WESTERN REGION

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

1996

MATHEMATICS

3 Unit (Additional)
and
3/4 Unit (Common)

Time allowed - TWO hours
(Plus 5 minutes reading time)

DIRECTIONS TO CANDIDATES

- Attempt ALL questions.
- ALL question are of equal value.
- ALL necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Standard integrals are printed on the last page. These may be removed for your convenience.
- Board-approved calculators may be used.
- Each question should be started on a new page.

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$

Question 1 (Start a new page) **Marks**

a. Find the exact value of $\int_0^4 \frac{dx}{x^2+16}$ 3

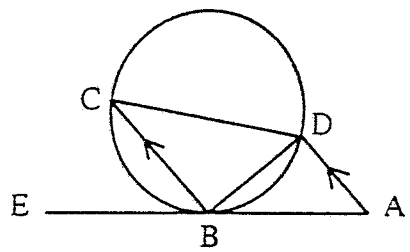
b. Find $\int (1-\cos x)^2 \, dx$ 3

c. Solve the inequality $\frac{3}{x-2} \geq 1 \quad x \neq 2$ 3

d. Find the first derivative of $y = \log_e\left(\frac{1}{\sqrt{\cos x}}\right)$ 3

Question 2 (Start a new page) **Marks**

a. 4



AB is a tangent at B and $AD \parallel BC$. Prove that $\Delta BCD \,|||\, \Delta DBA$.

b. Find $\int \frac{2x}{(x-1)^2} \, dx$ using the substitution $u = x - 1$ 4

c. Prove by the method of Mathematical Induction that 4

$$\sum_{r=1}^{n} 5^{r-1} = \frac{5^n - 1}{4}$$

Question 3 (Start a new page) Marks

a. If $^{12}P_r = 120.\,^{12}C_r$ find r. 3

b. The velocity of a particle moving in a straight line is given by 5

$$v^2 = 8x - 2x^2 \quad m/\sec$$

i. Show that the particle is moving in simple harmonic motion.
ii. Find the centre of the motion.
iii. Determine the two end points between which the particle is oscillating.
iv. Find the maximum speed of the particle.

c. A formula for the rate of change in population of a colony of bacteria, is given by $P = 3200 + 400e^{kt}$ 4

If the population doubles after 20 hours, how long would it take to triple the original population.

Question 4 (Start a new page) Marks

a. At what points on the curve $y = \cos^{-1} x$, is the gradient equal to $-\frac{2}{\sqrt{3}}$ 3

b. Find the middle term in the expansion of $\left(x^3 - \frac{1}{3x}\right)^8$ 4

c. A capsule is in the shape of a cylinder with hemispherical ends. The radius of the cylindrical section is $r$cm, and the volume of the capsule is 16cm^3. 5

i. If the height of the cylinder is 4cm show that $r^3 + 3r^2 = \frac{12}{\pi}$

ii. Show that one solution of the equation $r^3 + 3r^2 = \frac{12}{\pi}$ lies between 0 and 1.

iii. The equation $r^3 + 3r^2 = \frac{12}{\pi}$ has a root close to $0 \cdot 9$. Use one application of Newton's method to give a better approximation.

Question 5 (Start a new page) **Marks**

a. Solve the equation $3x^3 - 17x^2 - 8x + 12 = 0$ given that the product of two of the roots is 4 **3**

b. The probability that a vaccine succeeds is $\frac{29}{30}$. An experiment is conducted m times with white mice. **4**

i. What is the probability that the experiment will fail at least once?

ii. Show that if the probability that the experiment will fail at least once in m trials, is greater than than $\frac{9}{10}$ then $m > \frac{1}{\log_{10} 30 - \log_{10} 29}$

c. For a particular vessel, the rate of increase of the volume with respect to its depth, is given by $\frac{dV}{dh} = \frac{\pi(h+6)^2}{12}$ $0 \le h \le 10$ **5**

where $V\text{cm}^3$ is the volume and h is the depth of the water.

i. If the container is initially empty, show that the volume as a function of the depth is $V = \frac{\pi h}{36}\left(h^2 + 18h + 108\right)$

ii. Find the volume when the depth is 6cm.

iii. If water is being poured into the vessel at a constant rate of $8\text{cm}^3/\text{s}$ find an expression for the rate of increase in the depth of the water.

iv. At what rate is the depth increasing when the water level is 6 cm, and how long will it take to the nearest second to reach this level.

Question 6 (Start a new page) **Marks**

a. The letters of the word **R E P E T I T I O N** are arranged at random in a row. **3**

i. how many different arrangements are possible?

ii. what is the probability that one particular arrangement will have vowels and consonants alternating?

