WESTERN REGION

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

1998

MATHEMATICS

2/3 Unit (Common)

Time allowed - THREE hours
(Plus 5 minutes reading time)

DIRECTIONS TO CANDIDATES

- Attempt ALL questions.
- ALL questions are of equal value.
- ALL necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Standard integrals are printed on the last page. These may be removed for your convenience.
- Board-approved calculators may be used.
- Answer each question on a separate page.

Question One. *Start a new page* **Marks**

(a) Find the value of $\dfrac{7}{\sqrt{2}-5}$ correct to 3 significant figures. **1**

(b) The point $(k\,, 3)$ lies on the line $x + 5y = 10$. Find the value of k. **1**

(c) Simplify $(2a-1)^2 - 4(1-a)$. **2**

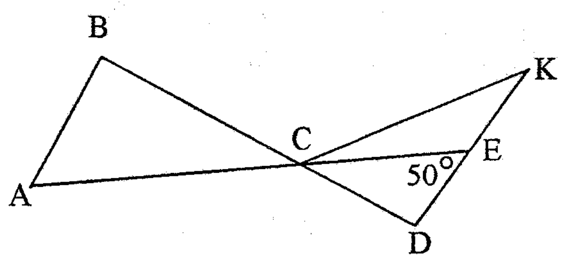
(d) Determine $f'(x)$ where $f(x) = 3x - \frac{3}{x}$. **2**

(e) Find the value of $|x-2| - |x+2|$ when $x = 1\frac{1}{2}$. **2**

(f) In the accompanying figure **2**

$AB \parallel DK$ and $\angle ABD = 90°$.
AE and BD intersect at C.
$EC = EK$ and $\angle DEC = 50°$.

Find the size of $\angle KCD$.



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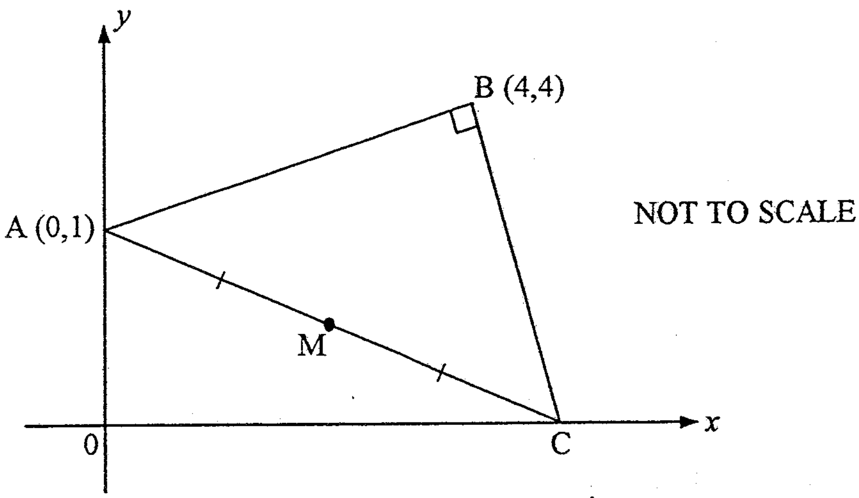
(g) Graph the solution to $5 - 2x < 8$ on a number line. **2**

Question Two. *Start a new page* **Marks**

(a) **6**



In the diagram, C lies on the x axis and AB is perpendicular to BC.
M is the midpoint of AC.

Copy the diagram onto your answer sheet.

(i) Find the gradient of AB.

(ii) Show that the equation of BC is $4x + 3y - 28 = 0$.

(iii) Show that the coordinates of C are (7, 0).

(iv) It is known that a circle with centre at M can be drawn to pass through the points A, B and C.
Calculate the radius of this circle, leaving your answer in surd form.

(b) Differentiate the following : **6**

(i) $f(x) = \log_e (2x - 1)$.

(ii) $f(x) = \sin\ x^2$.

(iii) $f(x) = \dfrac{x}{x+2}$.

(iv) $f(x) = x\,e^x$.

