WESTERN REGION

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

1997

MATHEMATICS

2/3 Unit (Common)

Time allowed THREE hours
(Plus 5 minutes reading time)

DIRECTIONS TO CANDIDATES

- Attempt ALL questions.
- ALL questions are of equal value.
- ALL necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Standard integrals are printed on page 11 of this paper.
- Board-approved calculators may be used.
- Each Answer each question in a separate answer booklet.
- You may ask for extra booklets if you need them.

Question 1 *Start a new page* **Marks**

(a) Factorise $20x^2 - 45y^2$ **2**

(b) Solve for x $3^x \times 9^{x+1} = \frac{1}{3}$ **2**

(c) Solve for x $|3x - 5| = 4$ **2**

(d) Express $x - \frac{1}{x}$ as an exact value, in simplest form if $x = 1 - \sqrt{3}$ **2**

(e) Simplify $(x+2)^2 - (x+3)(x-3)$ **2**

(f) Find the primitive function of $x^2 + x^{-4}$ **2**

Question 2 *Start a new page* **Marks**

The points A, B and O have co-ordinates (5,4), (-3,3) and (0,0) respectively and M is the midpoint of AB.

(a) Sketch the points A,B,O and M on a number plane, giving the co-ordinates of M **2**

(b) Show that the line k, which passes through A and is parallel to OB has equation $x + y - 9 = 0$ **2**

(c) Show that the equation of the line ℓ, passing through O and M has equation $7x - 2y = 0$ **2**

(d) Find the co-ordinates of the point C, the intersection of the lines k and ℓ **2**

(e) Calculate the distance AC **1**

(f) Calculate the area of the triangle AOC **2**

(g) On your diagram, shade the region in which the following inequalities hold simultaneously **1**

$7x - 2y \le 0$

$x + y - 9 \le 0$

$x \ge 0$

Question 3 *Start a new page* **Marks**

(a) Differentiate with respect to x 4

(i) $(5x+8)^6$

(ii) $\dfrac{e^x+2x}{x}$

(iii) $\ln(x^2-5)$

(b) For a certain arithmetic series the sum of the first six terms, S_6, is 84 and the sum of the first fourteen terms, S_{14}, is 420. 4

(i) Find the first term and common difference

(ii) Find an expression for the sum of n terms, S_n

(iii) Find an expression for the nth term, T_n

(c) (i) Find $\displaystyle\int \cos 2x \, dx$ 2

(ii) Find $\displaystyle\int \left(2x^2 - \frac{2}{x^2}\right) dx$

(d) Evaluate $\displaystyle\int_0^{\ln 4} e^{3x} dx$ 2

Question 4 *Start a new page* **Marks**

(a) Two yachts sail from a lighthouse L. Yacht A sails 12 km along a bearing of 038° and yacht B sails 16km along a bearing of 130°. 5

(i) Draw a diagram showing this information

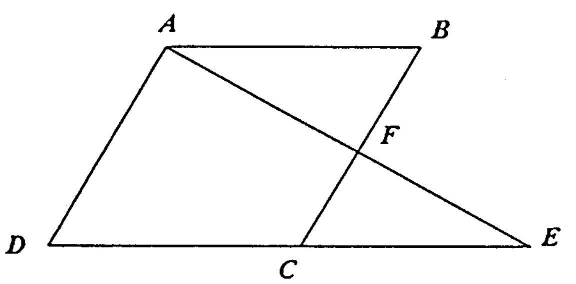
(ii) Calculate the distance between the two yachts

(iii) What is the bearing of yacht A as seen from yacht B, to the nearest minute?

(b) $ABCD$ is a parallelogram. $AD = 12cm$, $CE = 4cm$ and $BF = 7cm$. 4

(i) Show that ΔABF is similar to ΔECF

(ii) Find the length of AB



(c) The values of a function $f(t)$ for certain values of t are given below 3

t	0	0·5	1·0	1·5	2·0
$f(t)$	1·76	2·04	2·39	1·93	1·62

(i) Use Simpson's Rule with 3 function values to evaluate

$$\int_0^2 f(t)\, dt$$

(ii) Use Simpson's Rule with 5 function values to evaluate

$$\int_0^2 f(t)\, dt$$

(iii) Which approximation is more accurate, and why?