(Question 6 continued on page 4)

Question 6 Continued Marks

b. i. Write the general expansion of $(1+x)^n$ 3

ii. Hence or otherwise prove that

$$^nC_0 + \tfrac{1}{2}\,{}^nC_1, \; + \tfrac{1}{3}\;{}^nC_2 + \ldots\ldots + \frac{1}{n+1}\;{}^nC_n = \frac{2^{n+1}-1}{n+1}$$

c. The curve $y = \sin^{-1} x$ intersects the curve $y = \cos^{-1} x$ at P, and the latter intersects the x axis at Q. 6

i. Draw a neat sketch of this information.

ii. Verify that P has co-ordinates $\left(\frac{1}{\sqrt{2}}\;.\;\frac{\pi}{4}\right)$

iii. Prove $\frac{d}{dx}\left(x\sin^{-1}x + \sqrt{1-x^2}\right) = \sin^{-1}x$

iv. If O is the origin, find the area enclosed by the arcs OP and PQ and the x axis using the results in (iii) and the fact that
$\frac{d}{dx}\left(x\cos^{-1}x - \sqrt{1-x^2}\right) = \cos^{-1}x$

Question 7 (Start a new page) Marks

A projectile fired with velocity V and at an angel $45°$ to the horizontal, just clears the tops of two vertical posts of height $8a^2$, and the posts are $12a^2$ apart. There is no air resistance, and the acceleration due to gravity is g.

a. If the projectile is at the point (x, y) at time t, derive expressions for x and y in terms of t. 3

b. Hence show that the equation of the path of the projectile is $y = x - \frac{gx^2}{V^2}$ 2

c. Using the information in (b) show that the range of the projectile is $\frac{V^2}{g}$ 2

d. If the first post is b units from the origin, show 2

i. $\frac{V^2}{g} = 2b + 12a^2$

ii. $8a^2 = b - \frac{gb^2}{V^2}$

e. Hence or otherwise prove that $V = 6a\sqrt{g}$ 3

Western Region
Trial Higher School Certificate Examination
1996

MATHEMATICS
3/4 UNIT COMMON

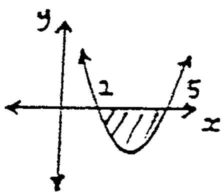
Solutions and Marking Scheme

Please Note:

* These are suggested solutions. They are not intended to specify the amount of working required or the method to be applied.

  Teachers should accept any valid method of solution providing adequate working is shown

WESTERN REGION 1996 TRIAL HSC MARKING SCHEME

SOLUTIONS	COMMENTS
QUESTION 1. – (12 marks)	
a.) $\int_0^4 \frac{dx}{x^2+16} = \int_0^4 \frac{dx}{x^2+4^2}$	
$= \left[\frac{1}{4}\tan^{-1}\frac{x}{4}\right]_0^4$	1 for integral
$= \left(\frac{1}{4}\tan^{-1}1\right) - \left(\frac{1}{4}\tan^{-1}0\right)$	1 for substitution
$= \frac{1}{4}\times\frac{\pi}{4} - 0$	
$= \boxed{\frac{\pi}{16}}$	1 answer
b.) $\int(1-\cos x)^2\,dx = \int 1 - 2\cos x + \cos^2 x\,dx$	1 for expansion
$= x - 2\sin x + \frac{1}{2}x + \frac{1}{4}\sin 2x$	1 for integral
$= \boxed{\frac{3x}{2} - 2\sin x + \frac{1}{4}\sin 2x + C}$	1 answer
c.) $\frac{3}{x-2} \geqslant 1$	
$\frac{(x-2)^{\cancel{2}}\cdot 3}{\cancel{x-2}} \geqslant 1\cdot(x-2)^2$	1 for method
$3x-6 \geqslant x^2-4x+4$	
$x^2-7x+10 \leqslant 0$	
$(x-5)(x-2) \leqslant 0$ 	1 for quadratic
$2 \leqslant x \leqslant 5$	
but since $x \neq 2$.	
$\therefore\ \boxed{2 < x \leqslant 5}$	1 answer

(1)(a) $\frac{\pi}{16}$

(b) $\frac{3x}{2} - 2\sin x + \frac{1}{4}\sin 2x + c$

(c) $2 < x \le 5$

(d) $\frac{1}{2}\tan x$

(2) (a) Proof (Equiangular triangles)

(b) $2\ln(x-1) - \frac{2}{x-1} + c$

(c) Proof

(3)(a) $r = 5$

(b) (i) Show that $\ddot{x} = -2(x-2)$

(ii) Centre at $x = 2$

(iii) $0 \le x \le 4$

(iv) $2\sqrt{2}$ ms^{-1}

(c) 25 hrs 35 mins

(4) (a) $\left(\frac{1}{2}, \frac{\pi}{3}\right)$ and $\left(-\frac{1}{2}, \frac{2\pi}{3}\right)$

