WESTERN REGION

2005
TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics

General Instructions

- Reading Time - 5 minutes.
- Working Time - 3 hours.
- Write using a blue or black pen.
- Approved calculators may be used.
- A table of standard integrals is provided at the back of this paper.
- All necessary working should be shown for every question.
- Begin each question on a fresh sheet of paper.

Total marks (**120**)

- Attempt Questions 1-10.
- All questions are of equal value.

Question 1 **(12 Marks)** Use a Separate Sheet of paper **Marks**

(a) Factorise

i) $16x^2 - 9$ **1**

ii) $5a^2 + 10a$ **1**

(b) Express $3\sqrt{5} + \sqrt{20}$ in the form $\sqrt{a}$. **2**

(c) Simplify $2x^2y - yx^2 + xy^2 + 2y^2x$ **1**

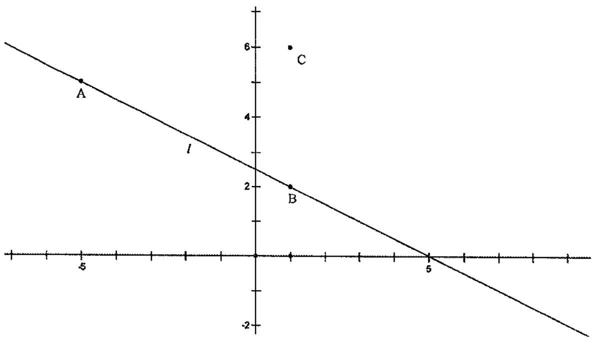
(d) Evaluate $\lim_{x \to \infty} \frac{3x^2 - 4x + 5}{x^2}$ **2**

(e) Solve $\frac{1}{x} = x - 1$ leaving your answer in exact form. **2**

(f) Express $0\cdot\dot{4}\dot{2}$ as a rational number in simplest form. **2**

(g) Express $\log_e 5$ as a decimal correct to 2 decimal places. **1**

Question 2 **(12 Marks)** Use a Separate Sheet of paper **Marks**



The line l contains the points A (-5, 5) and B (1, 2).

C (1, 6) is another point which is not on the line l.

(a) Calculate the distance AB. **2**

(b) Find the gradient of AB and hence, the acute angle the line AB makes with the x-axis. **2**

(c) Show that the equation of the line AB is $x + 2y - 5 = 0$. **2**

(d) Find the perpendicular distance of C from l. **2**

(e) Use your answers from a) and d) to show the area of $\Delta ABC = 12u^2$. **1**

(f) Show how you can calculate the area of ΔABC in another way. **1**

(g) A fourth point D forms a parallelogram with A, B and C. Write all the possible positions of D. **2**

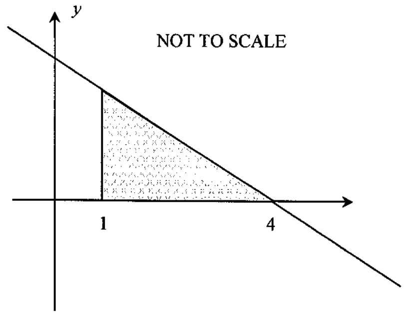
Question 3 **(12 Marks)** Use a Separate Sheet of paper **Marks**

(a) Differentiate the following with respect to x.

(i) $\sqrt[4]{x^3}$ **2**

(ii) $\sin x.\ln x$ **2**

(iii) $\dfrac{\sin x}{e^x}$ **2**



(b) $y = 4 - x$ is shown on the graph. **3**

Calculate the volume of the solid formed when the area bounded by the function, x axis and $x = 1$ is rotated around the y axis.

(c) $g'(x) = 3x^2 - 4 + \dfrac{1}{x^2}$ **3**

$g(x)$ takes the value 4 when $x = 1$. Find $g(x)$.

Question 4 **(12 Marks)** Use a Separate Sheet of paper **Marks**

(a) Bernice contributes to a superannuation fund. She contributes $250 at the start of every quarter. The investment pays 8%pa interest, compounding quarterly. She continues making contributions for 30 years.