Question 5 *Start a new page* **Marks**

(a) Express $x^2 - 3$ in the form $a(x-2)^2 + b(x-2) + c$ **3**

(b) (i) Simplify $\log_5 25$ **3**

(ii) Solve $4^m = 7$ correct to 2 decimal places

(c) Dr Martin's sock drawer contains 10 pairs of socks which have become separated. Two pairs are grey, three pairs are blue and five pairs are white. If she randomly selects 2 socks from the drawer without replacement, what is the probability that: **6**

(i) the first sock is grey?

(ii) the first sock is grey and the second sock is white?

(iii) one sock is grey and the other sock is white?

(iv) both socks are the same colour?

Question 6 *Start a new page* **Marks**

(a) (i) Draw a sketch of the curve $y = \cos x$ for $0 \le x \le \frac{3\pi}{4}$ **4**

(ii) Calculate the area bounded by the curve $y = \cos x$, the x axis and the lines $x = 0$ and $x = \frac{3\pi}{4}$

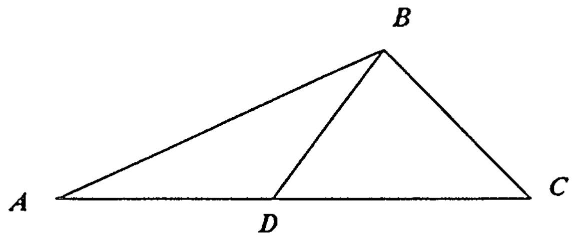
(b) (i) Show that $1 + \tan^2\theta = \sec^2\theta$ **5**

(ii) Hence prove that $\frac{1 - \tan^2\theta}{1 + \tan^2\theta} + \sin^2\theta = \cos^2\theta$

(c) Find the equation of the normal to the curve $y = e^{-2x}$ at the point where $x = \frac{1}{2}$ **3**

Question 7 *Start a new page* **Marks**

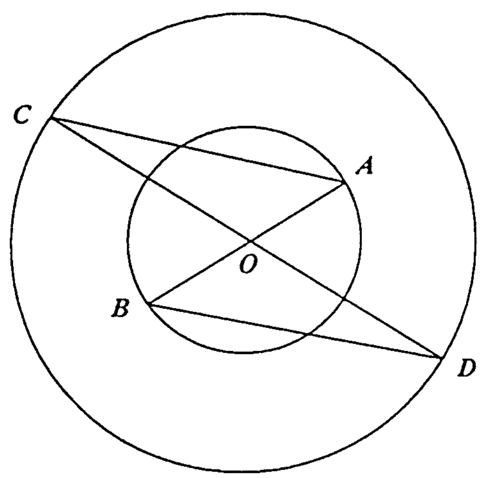
(a) In the figure shown below, $AD = BD = CD$ and $\angle BAD$ is 39°. **3**



(i) Copy the diagram and mark the above information on it.

(ii) Find the size of $\angle BCD$, giving full reasons.

(b) In the diagram below, O is the centre of both circles, AB is a diameter of the smaller circle and CD is a diameter of the larger circle. **3**



(i) Copy the diagram onto your paper and show that triangles AOC and BOD are congruent.

(iii) Hence show that $AC \parallel BD$

(c) (i) Sketch the curve $y = 2x^2 - x^4 + 10$ for the domain $-2 \le x \le 2$ **6**

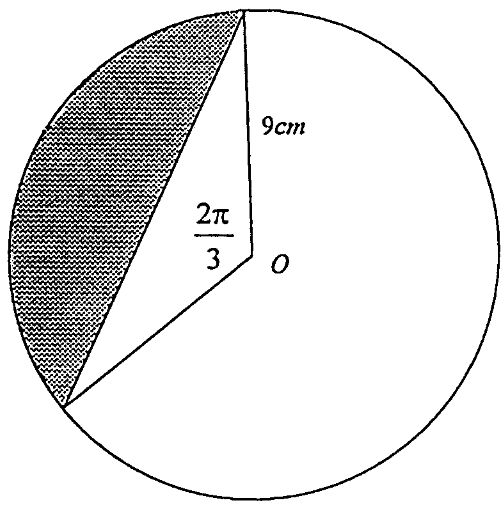
(ii) For what values of x within the domain is the curve concave down?

(iii) What is the absolute maximum within the domain?