(b) $\frac{70}{81}x^8$

(c) (i) Proof

(ii) $f(0) < 0$ and $f(1) > 0$, root exists

(iii) 0.9843

(5) (a) $6, \frac{2}{3}$ or -1

(b) (i) $1 - \left(\frac{29}{30}\right)^m$

(ii) Proof

(c) (i) Proof

(ii) 42π cm^3

(iii) $\frac{dh}{dt} = \frac{96}{\pi(h+6)^2}$

(iv) $\frac{2\pi}{3}$ cms^{-1}, 16.49s

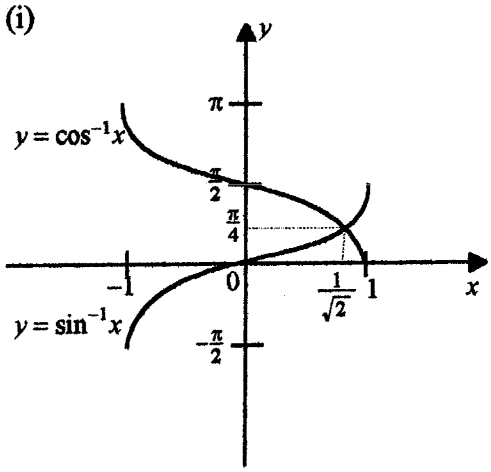
(6)(a) (i) 453 600

(ii) $\frac{1}{126}$

(b) (i) $\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \dots + \binom{n}{n}$

(ii) Proof

(c) (i)



(ii) Verify by subtituting $x = \frac{1}{\sqrt{2}}$

(iii) Proof

(iv) $\sqrt{2} - 1$ units^2

(7) (a) $x = \frac{Vt}{\sqrt{2}}$, $y = -\frac{gt^2}{2} + \frac{Vt}{\sqrt{2}}$

(b), (c), (d) and (e) Proofs

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SOLUTIONS	COMMENTS
d.) $y = \log_e \left(\frac{1}{\sqrt{\cos x}} \right)$	
$= \log_e 1 - \log_e \cos x^{\frac{1}{2}}$	1 for splitting. $\log(\frac{a}{b})$
$= \log_e 1 - \frac{1}{2} \log_e \cos x$	
$\frac{dy}{dx} = 0 - \frac{1}{2} \cdot \frac{-\sin x}{\cos x}$	1 for $\frac{dy}{dx}$
$= \boxed{\frac{1}{2} \tan x}$	1 answer.

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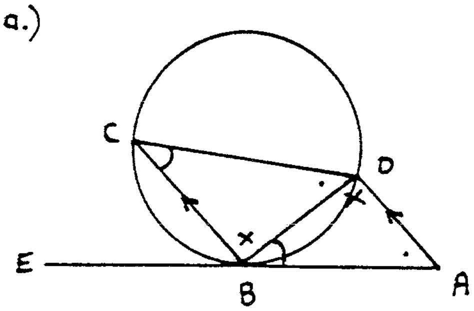
SOLUTIONS	COMMENTS
QUESTION 2 - (12 marks)	
a.) 	
$\angle DBA = \angle BCD$ (angle between tangent and chord = to $\angle$ in alternate segment)	1 (reason 1)
$\angle CBD = \angle BDA$ (alternate $\angle$'s in $\cancel{//}$ $BC // AD$)	1 (reason 2)
$\angle CDE = \angle DAB$ (third $\angle$ in Δ $180°$)	1 (reason 3)
$\therefore$ $\boxed{\Delta BCD \parallel\!\mid \Delta DBA \text{ (equiangular triangles)}}$	1 answer
b.) $\int \frac{2x}{(x-1)^2}\,dx$ $u = x-1$, $du = dx$	1 for sub and $x = u+1$
$\int \frac{2u+2}{u^2}\,du$ $x = u+1$	1 for $\int \frac{2u+2}{u^2}\,du$
$\int \frac{2}{u} + 2u^{-2}\,du$	
$= 2\ln u - 2u^{-1} + c$	1 for integration
$= 2\ln u - \frac{2}{u} + c$	
$= \boxed{2\ln(x-1) - \frac{2}{x-1} + c}$	1 for answer
c.) STEP 1: Prove for $n=1$	
LHS $= 5^{n-1} = 5^0 = 1$ RHS $= \frac{5^n - 1}{4} = \frac{4}{4} = 1$	
$\therefore$ LHS = RHS	
$\therefore$ True for $n=1$	1 for step 1