Question Three. *Start a new page* **Marks**

(a) Find the value of x if $\sqrt{x} = \sqrt{50} - \sqrt{18}$. **2**

(b) (i) Using any suitable method, show that the graphs of **4**

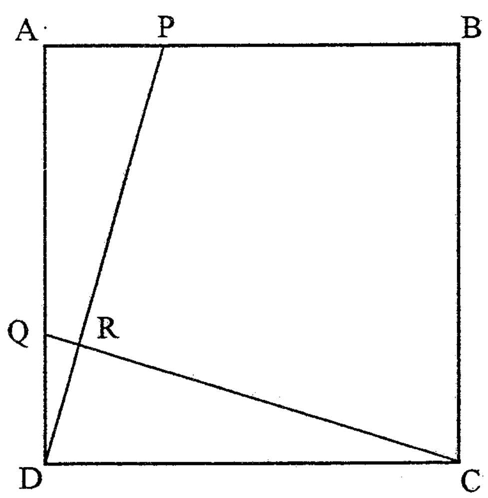
$$y = 3x - x^2 \quad \text{and}$$

$$y = x$$

intersect at (0, 0) and (2, 2).

(ii) Calculate the area enclosed between the curve $y = 3x - x^2$ and the line $y = x$.

(c) **6**



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ABCD is a square. P lies on AB and Q lies on AD such that AP = DQ.

(i) Prove $\Delta APD \equiv \Delta DQC$.

(ii) Show that $\angle PDC = \angle DQC$.

(iii) If PD and QC intersect at R show that $\angle PRC = 90^\circ$.

Question Four. *Start a new page* **Marks**

(a) An arc of a circle has a length of 12 cm. This arc subtends an angle of 60° at the centre of the circle. **3**

Calculate :

(i) the angle at the centre in terms of π,

(ii) the area of the sector in terms of π.

(b) Solve $2\cos\theta\sin\theta + \cos\theta = 0$ over the domain $0 \le \theta \le 2\pi$. **3**

(c) A ladder tapers in from bottom to top, as shown in the diagram. The ladder has two side rails and twenty steps. **6**

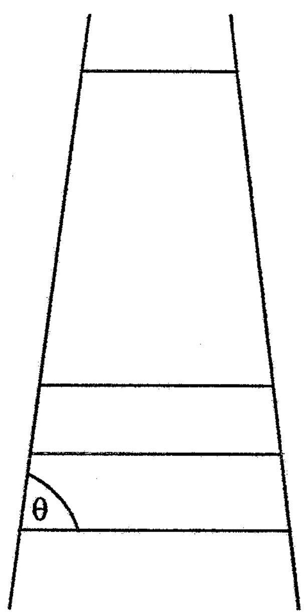
The bottom step is 600 mm long.
Each subsequent step is 15 mm shorter than the one below.

The perpendicular distance between each step is 250 mm.

(i) Calculate the length of the top step.

(ii) Calculate the total length of all twenty steps.

(iii) The angle formed between the side rail and each step has been labelled θ. Calculate the size of this angle to the nearest whole degree.



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Question Five. *Start a new page* **Marks**

(a) (i) Find the value of a if $\log_a 81 = 2$. **5**

(ii) Determine the value of x correct to 2 significant figures if $e^x = 7$.

(iii) Simplify the expression $\log_x\left(\frac{x^2}{y}\right) + \log_x\left(\frac{y}{x}\right)$.

(b) Find : **3**

(i) $\int \sqrt{x}\ dx$

(ii) $\int \frac{4x}{x^2 - 1}\ dx$

(c) A brand of rechargeable battery provides power for 16 hours when new. After recharging for the first time it only provides power for a further 12 hours. After the second recharge, power is only provided for another 9 hours. Each subsequent recharging results in the power output of the battery being 75 % of the previous power value. **4**

(i) What power is available in the battery after the third recharge ?

(ii) Calculate the total useful life of the battery.

Question Six. *Start a new page* **Marks**

(a) The pulse rate for an athlete during an exercise routine, in beats per minute, is given by **4**

$$P(t) = 65 + 2t^2 - t$$

where t is the time in minutes measured from the beginning of the routine.

(i) What was the athlete's pulse rate at the commencement of the routine ?

(ii) What is the change in the athlete's pulse rate in the first 5 minutes of the exercise routine ?

(iii) What is the rate of change in the pulse rate after 5 minutes ?