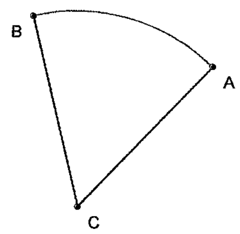
(i) How much does she contribute altogether? **1**

(ii) What is the value of her initial $250 investment at the end of the 30 years? **1**

(iii) Find the total value of her superannuation. **3**

(iv) How much of her superannuation lump sum is interest? **1**

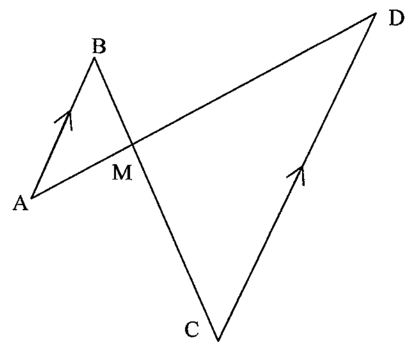
(b) The sector ABC has $r = 5$cm and the arc length BA = 5cm. Calculate the area of the sector. **2**



(c) Find the equation of the tangent to $y = 2e^x$ at the point (0, 2). **2**

(d) Sketch the graph of $y = \ln(x-2)$, showing the x intercept and the asymptote. **2**

Question 5 **(12 Marks)** Use a Separate Sheet of paper **Marks**



(a) The quadrilateral ABCD has AB || CD. It also has the feature that DA intersects BC at M.

(i) Prove $\Delta AMB \;|||\; \Delta DMC$. **3**

(ii) If AB:CD = 2:5 and area $\Delta AMB = 10u^2$, find the total area of the quadrilateral. **1**

(b) (i) Find $\int \cos(4x)dx$ **1**

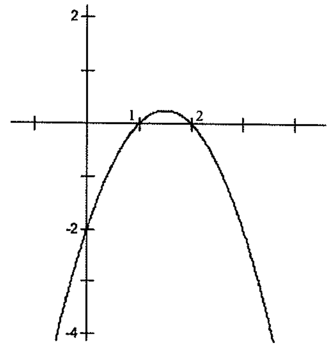
(ii) Evaluate $\int_1^4 \frac{x}{x^2+4}dx$ **2**

(c) On the Cartesian Plane, sketch the region satisfying the inequalities **3**

$x \geq 2 \quad y \geq 4 \quad$ and $\quad y \leq 8 - x$

(d) A hat contains 3 white marbles, 4 black marbles, 9 red marbles and 4 green marbles. 2 marbles are drawn out without replacement. What is the probability that they are both red? **2**

Question 6 **(12 Marks)** Use a Separate Sheet of paper **Marks**



(a) The graph depicts the gradient function $f'(x)$, for the function $y = f(x)$

(i) What x values provide the stationary points of $f(x)$? **1**

(ii) What feature of the graph of $f(x)$ will occur at $x = 1{\cdot}5$? **1**

(iii) If $f(0) = 3$ draw $y = f(x)$ between $x = -1$ and $x = 4$. **2**

iv) Explain why $f(x)$ has only one real root? **1**

(b) Consider the functions $y = 3 - \dfrac{x}{2}$ and $y = \dfrac{1}{2}x^2 - 2x + 1$

(i) Find the points x values where the curves intersect. **2**

(ii) Find the area between the curves. **2**

(c) Michael plays a match made up of 3 sets. In each set Michael has a 0.4 chance of winning that set. Find the probability that Michael will :

(i) Win all three sets. **1**

(ii) Not win any sets. **1**

(iii) Win at least one set, but not all three sets **1**

Question 7 **(12 Marks)** Use a Separate Sheet of paper **Marks**

(a) A population of bacteria in a medium are growing at a rate proportional to the current population. The population obeys the model $P = P_0e^{kt}$, where P_0 is the population of bacteria at noon on 1 August and t is measured in hours. When $t = 6$ the population has grown from 900 000 to 1.4 million.