SOLUTIONS	COMMENTS
STEP 2 : Assume true for $n=k$ ie. $1+5+5^2+\ldots\ldots+5^{k-1} = \dfrac{5^k-1}{4}$	1 statements
and prove true for $n=k+1$ ie. $1+5+5^2+\ldots+5^{k-1}+5^k = \dfrac{5^{k+1}-1}{4}$	
now $1+5+5^2+\ldots+5^{k-1}+5^k$ $= \dfrac{5^k-1}{4} + 5^k$ $= \dfrac{5^k-1}{4} + \dfrac{4.5^k}{4}$ $= \dfrac{5.5^k-1}{4}$ $= \dfrac{5^{k+1}-1}{4}$	1 for working
STEP 3 : $\therefore$ If true for $n=k$ then true for $n=k+1$, but it is true for $n=1$. $\therefore$ true for $n=1+1=2$ $n=2+1=3$ etc. $\therefore$ $\boxed{\text{By Induction } 1+5+\ldots 5^{n-1} = \frac{5^n-1}{4}}$	1 for step 3

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SOLUTIONS	COMMENTS
QUESTION 3 - (12 marks) a.) $^{12}P_r = 120 . {}^{12}C_r$ $^{12}P_r = 120 . \dfrac{^{12}P_r}{r!}$ $r! = 120 . \dfrac{^{12}P_r}{^{12}P_r}$ $r! = 120$ $\boxed{r = 5}$	1 for $^{12}C_r = \dfrac{^{12}P_r}{r!}$ 1 solving. 1 answer
b.) i.) $V^2 = 8x - 2x^2$ $\frac{1}{2}v^2 = 4x - x^2$ $a = \dfrac{d}{dx}\left(\frac{1}{2}v^2\right) = 4 - 2x$ $\boxed{= 2(2-x)}$ $\therefore$ the particle is in S.H.M. as the acceleration is proportional to the distance from the centre of motion.	1 for method 1 for statement of S.H.M.
b.) ii.) when $a = 0$ $\therefore \quad 0 = 2(2-x)$ $\boxed{x = 2}$ $\therefore$ the centre of motion is 2m to the right of the origin.	1 for centre.
b.) iii.) when $v = 0$ $\therefore \quad 0 = 8x - 2x^2$ $0 = 2x(4-x)$ $\boxed{x = 0, 4}$ $\therefore$ the two endpoints are the origin and 4m to the right of the origin.	1 for endpoints

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SOLUTIONS	COMMENTS
b.) iv.) Max speed at centre of motion $\therefore \quad x = 2$ $V^2 = 8x - 2x^2$ $= 16 - 8$ $= 8$ $V = \pm\sqrt{8} = \pm 2\sqrt{2}$ $\therefore$ $\boxed{\text{the maximum velocity is } 2\sqrt{2} \text{ m/s}}$	1 for velocity
c.) $P = 3200 + 400e^{kt}$ when $t = 0$ $P = 3200 + 400e^0 = 3600$ $\therefore$ initial population is 3600. when $t = 20$ $\quad P = 7200$ $\therefore \; 7200 = 3200 + 400e^{20k}$ $4000 = 400e^{20k}$ $10 = e^{20k}$ $\ln 10 = 20k$ $k = \dfrac{\ln 10}{20} \doteqdot 0.115$ $\therefore \; P = 3200 + 400e^{0.115t}$ when $P = 10800$ $10800 = 3200 + 400e^{0.115t}$ $7600 = 400e^{0.115t}$ $19 = e^{0.115t}$ $\ln 19 = 0.115t$ $t = \dfrac{\ln 19}{0.115} \doteqdot 25.575$ $\therefore$ $\boxed{\text{to triple } t \doteqdot 25\text{hr } 35\text{mins}}$	1 for initial pop. 1 for k. 1 for setting up equation 1 for answer.

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SOLUTIONS	COMMENTS
QUESTION 4 - (12 marks)	
a.) $y = \cos^{-1} x$ $\quad m = \frac{-2}{\sqrt{3}}$	
$\frac{dy}{dx} = \frac{-1}{\sqrt{1-x^2}}$	1 for derivative
$\frac{-2}{\sqrt{3}} = \frac{-1}{\sqrt{1-x^2}}$	
$-2\sqrt{1-x^2} = -\sqrt{3}$	
$4(1-x^2) = 3$	
$4 - 4x^2 = 3$	
$4x^2 = 1$	
$x^2 = \frac{1}{4}$	
$x = \pm \frac{1}{2}$	1 for solving.
when $x = -\frac{1}{2}$ $\quad y = \cos^{-1}(-\frac{1}{2}) = \frac{2\pi}{3}$	
when $x = \frac{1}{2}$ $\quad y = \cos^{-1}(\frac{1}{2}) = \frac{\pi}{3}$	
$\therefore$ points are $\boxed{(\frac{1}{2}, \frac{\pi}{3}) \text{ and } (-\frac{1}{2}, \frac{2\pi}{3})}$	1 for points
b.) $(x^3 - \frac{1}{3x})^8$	
Note :- middle term is the fifth term.	1 for 5th term
$T_{k+1} = {}^nC_k\, (a)^{n-k} . b^k$	1 for Theorem
$T_{4+1} = {}^8C_4\, (x^3)^4 . (\frac{-1}{3x})^4$	
$T_5 = 70\, x^{12} . \frac{1}{81x^4}$	
$= \frac{70x^8}{81}$	1 for substitution
$\therefore$ $\boxed{\text{the middle term is } \frac{70}{81}x^8}$	1 for answer