Question 8 *Start a new page* **Marks**

(a) (i) Solve $2\sin 2x = \sqrt{3}$ for $0 \le x \le 2\pi$ 4

(ii) Find the area of the shaded segment of the circle shown below.
(Angles are given in radians)



(b) Marcus starts a business, which in its first month has a profit of \$200. He finds that in the second month, the profit is \$210, an increase of 5%. He decides to aim to increase his monthly profit by 5% every month. If he is successful in this aim: 4

(i) What will be his profit in the twelfth month of operation?

(ii) What will be his total profit for the first twelve months?

(iii) In which month will his monthly profit first exceed \$1 000?

(c) The point $P(x, y)$ moves so that its distance from the line $y = -2$ is equal to its distance from the point $S(0,2)$ 4

(i) Show that the locus of P is a parabola with equation $x^2 = 8y$

(ii) What is the vertex of this parabola?

(iii) What is the focal length of this parabola?

Question 9 *Start a new page* **Marks**

(a) By considering the rotation of the semi-circle $y = \sqrt{a^2 - x^2}$ about the x axis, show that the volume of a sphere with radius a units is given by 5

$$V = \frac{4}{3}\pi\, a^3$$

(b) A particle moves in a straight line with a constant acceleration of $2ms^{-2}$ toward the right. It is initially at rest, 2 metres to the right of the origin. 7

(i) Find an expression for the velocity after t seconds.

(ii) Find the velocity after 3 seconds.

(iii) When is the velocity $10ms^{-1}$?

(iv) Find an expression for the displacement after t seconds.

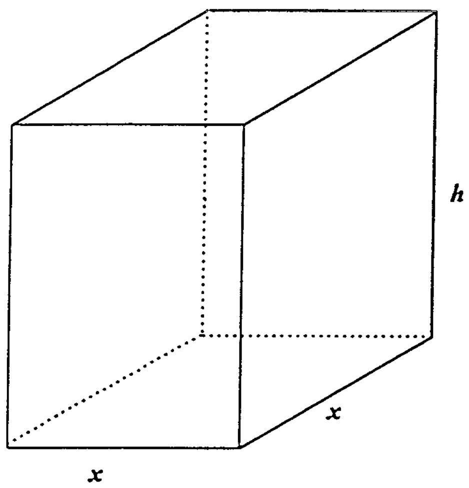
(v) When is the particle 6 metres to the right of the origin?

(vi) How far does the particle move in the third second?

(vii) Describe the motion of the particle in words.

Question 10 *Start a new page* **Marks**

(a) The square prism shown below is made so that the sum of the length, width and height is 6 metres. 5

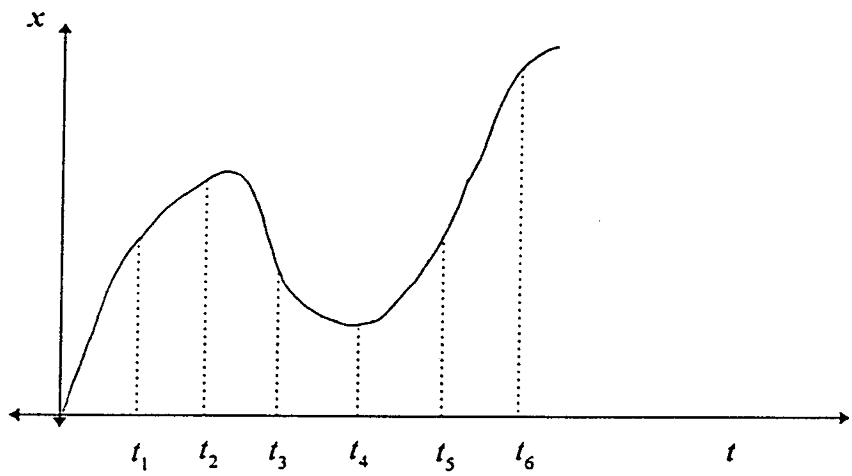


(i) If the length of the base is x metres, find the height h in terms of x.

(ii) Show that the volume is given by $V = 6x^2 - 2x^3$

(iii) Find the maximum volume of the prism.

(b) The graph shows the displacement, x, of a particle from the origin at time t 3



(i) At which time is the particle moving faster, t_1 or t_2?

(ii) What is the velocity of the particle at time t_4? Explain why.

(iii) Sketch the graph showing the velocity of the particle at time t.

(c) A spherical balloon which initially has a radius of 20 *cm*, is inflated so that its volume increases at a constant rate of $100\ cm^3$ / second. 4

(i) Find an expression for the volume V at any time t.