(i) Show that $\dfrac{dP}{dt} = kP$ **1**

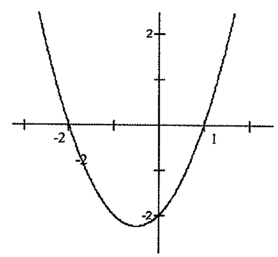
(ii) What is the value of k? **2**

(iii) What will the population be when $t = 10$? **1**

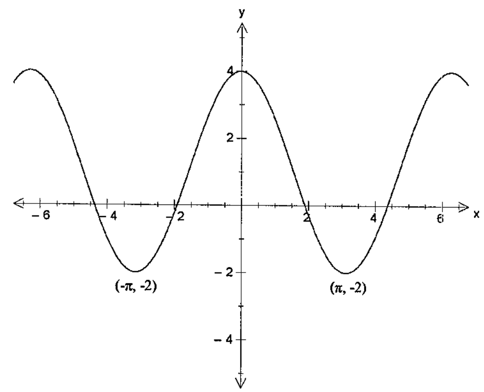
(iv) When will the population reach 3 million? **1**

(b) Write the functions represented by the following graphs

(i) **1**



(ii) **2**



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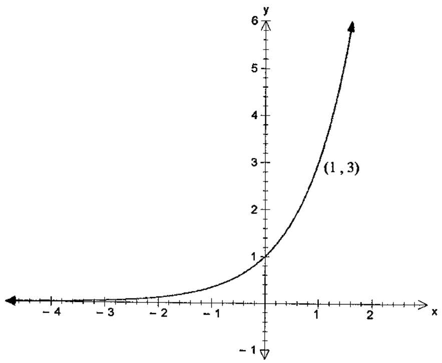
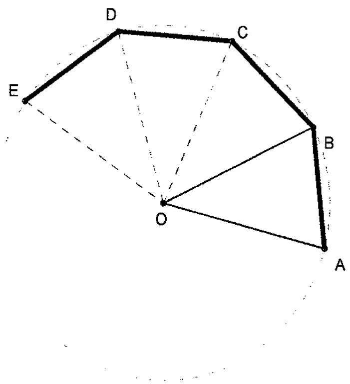
Question 7 continued **Marks**

(iii) 1



(c) ABCDE……. is a regular polygon with n sides inscribed within a unit circle with centre O.



(i) Explain why $A\hat{O}B = \dfrac{2\pi}{n}$ 1

(ii) Write an expression for the area of the polygon ABCDE… in terms of n. 1

(iii) Show $\lim\limits_{n \to \infty}\left(\dfrac{n}{2}\sin\left(\dfrac{2\pi}{n}\right)\right) = \pi$ 1

Question 8 **(12 Marks)** Use a Separate Sheet of paper **Marks**

(a) (i) Show that $y = mx - 2m^2$ is tangent to the parabola $x^2 = 8y$ 2

(ii) Find the two values of m for which the tangent passes through (2, -4) 2

(b) (i) Use Simpson's rule with 5 function values to evaluate 3

$$\int_0^4 \frac{\sqrt{144 - 9x^2}}{4}\,dx$$

ii) The formula $A = \dfrac{\pi ab}{4}$ where a=OA and b = OB, gives the exact value of the integral above. Comment on the accuracy of your answer from (i) compared to the exact answer. 1

(c) Consider the parabola $2y = x^2 - 4x$.

i) Rewrite it in the form $4a(y-k) = (x-h)^2$ 2

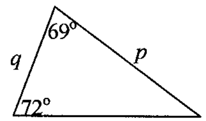
ii) Give the coordinates of the focus. 1

iii) Give the equation of the directrix. 1

Question 9 **(12 Marks)** Use a Separate Sheet of paper **Marks**

(a) Find the sum of the first 100 multiples of 5 **2**

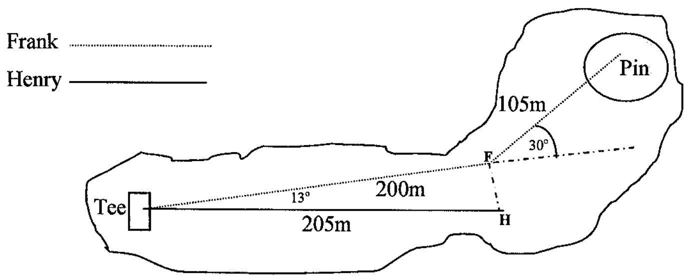
(b) Using the information in the diagram below to calculate the value of the ratio $\frac{p}{q}$ as a decimal correct to 3 decimal places **2**



(c) The 18th hole at Royal Maples is a dogleg to the left. Frank hits a 200m drive then turns left 30° and hits a 105m shot to the pin.