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SOLUTIONS	COMMENTS
c.) i.) $V = 16$ $\therefore \quad \frac{4}{3}\pi r^3 + 4\pi r^2 = 16$ $\frac{1}{3}\pi r^3 + \pi r^2 = 4$ $\pi r^3 + 3\pi r^2 = 12$ $r^3 + 3r^2 = \frac{12}{\pi}$ $\therefore \quad \boxed{r^3 + 3r^2 = \frac{12}{\pi}}$	1 for showing.
c.) ii.) $r^3 + 3r^2 = \frac{12}{\pi}$ $f(r) = r^3 + 3r^2 - 12/\pi$ $f(0) = -12/\pi$ $f(1) = 0.18028$ change in sign. $\therefore$ one root lies between 0 and 1.	1 for sub 0 and 1 1 for change of sign and reason
c.) iii.) $f(r) = r^3 + 3r^2 - 12/\pi$ $f'(r) = 3r^2 + 6r$ if $a = 0.9$ then a closer approx of a is given by $a_1 = a - \frac{f(a)}{f'(a)}$ $a_1 = 0.9 - \frac{f(0.9)}{f'(0.9)}$ $a_1 = 0.9 - \left(\frac{-0.6607186}{7.83}\right)$ $a_1 = 0.9 + 0.0843$ $a_1 = 0.9843$ $\therefore$ 0.9843 is a better approx	1 for formula and substitution 1 for calculation

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SOLUTIONS	COMMENTS
QUESTION 5 - (12 marks)	
a.) Let the roots be α, β and γ Product of roots $\therefore \quad \alpha\beta\gamma = \frac{-12}{3} = -4$	1 for product
but $\quad \alpha\beta = 4 \quad$ or $\quad \beta = \frac{4}{\alpha}$ $\therefore \quad 4\gamma = -4$ $\gamma = -1$	
Sum of roots $\alpha + \beta + \gamma = \frac{17}{3}$	1 for sum
$\alpha + \beta - 1 = \frac{17}{3}$ $\alpha + \frac{4}{\alpha} = \frac{20}{3}$ $3\alpha^2 - 20\alpha + 12 = 0$ $(3\alpha - 2)(\alpha - 6) = 0$	
$\alpha = 6$ or $\frac{2}{3}$ $\therefore$ the roots are $6, \frac{2}{3}$ or -1 $\boxed{\text{ie. } x = 6, \frac{2}{3}, -1}$	1 for 3 solutions
b.) i.) P(fail at least once) $= 1 -$ P(doesn't fail)	1 for $P(E) = 1 - P(\bar{E})$
$= 1 - \left(\frac{29}{30}\right)^m$	1 for expression
b.) ii.) P(fail at least once) $> \frac{9}{10}$ $\therefore \quad 1 - \left(\frac{29}{30}\right)^m > \frac{9}{10}$	1 for setting up equation
$\left(\frac{29}{30}\right)^m < \frac{1}{10}$ $\left(\frac{30}{29}\right)^m > 10$	
$\log_{10}\left(\frac{30}{29}\right)^m > \log_{10} 10$ $m(\log_{10} 30 - \log_{10} 29) > 1 \qquad \therefore m > \frac{1}{\log_{10} 30 - \log_{10} 29}$	1 for working

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SOLUTIONS	COMMENTS
c.) i.) $\frac{dV}{dh} = \frac{\pi}{12}(h^2+12h+36)$ $V = \frac{\pi}{12}\int h^2+12h+36\,dh$ $= \frac{\pi}{12}\left(\frac{h^3}{3}+6h^2+36h\right)+c$ $= \frac{\pi}{36}(h^3+18h^2+108h)+c$ $= \frac{\pi h}{36}(h^2+18h+108)+c$ when $h=0$, $V=0$ and $c=0$ $\therefore \boxed{V = \frac{\pi h}{36}(h^2+18h+108)}$	integral 1 for correct form
c.) ii.) when $h=6$ $V = \frac{6\pi}{36}(252) = \boxed{42\pi \text{ cm}^3}$	1 for V when $h=6$
c.) iii.) $\frac{dV}{dt} = 8$ $\frac{dh}{dt} = \frac{dV}{dt} \times \frac{dh}{dV}$ $= 8 \times \frac{12}{\pi(h+6)^2}$ $\boxed{\frac{dh}{dt} = \frac{96}{\pi(h+6)^2}}$	1 for chainrule and $\frac{dh}{dt}$
c.) iv.) when $h=6$ $\frac{dh}{dt} = \frac{96}{144\pi} = \frac{2\pi}{3}$ $\therefore$ the depth is increasing at a rate of $\boxed{\frac{2\pi}{3} \text{ cm/sec.}}$ (see over page)	1 for rate