(ii) Calculate the time required, to the nearest second, for the volume to reach 100 litres. ($1000\ cm^3 = 1\ litre$)

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$

(1) (a) $5(2x-3y)(2x+3y)$

(b) $x=-1$

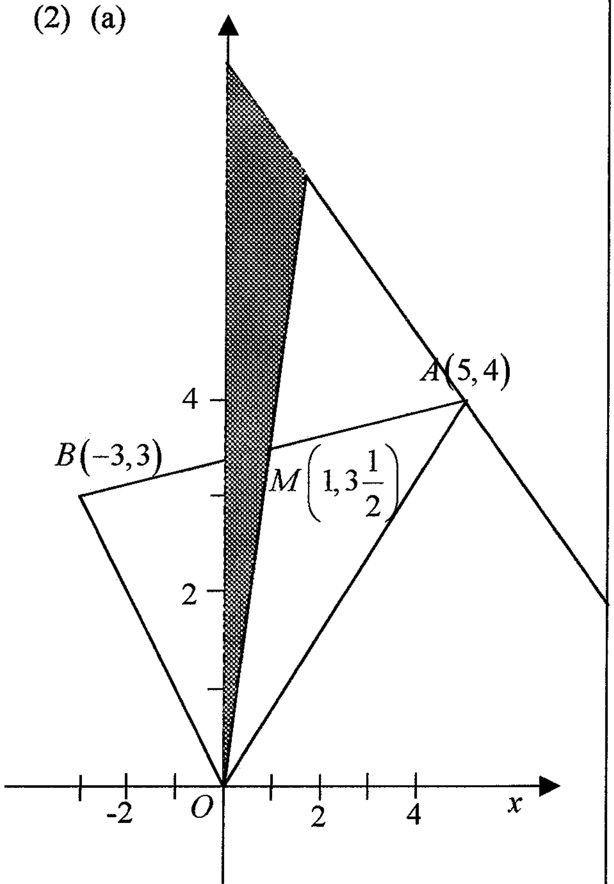
(c) $x=\frac{1}{3},3$.

(d) $\frac{3-\sqrt{3}}{2}$

(e) $4x+13$

(f) $\frac{x^3}{3}-\frac{1}{3x^3}+c$

(2) (a)



(b) Proof

(c) Proof

(d) $C(2,7)$

(e) $3\sqrt{2}$

(f) Area $=13.5$ units^2

(g) See shaded region in diagram (a).

(3) (a) (i) $30(5x+8)^5$

(ii) $\frac{e^x(x-1)}{x^2}$

(iii) $\frac{2x}{x^2-5}$

(b) (i) $a=4, d=4$

(ii) $S_n=2n(1+n)$

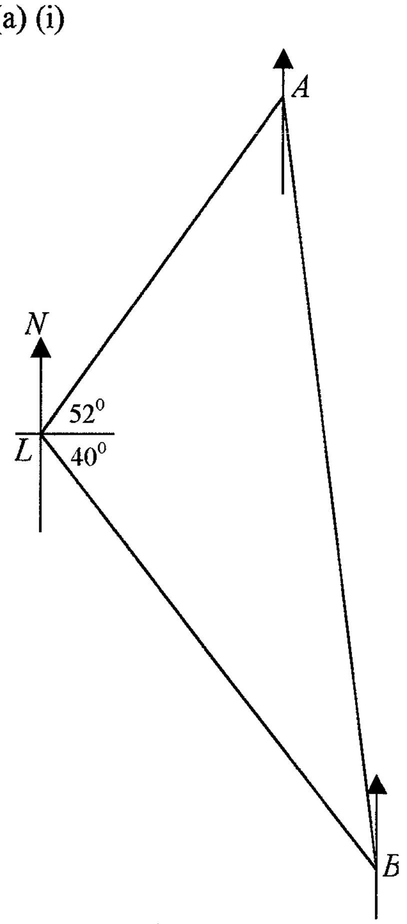
(iii) $T_n=4n$

(c) (i) $\frac{1}{2}\sin 2x+c$

(ii) $\frac{2x^3}{3}+\frac{2}{x}+c$

(d) 21

(4) (a) (i)



(ii) 20.33 km

(iii) 346^0 9'

(b) (i) Proof

(ii) 5.6 cm

(c) (i) 4.31 u^2 (to 2d.p.)