(i) What is the straight line distance from the tee to the pin? **2**

(ii) Henry hits his drive a distance of 205m and to the right of Frank's drive line by 13°. Show that the triangle formed by the two initial drives is approximately right angled. **2**



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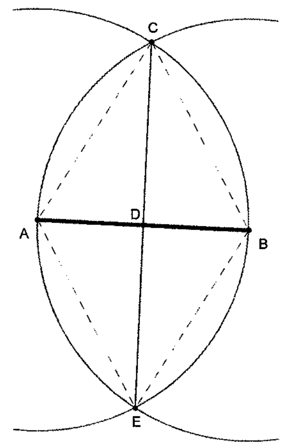
Question 9 continued **Marks**

(d) Two circles of equal radius are drawn. One has it's centre at A and the other has it's centre at B.

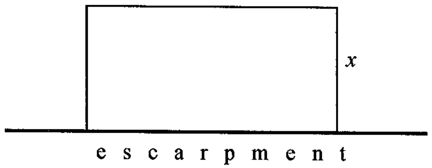
(i) Prove $\Delta AEC \equiv \Delta BEC$ **2**

(ii) Hence show CE bisects AB **2**



Question 10 (12 Marks) Use a Separate Sheet of paper **Marks**

(a) A farmer wishes to build a rectangular enclosure for his sheep.
Fortunately he can use a sandstone escarpment as one side of his rectangle.



He has sufficient fencing material for 200m of fence.

(i) If we let one side of the rectangle be x, write an expression for the area of the enclosure in terms of x. **2**

(ii) Find the maximum area enclosure the farmer can build. Be sure to justify that this area is a maximum. **3**

(b) A particle is moving in a straight line with velocity $v = 3e^t + 6e^{-t}$
It begins its motion at origin. t is measured in minutes and v in ms^{-1}.

(i) What is the velocity initially? **1**

(ii) Find an equation for x, the displacement of the particle **2**

(iii) When $x = 10$ show that $3e^{2t} - 7e^t - 6 = 0$ **1**

(iv) Find t when $x = 10$ **1**

(c) The number line graph represents the solution to an inequality of the type $|x - a| \le b$. Find the value of a and b. **2**

STANDARD INTEGRALS

$\int x^n dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0 \text{ if } n < 0$

$\int \frac{1}{x} dx = \ln x, \quad x > 0$

$\int e^{ax} dx = \frac{1}{a}e^{ax}, \quad a \neq 0$

$\int \cos ax dx = \frac{1}{a}\sin ax, a \neq 0$

$\int \sin ax dx = -\frac{1}{a}\cos ax, a \neq 0$

$\int \sec^2 ax dx = \frac{1}{a}\tan ax, a \neq 0$

$\int \sec ax \tan ax dx = -\frac{1}{a}\sec ax, a \neq 0$

$\int \frac{1}{a \ + x^2} dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, a \neq 0$

$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\frac{x}{a}, a > 0, -a < x < a$

$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln(x + \sqrt{x^2 - a^2}), x > a > 0$

$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln(x + \sqrt{x^2 + a^2})$

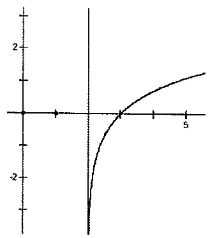
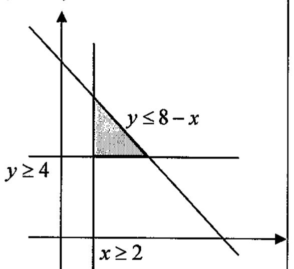
Note: $\ln x = \log_e x, x > 0$

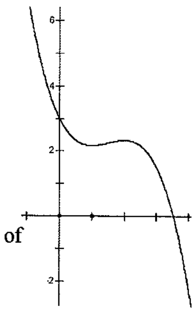
Western Region Trial HSC Examinations 2005 Mathematics

These are suggested answers only. Any reasonable solution should be accepted.