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SOLUTIONS	COMMENTS
c.) iv.) con't.	
$\frac{dh}{dt} = \frac{96}{\pi(h+6)^2}$	
$\frac{dt}{dh} = \frac{\pi}{96}(h^2+12h+36)$	
$t = \frac{\pi}{96}\int h^2+12h+36\ dh$	
$t = \frac{\pi}{96}\left(\frac{h^3}{3}+6h^2+36h\right)+c$	
when $h=0$, $t=0$ and $c=0$	
$\therefore\ t = \frac{\pi}{288}(h^3+18h^2+108h)$	
when $h=6$	
$t = \frac{\pi}{288}\left((6)^3+18(6)^2+108(6)\right)$	
$= \boxed{16\cdot 49 \text{ seconds}}$	1 for working to 16·49 se

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SOLUTIONS	COMMENTS
QUESTION 6 - (12 marks) a.) i.) number of arrangements $n = \dfrac{10!}{2!\,2!\,2!}$ $= \boxed{453600}$	1 for arrangements
a.) ii.) P (vowels and consonants alt) $= \dfrac{2 \times \frac{5!}{2!2!} \times \frac{5!}{2!}}{453600}$ $= \dfrac{360\cancel{0}}{45360\cancel{0}}$ cvcvcvcvcv or vcvcvcvcvc $= \dfrac{1}{126}$ $\therefore \boxed{P(\text{V and C alternate}) = \frac{1}{126}}$ Note:- $\dfrac{5!}{2!2!} \rightarrow$ Arrange 5 vowels with 2 vowels repeated. $\dfrac{5!}{2!} \rightarrow$ Arrang 5 consonants with 1 repeated.	1 for accounting for 2 combinations 1 for correct prob.
b.) i.) $\boxed{(1+x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + \dots + {}^nC_n x^n}$	1 for expression
b.) ii.) $(1+x)^n = {}^nC_0 + {}^nC_1 x + \dots\dots + {}^nC_n x^n$ integrate both sides w.r.t x. $\dfrac{(1+x)^{n+1}}{n+1} = {}^nC_0 x + {}^nC_1 \dfrac{x^2}{2} + \dots\dots + \dfrac{{}^nC_n x^{n+1}}{n+1} + C$ Let $x=0$ $\dfrac{1}{n+1} = C$ $\therefore \dfrac{(1+x)^{n+1}}{n+1} = {}^nC_0 x + {}^nC_1 \dfrac{x^2}{2} + \dots + \dfrac{{}^nC_n x^{n+1}}{n+1} + \dfrac{1}{n+1}$	1 for integration

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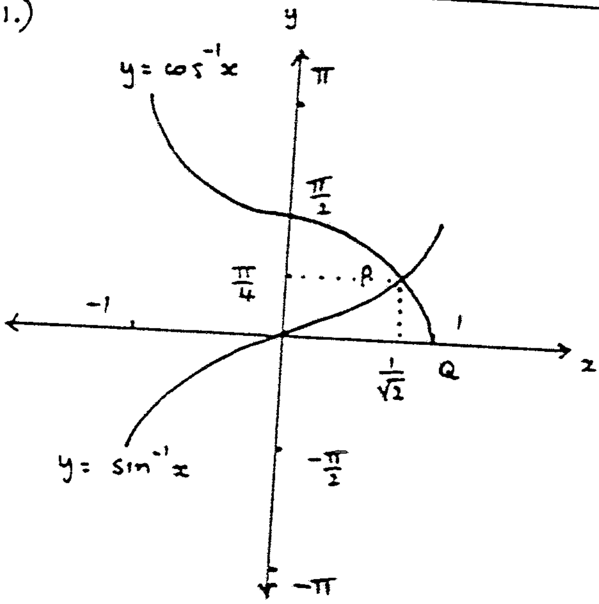
SOLUTIONS	COMMENTS
b.) ii.) Con't Let $x = 1$ $\therefore$ $\dfrac{2^{n+1}}{n+1} = {}^nC_0 + \frac{1}{2}\,{}^nC_1 + \frac{1}{3}\,{}^nC_2 + \dots + \frac{1}{n+1}\,{}^nC_n + \frac{1}{n+1}$ $\dfrac{2^{n+1}-1}{n+1} = {}^nC_0 + \frac{1}{2}\,{}^nC_1 + \frac{1}{3}\,{}^nC_2 + \dots + \frac{1}{n+1}\,{}^nC_n.$ $\boxed{\dfrac{2^{n+1}-1}{n+1} = {}^nC_0 + \frac{1}{2}\,{}^nC_1 + \frac{1}{3}\,{}^nC_2 + \dots + \frac{1}{n+1}\,{}^nC_n}$	1 for working.
c.) i.) 	1 for correct graphs
c.) ii.) when $x = \frac{1}{\sqrt{2}}$ $y = \cos^{-1} x = \cos^{-1} \frac{1}{\sqrt{2}} = \frac{\pi}{4}$ $y = \sin^{-1} x = \sin^{-1} \frac{1}{\sqrt{2}} = \frac{\pi}{4}$ $\therefore \boxed{P\left(\frac{1}{\sqrt{2}}, \frac{\pi}{4}\right)}$ Thus $y = \cos^{-1} x$ and $y = \sin^{-1} x$ intersect at $\left(\frac{1}{\sqrt{2}}, \frac{\pi}{4}\right)$	1 for substitution