(b) Explain why $f(x) = 2^x$ is a positive definite function. **2**

(c) Evaluate the following definite integrals. **6**

(i) $\displaystyle\int_{-1}^{1} (x^2 + 2x)\ dx$

(ii) $\displaystyle\int_{0}^{\frac{\pi}{3}} \sin 3x\ dx$

(iii) $\displaystyle\int_{-1}^{2} (x+2)^2\ dx$

Question Seven. *Start a new page* **Marks**

(a) The point P(2, 4) lies on the parabola $y = x^2$. **6**
The tangent at P cuts the x axis at A and the y axis at B.
The normal to the tangent at P cuts the x axis at C and the y axis at D.

(i) Draw a diagram to show the above information.

(ii) Show that the tangent at P has equation $4x - y - 4 = 0$ and the normal at P has equation $x + 4y - 18 = 0$.

(iii) Calculate the area of triangle APC.

(iv) Determine the area of quadrilateral OAPD, where O is the origin.

(b) The position x, of a particle moving along a straight line can be found using the expression : **6**

$$x = t^3 - 5t^2 + 7t + 6$$

where x is in centimetres and t is in seconds.

(i) What is the starting position of the particle ?

(ii) What is the initial velocity and acceleration of the particle ?

(iii) Show the particle comes to rest on two occasions.

(iv) Hence, determine the distance travelled in the first second.

Question Eight. *Start a new page* **Marks**

(a) A house is protected by two alarm systems which work independently. One system covers the exterior of the house and can detect an intruder with a 75 % success rate. The second system covers the interior of the house and can detect an intruder with a 90 % success rate. **5**

(i) What is the probability, expressed as a percentage, that an intruder will be detected by both alarm systems ?

(ii) What is the probability of an intruder being undetected by both alarm systems ?

(iii) The security firm servicing this house requests the owners to upgrade the external alarm system so there is a **99** % probability an intruder will be detected by either or both systems. If the rating for the internal system remains unchanged, what would the new rating of the external system need to be ?

(b) A small farm business decides to manufacture woollen pullovers. It is estimated that the cost of manufacturing x pullovers per week can be found from the cost function **7**

$$C(x) = 0{\cdot}3x^2 + 54x + 44 \ .$$

The total selling price of the x pullovers can be found from the selling function

$$S(x) = 142\,x - \frac{4x^2}{5} \ .$$

If $P(x)$ represents the profit made from selling the x pullovers ;

(i) show that $P(x) = 88x - 1{\cdot}1x^2 - 44$

(ii) calculate the number of pullovers needed to be manufactured each week so as to return a maximum profit

(iii) hence, determine the selling price of each pullover to achieve this maximum profit.

Question Nine. *Start a new page* **Marks**

(a) A triangle has sides of 6 cm, 3 cm and $3\sqrt{7}$ cm. Calculate the size of the largest angle. **2**

(b) (i) Explain why the y axis is an asymptote for the graph of $y = \frac{x}{4} + \frac{1}{x}$. **5**

(ii) Find the two stationary points on the graph of $y = \frac{x}{4} + \frac{1}{x}$.

(iii) Hence draw a neat sketch of the graph of $y = \frac{x}{4} + \frac{1}{x}$.

(c) (i) Show that $(e^{x} + e^{-x})^2 = e^{2x} + 2 + e^{-2x}$. **5**

(ii) The arc of the curve on $y = e^{x} + e^{-x}$ between $x = 0$ and $x = 1$ is rotated about the x axis.

Find the volume of the solid of revolution formed, correct to 3 significant figures.

Question Ten. *Start a new page* **Marks**

(a) A container filled with liquid is being emptied. The volume, V cm ^{3}, at any time t minutes is given by **5**

$$V = 5000e^{-kt}$$, where k is a constant.

(i) Show that $\dfrac{dV}{dt} = -kV$.

(ii) If the volume in the container after 5 minutes is 2200 cm ^{3} find the value of k correct to three decimal places.

(iii) Determine the rate of change in the volume after 5 minutes.

(b) (i) Sketch, on the same set of axes, the graphs of **7**

$y = \sin x$ and $y = 1 + \cos x$

over the domain $0 \le x \le \pi$.

(ii) Write down the values of x for which $\sin x = 1 + \cos x$ in this domain.

(iii) Evaluate the integral $\displaystyle\int_0^{\pi} (1 + \cos x - \sin x) \; dx$

(iv) Calculate the area between the two curves for the given domain.

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$

(1)(a) -1.95 (to 3 s.f.)