Solutions	**Marks/Comments**
1 a) i) $(4x+3)(4x-3)$	1
ii) $5a(a+2)$	1
b) $3\sqrt{5}+\sqrt{4}\sqrt{5}=3\sqrt{5}+2\sqrt{5}$	1
$5\sqrt{5}=\sqrt{25}\sqrt{5}=\sqrt{125}$	1
c) x^2y+3xy^2	1
d) $\lim_{x\to\infty}\left(3-\frac{4x}{x^2}+\frac{5}{x^2}\right)$	1
$=3$	1
e) $x^2-x-1=0$	1
$\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{1\pm\sqrt{1+4}}{2}=\frac{1\pm\sqrt{5}}{2}$	1
f) $x=0\cdot\dot{4}\dot{2}$ $100x=42\cdot\dot{4}\dot{2}$	1
$99x=42 \qquad x=\frac{42}{99}=\frac{14}{33}$	1
g) $1{\cdot}61$	1
	/12
2 a) $d=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}=\sqrt{6^2+3^2}$	1
$=\sqrt{45}$ $=3\sqrt{5}$	1 simplification not reqd
b) $m=\frac{-3}{6}=\frac{-1}{2}$	1
$\theta=\tan^{-1}\left(\frac{1}{2}\right)=26°34'=27°$	1 ignore rounding error
c) subbing $x=-5, y=5$ or $y-2=-\frac{1}{2}(x-1)$	1
$-5+2\times5-5=0$ true $\qquad x+2y-5=0$ subbing $x=1, y=2$ $1+2\times2-5=0$ true	1

d) $d=\frac{\lvert Ax_1+By_1+C\rvert}{\sqrt{A^2+B^2}} \qquad x_1=1, y_1=6$	1
$d=\frac{\lvert 1+12-5\rvert}{\sqrt{5}}=\frac{8}{\sqrt{5}}$	1
e) $A=\frac{bh}{2}=3\sqrt{5}\times\frac{8}{\sqrt{5}}\div2=12$	1
f) call BC =4 the base and the perpendicular distance to A = 6, the height. 4 × 6 ÷ 2 = 12	1
g) (-5, 1), (-5, 9), (7, 3)	1 for first, 1 more for all 3 /12
3. a) i) note $\sqrt[4]{x^3}=x^{\frac{3}{4}}$	1
$\frac{d}{dx}\left(x^{\frac{3}{4}}\right)=\frac{3}{4}x^{\frac{-1}{4}}$	1
ii) $vu'+uv'$	1
$\cos x\ln x+\frac{\sin x}{x}$	1
iii) $\frac{vu'-uv'}{v^2}$	1
$\frac{e^x\cos x-e^x\sin x}{(e^x)^2}=\frac{\cos x-\sin x}{e^x}$	1
b) $V=\pi\int_a^b x^2dy$ noting $x=4-y$ i.e. $x^2=16-8y+y^2$	1
$V=\pi\int_0^3 16-8y+y^2.dy=\pi\left[16y-4y^2+\frac{y^3}{3}\right]_0^3$	1
$=\pi(48-36+9)$ $=21\pi$	1
c) $g(x)=\int g'(x).dx=x^3-4x-x^{-1}+c$	1
but $g(x)=4$ when $x=1$ i.e. $4=1-4-1+c$ so $c=8$	
and $g(x)=x^3-4x-\frac{1}{x}+8$	1 1 /12
4 a) i) $4\times250\times30=\$30\,000$	1
ii) $250\times1{\cdot}02^{120}=\$2\,691{\cdot}29$	1
iii) $250\times(1{\cdot}02^{120}+1{\cdot}02^{119}+1{\cdot}02^{118}+\ldots\ldots+1{\cdot}02)$	1