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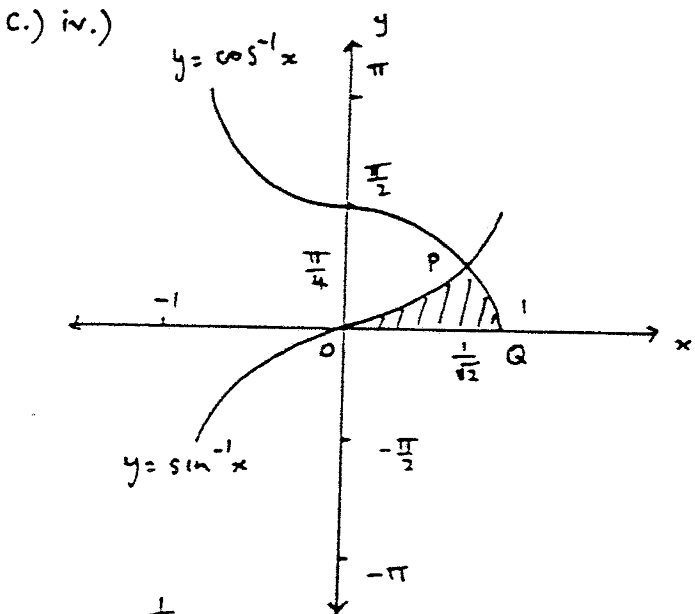
SOLUTIONS	COMMENTS
c.) iii) $\frac{d}{dx}\left(x\sin^{-1}x+\sqrt{1-x^2}\right) = \sin^{-1}x$ LHS $= \sin^{-1}x.(1) + \frac{x.1}{\sqrt{1-x^2}} + \frac{1}{\cancel{2}}(1-x^2)^{-\frac{1}{2}}.-\cancel{2}x$ $= \sin^{-1}x + \frac{x}{\sqrt{1-x^2}} - \frac{x}{\sqrt{1-x^2}}$ $= \sin^{-1}x$ $=$ RHS.	1 for derivative 1 for proof LHS = RHS.
c.) iv.)  Area $= \int_0^{\frac{1}{\sqrt{2}}}\sin^{-1}x\,dx + \int_{\frac{1}{\sqrt{2}}}^{1}\cos^{-1}x\,dx$ $= \left[x\sin^{-1}x+\sqrt{1-x^2}\right]_0^{\frac{1}{\sqrt{2}}} + \left[x\cos^{-1}x-\sqrt{1-x^2}\right]_{\frac{1}{\sqrt{2}}}^{1}$ $= \left[\left(\frac{1}{\sqrt{2}}\cdot\frac{\pi}{4}+\frac{1}{\sqrt{2}}\right)-(1)\right] + \left[(0)-\left(\frac{1}{\sqrt{2}}\cdot\frac{\pi}{4}-\frac{1}{\sqrt{2}}\right)\right]$ $= \frac{\pi}{4\sqrt{2}}+\frac{1}{\sqrt{2}}-1-\frac{\pi}{4\sqrt{2}}+\frac{1}{\sqrt{2}}$ $= \frac{2}{\sqrt{2}}-1 = \frac{2-\sqrt{2}}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}-2}{2}$ $= \boxed{\sqrt{2}-1 \text{ units}^2 \text{ is area.}}$	1 for integral 1 for sub and area