(b) $k=-5$

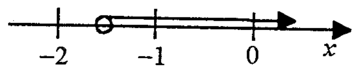
(c) $4a^2-3$

(d) $f'(x)=3+\frac{3}{x^2}$

(e) -3

(f) $\angle KCD=65^0$

(g) $x>-\frac{3}{2}$



(2)(a)(i) $\frac{3}{4}$ (ii) $4x+3y-28=0$

(iii) Proof (iv) $C(7,0)$

(v) Radius $=\frac{5\sqrt{2}}{2}$

(b)(i) $\frac{2}{2x-1}$ (ii) $2x\cos(x^2)$

(iii) $\frac{2}{(x+2)^2}$ (iv) $e^x(x+1)$

(3)(a) $x=8$

(b)(i) Solve simultaneously;

(ii) Area= $1\frac{1}{3}$ sq. units

(c)(i),(ii),(iii) Proofs

(4)(a)(i) $\frac{\pi}{3}$ radians (ii) $\frac{216}{\pi}$ cm^2

(b) $\theta=\frac{\pi}{2},\frac{3\pi}{2},\frac{7\pi}{6},\frac{11\pi}{6}$

(c)(i) 315 mm (ii) 9150 mm

(iii) $\theta\approx 88^0$

(5)(a)(i) $a=9$ (ii) $x\approx 1.9$ (iii) 1

(b)(i) $\frac{2\sqrt{x^3}}{3}+c$ (ii) $2\ln(x^2-1)+c$

(c)(i) 6.75 hours (ii) 64 hours

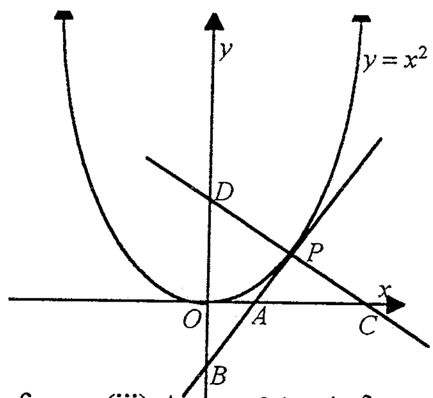
(6)(a)(i) 65 (ii) 45 beats/min

(iii) 19 beats/min

(b)(i) $y=2^x$ is always >0

(c)(i) $\frac{2}{3}$ (ii) $\frac{2}{3}$ (iii) 21

(7)(a)(i)



(ii) Proof (iii) Area = 34 units^2

(iv) Area = 6.5 units^2

(b)(i) $x=6$ cm

(ii) $v_I=7$ cm/s; $a_I=-10$ cm/s^2

(iii) $t=1$ s and $2\frac{1}{3}$ s

(iv) Distance travelled = 3 cm

(8)(a)(i) 67.5% (ii) 2.5% (iii) 90%

(b)(i) Proof

(ii) $x=40$

(iii) $110

(1)(a) -1.95 (to 3 s.f.)

(b) $k = -5$

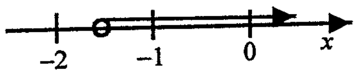
(c) $4a^2 - 3$

(d) $f'(x) = 3. + \dfrac{3}{x^2}$

(e) -3

(f) $\angle KCD = 65^0$

(g) $x > -\dfrac{3}{2}$



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(iii) $\dfrac{2}{(x+2)^2}$ (iv) $e^x(x+1)$

(3)(a) $x = 8$

(b)(i) Solve simultaneously;

(ii) Area$= 1\frac{1}{3}$ sq. units

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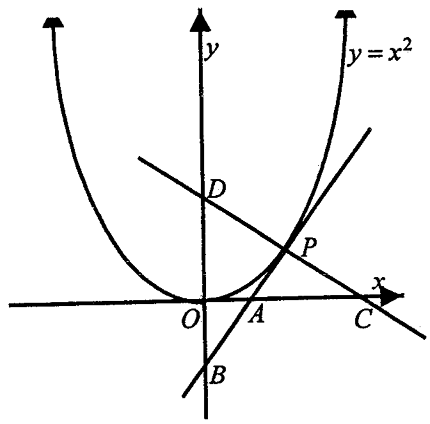

(ii) Proof (iii) Area = 34 units^2

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(iv) Distance travelled = 3 cm

(8)(a)(i) 67.5% (ii) 2.5% (iii) 90%

(b)(i) Proof

(ii) $x = 40$