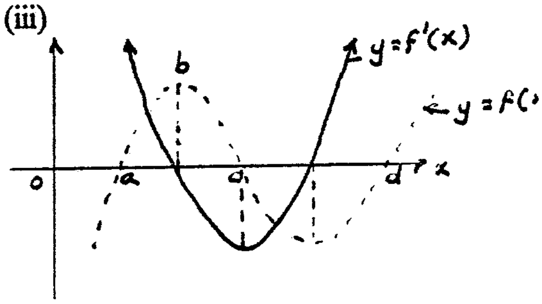

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