$250\times\frac{a(r^n-1)}{r-1}=250\times\frac{1\cdot02(1\cdot02^{120}-1)}{1\cdot02-1}$ $250\times498\cdot02328=\$124\ 505\cdot83$ iv) $124 505·82 - $30 000 = $94 505.83	1 1 1 for the subtraction
b) $\theta=1$ and $A=\frac{1}{2}r^2\theta$ $A=0\cdot5\times5^2\times1=12.5cm^2$	1 1
c) $f'(x)=2e^x$ $f'(0)=2$ slope of tangent $y-2=2x$ $y=2x+2$	1 1
d)  asymptote $x=2$ x intercept 3	1 1 /12
5 a) i) $B\hat{A}M=C\hat{D}M$ alternate angles parallel lines $C\hat{M}D=B\hat{M}A$ vertically opposite angles ΔAMB||ΔDMC equiangular ii) Ratio of areas = 6·25 (=2·5²) total area = 10 + 62·5 = 72·5	1 1 1 1
b) i) $I=\frac{1}{4}\sin(4x)+c$	1 ignore c
ii) $\int^{e^4}\frac{x}{x^2+4}.dx=\frac{1}{2}\int^{e^4}\frac{2x}{x^2+4}.dx$ $\frac{1}{2}\left[\ln(x^2+4)\right]^{e^4}=\frac{1}{2}(\ln(e^8+4)-\ln5)$ $\ln\sqrt{\frac{e^8+4}{5}}$	1 1
c) Notes as to what graph is what should appear on the graph 	1 for at least 1 area shown 1 for all 3 graphs shown 1 for correct area and annotation

5 d) $3+4+9+4=20$ $P=\frac{9}{20}\times\frac{8}{19}=\frac{18}{95}$	1 1 simplification not reqd /12
6 a) i) $x=1, x=2$ ii) point of inflexion iii)  iv) The graph of derivative indicates slope ≥ -2 between $x=0$ and $x=1$. The slope would have to be less than -3 To reach from $y=3$ to $y=0$ in the space of 1 unit	1 1 1 shape 1 intercept 1 Ignore inaccuracy such as crossing the axis before first min.
b) i) $3-\frac{x}{2}=\frac{x^2}{2}-2x+1$ $6-x=x^2-4x+2$ $0=x^2-3x-4$ $0=(x-4)(x+1)$ $x=-1$ or 4	1 1
ii) $I=\int_{-1}^{4}3-\frac{x}{2}-\left(\frac{x^2}{2}-2x+1\right).dx$	1
$\int_{-1}^{4}2+\frac{3x}{2}-\frac{x^2}{2}.dx=\left[2x+\frac{3x^2}{4}-\frac{x^3}{6}\right]_{-1}^{4}=8+12-\frac{32}{3}-\left(\frac{3}{4}+\frac{1}{6}-2\right)=10\frac{5}{12}$	1
c) (i) P(WWW) = $0.4^3=0.064$ (ii) P(LLL) = $0.6^3=0.216$ (iii) P(win at least 1 but not 3) = 1-(0.064 + 0.216) = 1 – 0.28 = 0.72	1 for 1/120 1 /12
7 a) i) $\frac{dP}{dt}=kP_0e^{kt}=kP$ as required ii) when $t=6, P=1400000, P_0=900000$, $1400000=900000e^{6k}$ $\frac{14}{9}=e^{6k}$ $6k=\ln(14/9)$ $k=0\cdot073638792$ iii) at $t=10$ $P=900000\times e^{10\times0.073638792}=1\ 879\ 541$	1 1 1 1