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SOLUTIONS	COMMENTS
QUESTION 7. - (12 marks)	
a.) Vertical Motion	
$\frac{d^2y}{dt^2} = -g$	
$\therefore \frac{dy}{dt} = -gt + c$	
when $t = 0$ $V = V\sin\theta = V\sin 45 = \frac{V}{\sqrt{2}}$	1 for integrals
$\therefore c = \frac{V}{\sqrt{2}}$	
$\therefore \frac{dy}{dt} = -gt + \frac{V}{\sqrt{2}}$	
$y = \frac{-gt^2}{2} + \frac{Vt}{\sqrt{2}} + c$	1 for $y = \frac{-gt^2}{2} + \frac{Vt}{\sqrt{2}}$
but $t = 0$ $y = 0$ $\therefore c = 0$	
$\therefore \boxed{y = -\frac{gt^2}{2} + \frac{Vt}{\sqrt{2}}}$	
Horizontal Motion	
$\frac{dx}{dt} = V\cos 45° = \frac{V}{\sqrt{2}}$	
$\therefore x = \frac{Vt}{\sqrt{2}} + c$ when $t = 0$, $x = 0$ $\therefore c = 0$	1 for $x = \frac{Vt}{\sqrt{2}}$
$\therefore \boxed{x = \frac{Vt}{\sqrt{2}}}$	

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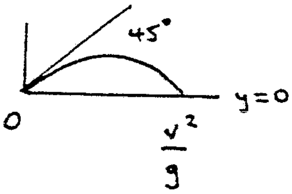
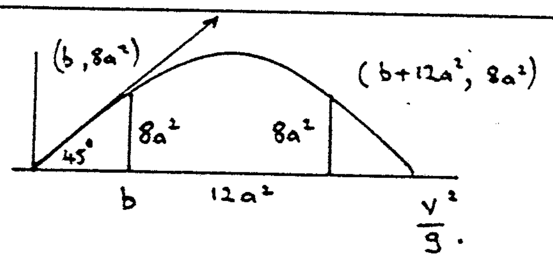
SOLUTIONS	COMMENTS
b.) $t = \dfrac{\sqrt{2}\,x}{v}$ $y = \dfrac{-gt^2}{2} + \dfrac{vt}{\sqrt{2}}$ $y = \dfrac{-g\left(\frac{\sqrt{2}\,x}{v}\right)^2}{\cancel{2}} + \dfrac{\cancel{v}\,\frac{\cancel{\sqrt{2}}\,x}{\cancel{v}}}{\cancel{\sqrt{2}}}$ $\boxed{y = x - \dfrac{gx^2}{v^2}}$	1 for substitution 1 for equation
c.) when $y = 0$  $0 = x - \dfrac{gx^2}{v^2}$ $0 = xv^2 - gx^2$ $0 = x(v^2 - gx)$ $x = 0$ or $v^2 = gx$ $x = \dfrac{v^2}{g}$ $\therefore$ $\boxed{\text{range of projectile } \dfrac{v^2}{g}}$	1 for solving equation 1 for correct range
d.) i)  $\therefore$ $b + 12a^2 + b = \dfrac{v^2}{g}$ $\boxed{\dfrac{v^2}{g} = 2b + 12a^2}$	(via diagram) ↑ 1 for showing $\dfrac{v^2}{g} = 2b + 12a^2$

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SOLUTIONS	COMMENTS
d.) ii.) the first post has co-ordinates $(b, 8a^2)$ $\therefore \quad y = x - \dfrac{gx^2}{V^2}$ sub $x = b$ $y = 8a^2$ $8a^2 = b - \dfrac{gb^2}{V^2}$ $\therefore \boxed{8a^2 = b - \dfrac{gb^2}{V^2}}$	1 for sub of (x,y) and getting equation
e.) $\dfrac{V^2}{g} = 2b + 12a^2 \quad$ (i) $8a^2 = b - \dfrac{gb^2}{V^2} \quad$ (ii) sub (1) into (2) $8a^2 = b - \dfrac{\cancel{g}b^2}{\cancel{g}(2b + 12a^2)}$	1 for substitution
$8a^2 = b - \dfrac{b^2}{2b + 12a^2}$ $b - 8a^2 = \dfrac{b^2}{2b + 12a^2}$ $(b - 8a^2)(2b + 12a^2) = b^2$ $2b^2 - 4a^2b - 96a^4 = b^2$ $b^2 - 4a^2b - 96a^4 = 0$ $(b - 12a^2)(b + 8a^2) = 0$ $b = 12a^2, \quad b = -8a^2$ (only positive for length). $\therefore b = 12a^2$	1 for solving equation
sub into (i) $\dfrac{V^2}{g} = 24a^2 + 12a^2 = 36a^2$ $V^2 = 36a^2 g \quad \therefore \boxed{V = 6a\sqrt{g}}$	1 for substitution and answer.