(ii) 4.01 u^2 (to 2 d.p)

(iii) Part (ii) is more accurate because it has more subintervals or function values.

(5)(a) $a = 1, b = 4, c = 1$

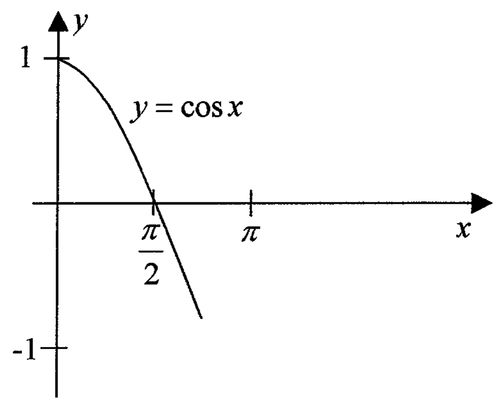
(b) (i) $x = 2$

(ii) $m = 1.40\,\text{m}$

(c) (i) $\frac{1}{5}$ (ii) $\frac{2}{19}$

(iii) $\frac{4}{19}$ (iv) $\frac{33}{95}$

(6) (a) (i)



(ii) Area $= 2 - \frac{1}{\sqrt{2}}$ units^2

(b) (i), (ii) Proofs

(c) $2e^2x - 4ey + 4 - e^2 = 0$

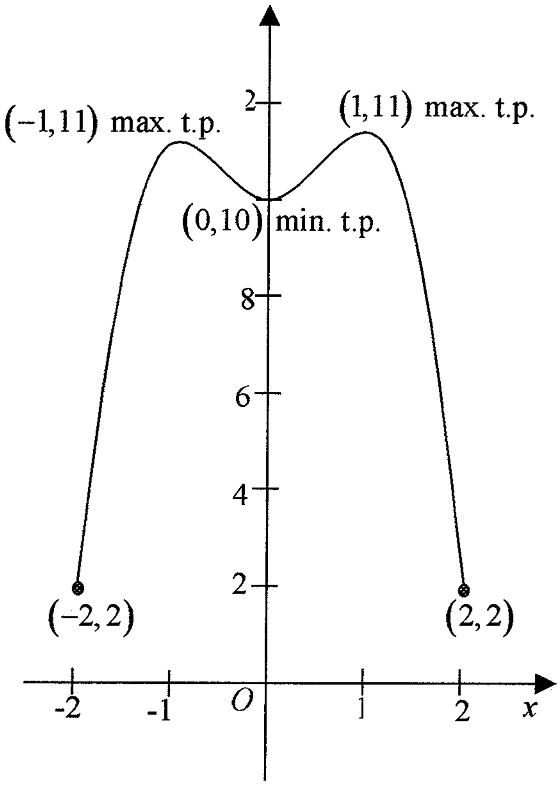
(7) (a) (i), (ii) $x = 51^0$

(b) (i), (ii) Proof

(c) (i) $(-1, 11)$ and $(1, 11)$ relative max.

$(0, 10)$ relative min.

Endpoints $(-2, 2)$ and $(2, 2)$



(ii) $-2 \le x < -\frac{1}{\sqrt{3}},\ \frac{1}{\sqrt{3}} < x \le 2$

(iii) $y = 11$

(8) (a) (i) $x = \frac{\pi}{6}, \frac{\pi}{3}, \frac{7\pi}{6}, \frac{4\pi}{3}$

(ii) Area $= 27\pi - \frac{81\sqrt{3}}{4}$ units^2

(b) (i) \$342.07

(ii) \$3113.43

(iii) 34th month

(c) (i) Proof

(ii) $V(0,0)$

(iii) 2

(9)(a) Use $V = 2\pi\int_0^a \left(\sqrt{a^2 - x^2}\right)^2 dx$

$$= 2\pi\left[a^2x - \frac{x^3}{3}\right]_0^a = \frac{4}{3}\pi a^3$$

(b) (i) $v = 2t$ (ii) $v = 6$ m/s

(iii) $t = 5$ s (iv) $x = t^2 + 2$

(v) $t = 2$ s (vi) $x = 5$ m

(vii) Particle keeps moving to the right with an increasing velocity.

(10)(a) (i) $h = 6 - 2x$

(ii) $V = 6x^2 - 2x^3$

(iii) $V = 8$ units^3

(b) (i) t_1 (ii)

$v = 0,\ \because \frac{dx}{dt} = 0$

(iii)