iv) $\frac{3000000}{900000} = e^{t\times 0.073638792}$ $\ln 3.\dot{3} = t \times 0.073638792 \quad t = 16hrs21'$	1
b) i) $y = (x+2)(x-1)$ ii) $y = 3\cos x + 1$ iii) $y = 3^x$	1 1 for 1 element 1 for the others 1
c) i) $360° = 2\pi$ therefore 2π by the number of sides n ii) $\frac{1}{2}ab\sin C = \frac{1}{2}\times 1\times 1\times \sin\left(\frac{2\pi}{n}\right)$ iii) As n increases the area of polygon approaches area of the circle i.e. $\pi\times 1\times 1 = \pi$	1 1 1 /12
8 a) i) Solving simultaneously $8mx - 16m^2 = x^2$ $x^2 - 8mx + 16m^2 = 0$ $(x-4m)^2 = 0$ As this has only 1 answer the line is a tangent ii) $x = 2, y = -4$ and $-4 = 2m - 2m^2$ $2m^2 - 2m - 4 = 0 \quad 2(m-2)(m+1) = 0 \quad m = -1$ or 2	1 1 1 1
b) i) $A \cong \frac{h}{3}(f(0) + f(4) + 2\times f(2) + 4\times(f(1)+f(3)))$ $\cong \frac{1}{3}\left(3 + 0 + 2\times\frac{\sqrt{108}}{4} + 4\times\left(\frac{\sqrt{135}}{4} + \frac{\sqrt{63}}{4}\right)\right)$ $= 9{\cdot}2507855$ ii) $3 \times 4 \times \pi \div 4 = 9{\cdot}424778$ the estimate slightly less than the true value	1 1 1 1 some effort to compare
c) i) $2y = x^2 - 4x \qquad 2y + 4 = x^2 - 4x + 4$ $4\times\frac{1}{2}(x+2) = (x-2)^2$ ii) Focus is (2, -1·5) iii) Directrix $y = -2{\cdot}5$	1 1 1 1 /12
9 a) $n = 100, \ a = d = 5$ $\frac{n}{2}(2a + (n-1)d) = 50\times(10 + 495) = 25250$	1 1
b) $\frac{p}{\sin 72} = \frac{q}{\sin 39}$ so … $\frac{p}{q} = \frac{\sin 72}{\sin 39} = 1{\cdot}511$	1 + 1

c) i) $c^2 = a^2 + b^2 - 2ab\cos C =$ $= 200^2 + 105^2 - 2\times 200\times 105\times \cos 150$ $=87398$ $c = 295.63$ ii) By Cos Rule $FH^2 = 200^2 + 205^2 - 2\times 200\times 205\times \cos 13°$ $FH = 45.06847$ $200^2 + (45.06847) = 42031$ $205^2 = 42025$ Square of hypotenuse $\approx$ sum of squares on shorter sides. By Pythagoras Thm, angle is approx 90° Or use cos rule again to find angle, $\cos\theta = \frac{200^2 + FH^2 - 205^2}{2\times 200\times FH}$ θ = 89°58' approximately a right angle. d) (i) AC=BC radii of equal circles AE=BE radii of equal circles CE common ∴ ΔAEC≡ΔBEC (SSS) rule (ii) There are several possibilities a.CE bisects apex of isosceles ΔACB (corresponding angles in congruent triangles) and hence bisects base AB b. ACBE is rhombus, ∴ diagonals bisect at right angles. c. Prove $\Delta ACD \equiv \Delta BCD$ (SAS), hence AD = DB Others possible.	1 1 ignore rounding errors 1 1 1 both equal radii 1 common side + conclusion 2 marks for any reasonable explanation /12
10 a) i) in this case the other side equals $200 - 2x$ Then $A = x(200 - 2x) = 200x - 2x^2$ ii) $\frac{dA}{dx} = 200 - 4x$ which $= 0$ when $x = 50$ then A = 100 × 50 = 5 000m² this is a maximum since if we take $w = 49$ then $l = 102$ and A = 4998	1 1 1 1 1
b) i) $t = 0 \quad v = 3 + 6 = 9$ ii) $\int v.dt = x = 3e^t - 6e^{-t} + c$ but $x = 0$ when $t = 0$ so $0 = 3 - 6 + c$ whence $c = 3$ then $x = 3e^t - 6e^{-t} + 3$ iii) If $e^t = 3$, $e^{-t} = \frac{1}{3}$ and so $x = 3\times 3 - 2 + 3 = 10$ Under these conditions $3e^{2t} - 7e^t - 6 = 0 \quad 3\times 9 - 7\times 3 - 6 = 0$ iv) $(3e^t + 2)(e^t - 3) = 0 \quad t = \ln 3 = 1$ minute 6 seconds	1 1 1 1 1
c) $a = 6$ $b = 3$	1 1 /12